Determination of rock acoustic properties at low frequency: A differential acoustical resonance spectroscopy device and its estimation technique

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This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1002/grl.50346

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Abstract

A laboratory device, Differential Acoustic Resonance Spectroscopy (DARS), has been developed to investigate the acoustic properties of rock materials below 1 KHz. The device is based on analyzing shifts of resonance frequency of a cavity perturbed by the presence of a small sample. Numerical and experimental studies have previously shown that this technique can be used to estimate the compressibility of samples. In this study, we adopt a nonlinear least-squares whole-curve-fitting inversion approach, which for the first time allows estimation of both the compressibility and density of rock samples. In comparison to previous estimation techniques, this inversion method provides more reliable estimation of rock acoustic properties. This research proves that the DARS laboratory device, in conjunction with the calibration technique proposed herein, is a useful tool to estimate the properties of small rock samples. In addition, the simultaneous estimation of compressibility and density can potentially provide information on porosity and, by extension, a link between porosity and acoustic modulus at low frequency.

1. Introduction

Seismic wave propagation in fluid-saturated rocks is usually frequency dependent. Many theories have been developed to predict or interpret this
frequency-dependency [Biot, 1956; Dvorkin and Nur, 1993; King and Marsden, 2002]. However, these theoretical models remain unconstrained by experimental data because of the scarcity of laboratory measurements at frequencies below MHz range. In fact, the acoustic properties measured by the ultrasonic techniques are usually directly applied to seismic data interpretation and inversion regardless of their dependency on frequency. As low-frequency measurements are not readily available, considerable effort is expended reconciling acoustic properties measured at different frequency bands. To measure acoustic properties at lower frequencies, typical approaches include the resonance bar and the forced oscillation methods. The resonance bar technique can operate in the kilohertz frequency range, which is close to that used in acoustic logging and seismic exploration [Cadoret et al., 1995; Yin et al., 1992]. The forced oscillation method, or stress-strain measurement, records deformation as cyclic loading is exerted on rock samples, and can measure acoustic properties at a frequency range as low as several hertz [Spencer 1981; Gribb and Cooper, 1998; Jackson et al., 2011; Batzle et al., 2006]. These methods have their own merits and limitations in terms of operation and sample preparation.

We have developed a new measurement system to investigate the acoustic properties of rock samples at frequencies below 700 Hz. The system is based on differential acoustic resonance spectroscopy (DARS), which has been explored in several recent papers [Harris et al., 2005; Xu et al., 2007; Wang et al., 2012]. The DARS concept is based on perturbation theory. The resonance frequency of a
fluid-filled cavity is dependent on the sonic velocity of the fluid and the length of the cavity. The introduction of a sample into the cavity perturbs the resonance properties of the system. Therefore, the acoustic properties of samples can be inferred from the resonant frequency shift between measurements with and without the sample present. Wang et al. [2012] used an amended DARS perturbation formula to determine system calibration coefficients. However, this calibration technique was based on a one-data-point fitting approach. Because it only used partial information from among abundant measurements of resonance frequencies for the estimation of the acoustic properties, this approach suffers from random error and loss of information. In this paper, we present an improved calibration technique, a whole-curve-fitting inversion method, to estimate the acoustic properties of rock samples. Using the measured resonance frequencies with the sample at various locations inside the resonance cavity, this inversion technique can provide more reliable estimation of acoustic properties than the one-data-point fitting method. Noticeably, only compressibility can be obtained with the one-data-point approach, whereas the proposed technique can be used to estimate not only compressibility but density of the samples. We apply the technique to the estimation of acoustic properties of six rock samples, two artificial and four drilled rock samples. Note that the simultaneous measurements of both compressibility and density of fluid-saturated rock provides a possibility to calculate porosity, which can be used to establish a relationship between rock acoustic properties and porosity in the same
experiment, which will be a great gain in understanding the properties of fluid-saturated rocks.

2. DARS Perturbation Equation

Based on the derivation of Wang et al. [2012], the DARS perturbation equation is

\[ \omega_s^2 - \omega_0^2 = -\omega_0^2 \frac{\kappa_s - \kappa_0}{\kappa_0} \frac{V_s}{V_c} A - \omega_0^2 \frac{\rho_s - \rho_0}{\rho_s} \frac{V_s}{V_c} B. \]  

(1)

where

\[ A = \frac{V_c}{V_s} \int_{r_s} p_1 p_2 dV / \int_{r_c} p_1 p_2 dV, \]
\[ B = \frac{V_c}{V_s} \frac{1}{k^2} \int_{r_s} \nabla p_1 \cdot \nabla p_2 dV / \int_{r_c} p_1 p_2 dV, \]
\[ k = \omega_0 / c. \]

In equation (1), \( V_C \) and \( V_S \) are the volumes of the cavity and test sample, respectively. We use \( \omega_0 \) and \( \omega_s \) to denote the resonant frequencies of the cavity with and without sample, \( \kappa_0 \) and \( \kappa_s \) to denote the compressibility parameters of the cavity fluid and test sample, and \( \rho_0 \) and \( \rho_s \) to denote the densities of the cavity fluid and test sample, respectively. The parameter \( c \) is the sonic velocity in the cavity fluid, and \( k = \omega_0 / c \) is the wave number. The acoustic pressure fields \( p_1 \) and \( p_2 \) are those for cavity conditions with and without a sample. The coefficients \( A \) and \( B \) give the ratio of the energy density of the acoustic pressure and particle velocity field inside the sample to that of the acoustic pressure field inside the cavity when resonance occurs, reflecting the acoustic properties of the measurement system. As shown in Figure 1, the acoustic pressure field inside the cavity has a
spatially varying but harmonic distribution when the fundamental mode resonance occurs. Theoretically, the acoustic pressure has a cosine function distribution along the longitudinal direction. There are two particular locations at which acoustic pressure or velocity reach their maxima. For the fundamental mode, an acoustic pressure node occurs where sonic velocity is at its maximum, and a velocity node with acoustic pressure at its maximum. At these locations, either A or B is zero.

Equation (1) shows that the compressibility contrast, \( \frac{\kappa_s - \kappa_0}{\kappa_0} \), and the density contrast, \( \frac{\rho_s - \rho_0}{\rho_s} \), between a test sample and the fluid inside the cavity, both contribute to the resonance frequency shift. Thus, the perturbation equation (1), in conjunction with DARS measurements, can be used to estimate the acoustic properties of a sample.

3. Algorithms to Determine Acoustic Properties

3.1. One-data-point-fitting Method

To measure compressibility, a sample is positioned at a velocity node, where \( \langle \nabla p_1 \cdot \nabla p_2 \rangle \) vanishes, and equation (1) can be reduced to

\[
\frac{0\omega\kappa_k \kappa_0}{\omega \kappa} = C' V_S \left( \frac{s - \rho_0}{\rho} \right),
\]

using approximation \( \omega_s \approx \omega_0 \). The calibration coefficient \( C' \), which is related to the cavity geometry and experimental conditions (i.e. temperature and ambient pressure), can be obtained from the measured resonance frequency shift and known compressibility and volume of a reference sample.

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The calculation of compressibility based on equation (2) is called the one-data-point method, in which only the resonance frequency measured at the velocity node is used. This inevitably causes some limitations because measurements at other positions have been made redundant, and errors may arise due to individual measurement uncertainties or inaccurate positioning of the sample. So far, the one-data-point approach has performed better with samples of high compressibility than with those of low and medium compressibility, in terms of the estimation accuracy of acoustic properties [Wang et al., 2012]. We expect that the DARS measurements with a sample at multiple locations inside the cavity will effectively suppress the errors caused by fluctuations in experiment conditions or measurement uncertainty, thus contributing to more accurate estimates for samples of low compressibility. Moreover, an important feature of multiple location-based DARS measurements is that both compressibility and density parameters can be obtained through a whole-curve-fitting inversion.

3.2. Whole-curve-fitting Inversion

The whole-curve-fitting inversion involves a nonlinear least-squares fitting technique, and it provides a solution to both compressibility and density of a sample. In this technique, the second term of the right side of equation (1) is no longer negligible, as the measurements at other positions along the cavity axis (See Figures 1, 2a) will be used for the estimation of the acoustic properties. Accordingly, the
coefficients \( A \) and \( B \) should be determined for each measurement location. At any location \( (z_i, i=1,2\ldots n) \), an expression relating the measured resonance frequency to the contrast in acoustic properties between the sample and the fluid can be obtained

\[
\omega_{n_i}^2 = \frac{\omega_0^2 - \omega_0^2(1-\rho_s\rho)B_i}{1 + \left(\frac{1}{\kappa_0} - 1\right)A_i}, \quad i = 1,2\ldots n, \tag{4}
\]

where \( A_i = \frac{V_c}{V_c} \bigg|_{z_i}, \quad B_i = \frac{V_c}{V_c} \bigg|_{z_i}, \quad \rho = \frac{1}{\rho_s}. \) Using a least-squares based nonlinear fitting technique, the measured data sets \( \{z_i, \omega_{n_i}^2\}, i=1,2\ldots n \), can be fitted by minimizing the objective function

\[
Y = \sum_{i=1}^{n} \left[ \frac{\omega_0^2 - \omega_0^2(1-\rho_s\rho)B_i}{1 + \left(\frac{1}{\kappa_0} - 1\right)A_i} - \omega_{n_i}^2 \right]^2.
\tag{5}
\]

The solution set \( \{\kappa_s, \rho\} \) is thus determined through a nonlinear fitting procedure.

Accurate estimation of the calibration coefficients, \( A_i \) and \( B_i \), is crucial for the non-linear inversion. We use two different approaches to determine the calibration coefficients, and compare their effects on the inversion by numerical and experimental studies.

For the frequency perturbation equation, the volume of a sample must be negligible compared to that of the cavity, namely, \( V_c \gg V_s \). Numerical modeling suggests that the sample volume should be less than 5\% of the cavity volume [Wang
et al., 2012]. Under these conditions, we have the approximation for the acoustic pressure field, \( p_1 \approx p_2 \approx \cos(k_0 z) \), where \( k_0 = \frac{\omega_0}{c} \) and \( \omega_0 = \frac{\omega}{L} \) for the fundamental mode resonance (with \( L \) as the cavity length).

When a sample is measured at a given location, \( z = \bar{z} \), as shown in Figure 1, the coefficients \( A \) and \( B \) can be obtained by calculating the volume integrals in equation (1). The results are

\[
A|_{z=\bar{z}} = 1 + \frac{\sin(2k_0z_2) - \sin(2k_0z_1)}{2k_0(z_2 - z_1)},
\]
\[
B|_{z=\bar{z}} = 1 - \frac{\sin(2k_0z_2) - \sin(2k_0z_1)}{2k_0(z_2 - z_1)}.
\]

where \( z_1 \) and \( z_2 \) denote the bottom and top ends of the sample, respectively. The determination of \( A \) and \( B \) with this approach is referred to as Method 1.

The approximation, \( p_1 \approx p_2 \approx \cos(k_0 \bar{z}) \), is reasonable to a great extent, especially if DARS measurements are under stable conditions (e.g., consistent room temperature) and with samples of high compressibility. The whole-curve-fitting inversion can then be implemented with these known coefficients \( A \) and \( B \). However, for samples of low compressibility (e.g. fluid-saturated rocks), and/or under variable temperature and pressure conditions, this approximation is more open to question.

Therefore, we propose an alternative method to obtain the coefficients, \( A \) and \( B \), at \( z = \bar{z} \) by using two reference samples instead of calculating the integrals in equation (1). In this method (hereafter referred to as Method 2), the resonance frequencies corresponding to two reference samples of known acoustic properties and volumes, can be measured at any location, thus, the calibration coefficients, \( A \)
and $B$, can be obtained by simply solving a binary linear equation system. Using the whole-curve-fitting inversion with the calibration coefficients obtained by Method 2, we conjecture that the estimation of both compressibility and density of samples can be more accurately achieved. Particularly, we anticipate that the estimation of the acoustic properties of samples with low compressibility may be significantly improved.

4. Implementation of whole-curve-fitting Inversion

4.1. Numerical study

A numerical study was conducted to investigate how the whole-curve-fitting inversion, with the determination of the calibration coefficients $A$ and $B$ using the two proposed methods, exhibits different performance in terms of the estimation accuracy of the compressibility and density of samples with a large range of compressibility. Commercial finite element simulation software, COMSOL, was used in the numerical modeling. Mathematically, the simulation of DARS measurements with COMSOL consists of solving an eigen-frequency problem with the following boundary conditions, $p = 0$ (free boundary) for the top and bottom (open) sides of the cavity, and $\nabla p \cdot n = 0$ (rigid boundary) for the side surface of the cavity [Wang et al., 2012]. A three-dimensional DARS model, shown in Figure 2a, is used in the simulation. The acoustic properties of 18 hypothetical samples used in the study, including the sonic velocity $v$, the density $\rho$, and the compressibility defined by $\kappa = (\rho c^2)^{-1}$, are listed in Table 1. All cylindrical
samples had the same diameter (0.028m) and length (0.05m). Resonance frequencies were simulated with the sample positioned at the same locations as in the real experiments. These resonance frequencies were then used to invert for both compressibility and density of a test sample using the whole-curve-fitting technique. Figures 2c and 2d show the estimated compressibility and density of the synthetic samples, using the two different methods outlined above. For comparison, Figure 2b shows the sample compressibility based on the one-data-point fitting method. The compressibility estimations obtained with the whole-curve-fitting method are comparable to those inverted using the one-data-point method. Figure 2d, which shows the results of the whole-curve-fitting method using the calibration parameters obtained from Method 2, demonstrates that the latter method can provide good density estimation for all samples. However, for samples of low compressibility, the densities estimated with this method using the calibration parameters determined by Method 1 deviate slightly from the true values as seen in Figure 2c. This is anticipated, since these coefficients are calculated based on the assumption that the pressure field obeys the cosine function distribution, which may be inaccurate when the sample has relatively low compressibility and high density, and when the boundary conditions do not apply [Wang et al., 2012]. Nevertheless, this assumption is useful when we do not have reference samples.

4.2. Laboratory measurements and discussion
Eight samples from different sources have been measured for their acoustic properties. A detailed description of these samples is given in the supplementary material. Of these samples, Al-1 and Lu-1 are two standard (reference) samples, consisting of aluminum and Lucite, with compressibilities of 0.01314 GPa$^{-1}$ and 0.17410 GPa$^{-1}$, and densities of 2.684 g/cm$^3$ and 1.181 g/cm$^3$, respectively; S10-1 and S10-2 are two artificial sandstone samples; and D7-1, D11-1, D13-2, and D36-1 are four sandstone samples from drilled cores. Their dimensions, density, and compressibility (ultrasonic measurements) in the saturated condition are listed in Table 2 (also see the supplement material for measurements under the dry condition). The compressibility of these samples was calculated from the P- and S-wave velocities obtained by ultrasonic measurements under room temperature and atmospheric pressure, and the density obtained directly from mass and volume measurements.

Each sample was measured at 75 positions along the cavity axis at intervals of 12 mm. The frequency range for searching the resonance frequency at each sample position was from 565 Hz to 665 Hz. Such measurements were conducted 10 times for each sample to further assess the measurement repeatability. Figure 3a shows the recorded resonance curves for the empty cavity and the two standard samples (Al-1 and Lu-1), for measurements at a fixed location. As shown in Figure 3a, the resonance frequency shift from the resonance frequency of the empty cavity caused by Al-1 was larger than that caused by Lu-1. Because the compressibility and
density of the other six rock samples are between those of aluminum and Lucite, it can be anticipated that their resonance curves (omitted in Figure 3a) should also lie between those of Al-1 and Lu-1. Using the Lorentzian curve-fitting technique [Xu, 2007], the resonance frequencies for each sample at different locations can be obtained. Figure 3b shows the resonance frequencies for the two standard samples at different positions.

We carried out the whole-curve-fitting inversions for the acoustic properties of the samples in two steps. First, we obtained the calibration coefficients $A$ and $B$ with Method 1 for the reference samples (Al-1 and Lu-1), which were then used for the whole-curve-fitting inversion. The resulting inverted compressibility and density values are listed in the supplementary material. For Lu-1, we obtained a resonance frequency curve perfectly fitted with the DARS measurements, as shown in Figure 3d. The inversion results, $\kappa = 0.17523$ (GPa$^{-1}$), and $\rho = 1.179$ (g/cm$^3$), agree well with the true values: $\kappa_{\text{real}} = 0.17410$ (GPa$^{-1}$), and $\rho_{\text{real}} = 1.181$ (g/cm$^3$). For Al-1, Figure 3c shows that the inversion curve matches the DARS measurements well, with some deviation at the extremes. The inverted density obtained with this approach, $\rho = 3.043$ (g/cm$^3$), is higher than its true value of 2.698 (g/cm$^3$), whereas the inverted compressibility, 0.01355 (GPa$^{-1}$), is comparable to the true value, 0.01307 (GPa$^{-1}$). Generally, the inversion results conform to the observation in the numerical study. For samples with extremely low compressibility, the whole-curve-fitting inversion approach with the calibration coefficients $A$ and $B$
obtained with Method 1 appeared to cause a larger error for density than for compressibility.

Next, Al-1 and Lu-1 were used as two reference samples to obtain the calibration coefficients $A$ and $B$ through Method 2, and the whole-curve-fitting inversions were carried out for the 6 rock samples. The rock samples were saturated with silicon oil, put in a vacuum for two days to extract air inside them, and in a container with 5 MPa pore pressure for two more days, before they were sealed on the outer wall (with both ends open). The resonance frequency measurements for the samples were made under normal room conditions. For each rock sample, we obtained the estimated compressibility and density when the objective function $Y$ in equation (5) was minimized. The estimated compressibility and density values of the 6 sandstone samples, and the relative errors between different techniques-estimated compressibilities and densities, are given in the supplementary material. Figure 4 shows a comparison between the compressibility and density inverted with the whole-curve-fitting technique and measured by the ultrasonic technique. Low standard deviations for both the estimated density and compressibility indicate that the DARS-based measurements, in conjunction with the whole-curve-fitting inversion approach, can give consistent and reliable estimates of the acoustic properties. In Figures 4a and 4b (also see the supplementary material), the relative error between the estimated and true densities is seen to be the largest for D7-1. The DARS-estimated compressibility for the 6 samples have large relative errors in
contrast to the ultrasonic measurements, but relatively low standard deviations, which also give us confidence that the DARS-estimated compressibility is reasonable considering the large frequency contrast between the ultrasonic and DARS measurements. Figure 4a shows the compressibility values obtained using the DARS inversion, the Gassmann fluid substitution (see supplementary material for input parameters), and the ultrasonic measurements. The frequency dependence of the compressibility is plotted in Figure 4(c). For the artificial sandstones (S10-1 and S10-2) with simple pore shapes, their compressibilities only show weak frequency dependency. This reflects that the wave-induced pore pressure can reach equilibrium at the measured frequency of DARS and even ultrasonic frequency, which satisfies the basic assumptions of the Gassmann equation [Gassmann et al., 1951]. Thus, for this kind of rock samples, even ultrasonic frequency can be considered in the low-frequency regime, which implies that the Gassmann equation is applicable in a wide frequency range. For the reservoir rocks, obvious discrepancy between the ultrasonic and DARS measurements can be observed, which is mainly attributed to complex pore shapes and the presence of clay. This complexity gives rise to a variety of dispersion mechanisms [King and Marsden, 2002; Batzle et al., 2006], such as squirt flow, which contributes to the apparent compressibility variation. Similar to the artificial sandstones, the DARS-estimated compressibilities for the four reservoir rock samples are closer to the values obtained using the Gassmann equation. However, for the specimens D11-1 and D13-2, the Gassmann equation
gives errors of about 10%, which are not negligible or simply attributed to measurement errors. Therefore, for some sedimentary rocks, the DARS frequency (about 600Hz) and even seismic frequency would not be in the low-frequency regime. Under the circumstances, the Gassmann equation would lose some validity, and the direct low-frequency measurement is essential.

5. Conclusion

A DARS laboratory device, operating in a low frequency range, has been developed to estimate the acoustic properties of rock samples. In comparison with other important low-frequency measurement techniques, the DARS-based measurement is easy and less time-consuming to operate.

The proposed whole-curve-fitting technique, which makes full use of all resonance frequencies measured for samples at different positions in the cavity, can be used to estimate both compressibility and density of the sample, in contrast to the one-data-point approach which only estimates compressibility. A critical aspect of this technique is the improved accuracy of the calibration coefficients, which yields more reliable estimation of the acoustic properties of samples, particularly those with relatively low compressibility. The numerical and experimental studies in this work show that the whole-curve-fitting technique, in conjunction with the calibration coefficients $A$ and $B$ obtained by measuring two reference samples, can achieve fairly good accuracy for both compressibility and density in the frequency range below 700 Hz. The DARS measurements have been applied to compressibility
and density estimates of fluid-saturated rock samples at the first time, and the differences in measured compressibility between the DARS method, ultra-sonic measurements, and the Gassmann estimation reflect substantial frequency dependence of the compressibility. Therefore, we conclude that the laboratory device, in conjunction with the whole-curve-fitting inversion, is a useful tool to estimate the properties of acoustically small rock samples in the low frequency range. Further, the DARS device and the inversion method could potentially be used to estimate the porosity of rock samples, hence establishing a relationship between porosity and other petrophysical properties of reservoir rocks. An improved version of DARS is being developed to apply high pressure and temperature, in order to simulate the \textit{in situ} response of reservoir rocks.

**Acknowledgement**

This work is sponsored by 973 Program “Fundamental Study on the Geophysical Prospecting of the Deep-layered Oil and Gas Reservoirs” (Grant No. 2013CB228600), CNPC 125 program “Multiple-frequency-band analysis of seismic rock physics” (2011A-3606), the Program for Changjiang Scholars and Innovative Research Team in University (PCSIRT), Science Foundation of China University of Petroleum, Beijing (KYJJ2012–05-02), and the Nation Natural Science Foundation of China Research (Grant No. 41274138).
Reference


Figure 1. DARS setup and the pressure field in the cavity. (a) Diagram of the DARS setup. A cylindrical cavity with open ends is immersed in a tank filled with silicon oil. Source and receiver are connected to a lock-in amplifier through a power amplifier and a preamplifier, respectively. A computer-controlled stepper motor is used to control sample positioning. (b) Acoustic pressure antinode (velocity node) and velocity antinode (pressure node) in the cavity.
Figure 2. The three-dimensional simulation of DARS measurements, and estimation of acoustic properties based on the whole-curve-fitting technique and the one-data-point fitting technique. The sonic velocity for the cavity fluid is 960 m/s, and the density is 908 kg/m$^3$. Eighteen synthetic samples, listed in Table 1, are used in the numerical study. (a) Each sample is moved along the axis inside the cavity, and the resonance frequency is obtained through the DARS simulation at each location. The resonance frequencies at all locations are used to estimate both compressibility and density parameters of a test sample through the whole-curve-fitting technique; (b) the density of 18 synthetic samples inverted using the one-data-point fitting technique; (c) and (d) show the inversion results of acoustic properties of the 18 synthetic samples. The results in (c) were obtained with the calibration coefficients $A$ and $B$ determined using equation (6), while those in (d) employed values for $A$ and $B$ from two reference samples (Sample 9 and 16).
Figure 3. Implementation of the whole-curve-fitting inversion for two standard samples, Al-1 and Lu-1. (a) Normalized pressure amplitude with scanning frequency, (b) resonance frequencies with the sample at the measurement locations along the cavity axis, and the whole-curve-fitting inversions for (c) Al-1, and (d) Lu-1.
Figure 4. Comparison of compressibility and density parameters between the ultrasonic and DARS measurements for six rock samples. Standard deviations of DARS-based compressibility and density parameters for the six samples (red bar) are calculated from the whole-curve-fitting inversions of repeated measurements (see text). Results employ A and B calculated by Method 2 using the two reference samples.
Table 1. Eighteen synthetic samples with their known compressibility and density values. The estimated acoustic properties inverted from the simulated resonance frequencies using the whole-curve-fitting technique (see text) are also listed. Samples No. 9 and 16 are used to obtain the calibration coefficients $A_i$ and $B_i$ in equation (4).

<table>
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<th>Sample No.</th>
<th>Velocity (m/s)</th>
<th>Density (kg/m$^3$)</th>
<th>Inversion results</th>
<th>$k$ (Gpa$^{-1}$)</th>
<th>Inversion results</th>
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<td>6000</td>
<td>2500</td>
<td>2503.700</td>
<td>0.02%</td>
<td>0.01111</td>
<td>0.01110 -0.10%</td>
</tr>
<tr>
<td>18</td>
<td>7000</td>
<td>2700</td>
<td>2711.300</td>
<td>0.02%</td>
<td>0.00756</td>
<td>0.00760 0.55%</td>
</tr>
</tbody>
</table>
Table 2. The dimensions and densities of two reference samples (aluminum and Lucite), two artificial sandstone samples, and four drilled rock samples. Ultrasonic transmission measurements were carried out for these samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Material</th>
<th>Diameter (mm)</th>
<th>Length (mm)</th>
<th>Mass (g)</th>
<th>Density (g/cm$^3$)</th>
<th>Vp (m/s)</th>
<th>Vs (m/s)</th>
<th>$\kappa$ (GPa$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-1</td>
<td>Aluminum</td>
<td>25.020</td>
<td>40.000</td>
<td>53.060</td>
<td>2.698</td>
<td>6399.70</td>
<td>3074.29</td>
<td>0.01307</td>
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<tr>
<td>Lu-1</td>
<td>Lucite</td>
<td>25.180</td>
<td>40.180</td>
<td>23.630</td>
<td>1.181</td>
<td>2773.25</td>
<td>1456.21</td>
<td>0.17410</td>
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<tr>
<td>S10-1</td>
<td>Artificial sandstone</td>
<td>25.181</td>
<td>41.502</td>
<td>45.698</td>
<td>2.211</td>
<td>3705.54</td>
<td>1900.11</td>
<td>0.05072</td>
</tr>
<tr>
<td>S10-2</td>
<td>Artificial sandstone</td>
<td>25.192</td>
<td>40.430</td>
<td>43.952</td>
<td>2.181</td>
<td>3485.35</td>
<td>1772.99</td>
<td>0.05763</td>
</tr>
<tr>
<td>D7-1</td>
<td>Claystone</td>
<td>25.412</td>
<td>41.126</td>
<td>53.231</td>
<td>2.552</td>
<td>4031.96</td>
<td>2531.48</td>
<td>0.05081</td>
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<td>D11-1</td>
<td>Shaly sand</td>
<td>25.319</td>
<td>39.550</td>
<td>51.295</td>
<td>2.576</td>
<td>4189.62</td>
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<td>Shaly sand</td>
<td>25.258</td>
<td>40.684</td>
<td>53.388</td>
<td>2.619</td>
<td>4101.41</td>
<td>2395.30</td>
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<td>Claystone</td>
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<td>39.718</td>
<td>52.666</td>
<td>2.621</td>
<td>4279.96</td>
<td>2658.15</td>
<td>0.04288</td>
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