Spectral sparse Bayesian learning reflectivity inversion

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ABSTRACT
A spectral sparse Bayesian learning reflectivity inversion method, combining spectral reflectivity inversion with sparse Bayesian learning, is presented in this paper. The method retrieves a sparse reflectivity series by sequentially adding, deleting or re-estimating hyper-parameters, without pre-setting the number of non-zero reflectivity spikes. The spikes with the largest amplitude are usually the first to be resolved. The method is tested on a series of data sets, including synthetic data, physical modelling data and field data sets. The results show that the method can identify thin beds below tuning thickness and highlight stratigraphic boundaries. Moreover, the reflectivity series, which is inverted trace-by-trace, preserves the lateral continuity of layers.

Key words: Reflectivity inversion, Sparse Bayesian learning, Thin bed, Bayesian inversion.

1 INTRODUCTION
One of the aims of seismic exploration is to obtain a subsurface image from reflected seismic data. However, reflected seismic data are band-limited, which results in limited resolution (Widess 1973). Thus, many techniques have been developed to remove the wavelet from the seismic data, for the purpose of retrieving the reflectivity series and identifying any thin layers.

The most popular method to retrieve reflectivity is inverse filtering within a least-squares framework (e.g., Berkhout 1977; Robinson and Treitel 1980; Bickel and Martinez 1983). This method is based on the assumption that the earth’s reflectivity has a spectrum of white noise, so that the autocorrelation of the wavelet can be replaced by that of seismic data. Therefore, the influence of the wavelet is removed. However, the fact that the wavelet is band-limited means that the results of the reflectivity inversion are non-unique.

In order to overcome or alleviate this ambiguity, it is common to assume that the reflectivity series is sparse. Based on such a sparsity constraint, many deconvolution or reflectivity inversion methods have been proposed. One such method is to use a non-linear function, for example, an L1 norm, entropy norm, Cauchy criterion or Huber criterion, as a regularization term to promote a sparse reflectivity estimation (e.g., Levy and Fullagar 1981; Sacchi, Velis and Cominguez 1994; Sacchi 1997). Another kind of method is to adopt a Bayesian framework, such as maximizing a likelihood function or maximizing a posteriori density function with the assumption that the reflectivity series obeys a certain prior distribution (e.g., Kormylo and Mendel 1983; Chi, Mendel and Hampson 1984; Debeye and van Riel 1990). This can be utilized to obtain sparse reflectivity. Besides, by pre-setting the number of non-zero reflectivity spikes, sparse reflectivity inversion methods based on global optimization algorithms, such as simulated annealing, particle swarm optimization or ant colony optimization, can also be introduced (e.g., Vestergaard and Mosegaard 1991; Velis 2008; Yuan, Wang and Tian 2009).

Recently, different methods based on matching pursuit (MP), orthogonal matching pursuit (OMP) or basis pursuit (BP) algorithms have been developed to implement sparse reflectivity inversion. Nguyen and Castagna (2010) presented a high-resolution seismic reflectivity inversion method based on a MP algorithm with the incorporation of well-log derived
patterns. Yang et al. (2011) also presented a MP reflectivity inversion method that utilized thin-bed models as a dictionary. For these two methods, a priori information such as the number, thickness and reflection strength of the thin-beds are used as constraint conditions to obtain a reliable result. Broadcom and Tonellot (2010) presented a sparse deterministic deconvolution method based on an OMP algorithm, a fast MP algorithm, by using the time-shift wavelets or the convolution matrix of the wavelet. Zhang and Castagna (2011) presented a seismic sparse-layer reflectivity inversion method based on a BP algorithm, by utilizing wedge-models as a dictionary.

Sparse Bayesian Learning (SBL) has been proposed and proven to be an effective and accurate method for a wide variety of regression and classification problems (Tipping 2001). The SBL paradigm performs parameter learning via type-II maximum likelihood or evidence maximization (Mackay 1992) where marginal likelihood maximization leads to automatic identification of the relevant kernels and hence sparsifies the model. The SBL model is known to enjoy many advantages, such as probabilistic prediction, the facility to utilize arbitrary basis functions and automatic estimation of nuisance parameters. This model has been successful in a wide range of applications such as visual tracking (Williams, Blake and Cipolla 2005), text classification (Silva and Ribeiro 2007), positron emission tomography (Peng et al. 2008) and dynamic light scattering (Nyeo and Ansari 2011).

In the context of signal processing, for basis vectors selection or sparse signal representation problems, SBL, similar to OMP, sequentially updates the basis vectors. The main difference between them is that SBL updates hyper-parameter controlling weights, while OMP directly updates weights. It is important to note that in the context of this paper the weights in the algorithm are the reflectivity of the earth. Compared with OMP, the main advantages (Ji, Xue and Carin 2008) for SBL are that 1) it deletes the basis vector that has been chosen to maintain a more concise signal representation; 2) it yields ‘error bars’ for the estimated weights, indicating the confidence. In contrast to the BP algorithm, the main differences are that 1) BP updates all weights for every iteration, while SBL mainly updates one weight for every iteration; 2) the result of BP is not exactly sparse due to noisy reconstruction, while that of SBL is strictly sparse without background oscillation in the model; 3) BP adopts a fixed sparsity-inducing prior to encourage sparsity, while SBL uses a parametrized prior to automatically control sparsity. In addition, SBL has the advantage of preventing any ‘structural errors’ (at least in the absence of noise) (Wipf and Rao 2004) and the structural errors mean that the global minimum of the cost function does not necessarily coincide with the sparsest solutions. Wipf and Rao (2006) described the worst-case scenario for SBL and demonstrated that it still outperforms BP and OMP.

Although, as described in the paragraph above, SBL has many properties that make it suitable for sparse reflectivity inversion, no attempt has been made, so far, to use it for this purpose. In this paper, we propose an SBL-based reflectivity inversion method in the frequency domain. Our approach controls the sparsity of reflectivity series by using a parametrized Gaussian prior with different hyper-parameters (or variances), slightly different from the popular Gaussian prior with a common hyper-parameter (or variance) in the form. The hyper-parameters controlling corresponding reflectivity spikes can be estimated by maximizing a marginal likelihood. Without pre-setting the number of non-zero reflectivity spikes, they can be automatically retrieved. Besides an addition operator, deletion and re-estimation operators are also adopted to optimize the inverted result. Unlike sparse deconvolution using a support vector machine (Rojo-Álvarez et al. 2008), which requires additional processing for kernel functions (or basis vectors), our approach inherently adapts to sparse reflectivity inversion with arbitrary basis vectors.

The first part of the paper describes the theory, including the spectral reflectivity inversion model (Section 2.1), SBL reflectivity inversion (Section 2.2) and marginal likelihood maximization (Section 2.3). Then we use 1D synthetic data (Section 3.1), 2D synthetic data (Section 3.2), 3D physical modelling data (Section 3.3) and 3D field data (Section 3.4) examples to test the performance of our method. Finally, some conclusions (Section 4) are drawn.

2 THEORY

2.1 Spectral reflectivity inversion model

The well-known seismic convolution model is described as

$$s(t) = w(t) * \sum_{k=1}^{K} r_k \delta(t - t_k) + n(t),$$

(1)

where $t$ is the time, $s(t)$ is the seismic data, $w(t)$ is the seismic wavelet, $K$ is the sample number, $r_k$ is the amplitude of the $k$-th reflectivity, $\delta(\cdot)$ is the delta function, $t_k$ is the time position of the $k$-th reflectivity and $n(t)$ is the noise.

By taking the Fourier transform for both sides of equation (1), we have

$$S(f) = W(f) \sum_{k=1}^{K} r_k \exp(-i2\pi t_k f) + N(f),$$

(2)

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where \( f \) is the frequency, \( S(f) \) is the complex spectrum of \( s(t) \), \( W(f) \) is the complex spectrum of \( w(t) \), \( \exp(•) \) is the exponential function, \( i \) is the imaginary unit and \( N(f) \) is the complex spectrum of \( n(t) \).

If we choose frequency components \( f_m \ (m = 1, 2, \ldots, M) \) within the available band-limited frequency band, equation (2) can be discretized and simplified as

\[
[S(f_m) - N(f_m)] / W(f_m) = \sum_{k=1}^{K} r_k \exp(-i2\pi t_k f_m).
\]

(3)

Defining \( R(f_m) \) as the complex spectrum of the reflectivity series, we have \([S(f_m) - N(f_m)] / W(f_m) \equiv R(f_m)\). By performing the inverse Fourier transform for both sides of equation (3), an estimated reflectivity series \( \hat{r}_k \) is obtained

\[
\hat{r}_k = \frac{1}{K} \sum_{m=1}^{M} R(f_m) \exp(i2\pi t_k f_m),
\]

(4)

with \( k = 1, 2, \ldots, K \). This estimation processing is essentially equivalent to classical least-squares inverse filtering or Wiener filtering.

In order to quantitatively evaluate how the estimated \( \hat{r}_k \) \((k = 1, 2, \ldots, K)\) is close to the true reflectivity series \( r_k \), we theoretically derive the intuitionistic expression for an arbitrary estimated reflectivity \( \hat{r}_c \) as:

\[
\hat{r}_c = \frac{1}{K} \sum_{m=1}^{M} \left[ \sum_{k=1}^{K} r_k \exp(-i2\pi t_k f_m) \right] \exp(i2\pi t_c f_m)
\]

\[
= \frac{1}{K} \sum_{m=1}^{M} \left[ r_c \exp(-i2\pi t_c f_m) + \sum_{k \neq c} r_k \exp(-i2\pi t_k f_m) \right]
\]

\[
\times \exp(i2\pi t_c f_m),
\]

\[
= \frac{M}{K} r_c + \frac{1}{K} \sum_{m=1}^{M} \sum_{k \neq c} r_k \exp[i2\pi(t_c - t_k) f_m],
\]

(5)

where \( c \in \{1, 2, \ldots, K\} \). Because the chosen frequency components are only a portion of the whole band from zero to Nyquist frequency, the estimated reflectivity series is neither equal nor proportionate to the true reflectivity series, as equation (5) shows. In fact, the result given by equation (4) or (5) is only one of the infinite solutions of equation (1).

In order to overcome or alleviate the ambiguity problem, we add a sparsity constraint by forcing most reflectivity \( r_k \) in equation (3) as zero and rewrite equation (3) as

\[
[S(f_m) / W(f_m)] = \sum_{k=1}^{K} r_k \exp(-i2\pi t_k f_m) + N(f_m) / W(f_m).
\]

(6)

or

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_M
\end{bmatrix}
\]

\[
\begin{bmatrix}
\exp(-i2\pi t_1 f_1) & \exp(-i2\pi t_2 f_1) & \cdots & \exp(-i2\pi t_K f_1) \\
\exp(-i2\pi t_1 f_2) & \exp(-i2\pi t_2 f_2) & \cdots & \exp(-i2\pi t_K f_2) \\
\vdots & \vdots & \ddots & \vdots \\
\exp(-i2\pi t_1 f_M) & \exp(-i2\pi t_2 f_M) & \cdots & \exp(-i2\pi t_K f_M)
\end{bmatrix}
\]

\[
\times
\begin{bmatrix}
r_1 \\
r_2 \\
\vdots \\
r_K
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\xi_1 \\
\xi_2 \\
\vdots \\
\xi_M
\end{bmatrix}
\]

(7)

where \( \alpha_m = S(f_m) / W(f_m) \) and \( \xi_m = N(f_m) / W(f_m) \). It is obvious that every \( \alpha_m \) is dependent on both the time position and amplitude of all non-zero reflectivity spikes.

For the sake of simplicity, we rewrite equation (7) in matrix/vector form:

\[
d = G_m + n = [G_1 \ G_2 \ \cdots \ G_K \ \cdots \ G_K] \\
\times [r_1 \ r_2 \ \cdots \ r_K]^T + n,
\]

(8)

where the superscript \( T \) denotes the transpose, \( d = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T \) represents the observed data, \( n = [\xi_1, \xi_2, \ldots, \xi_M]^T \) represents the noise and \( G_m = [\exp(-i2\pi t_1 f_1), \exp(-i2\pi t_2 f_2), \ldots, \exp(-i2\pi t_K f_M)]^T \) represents the basis vector.

Since the sparsity constraint is used to control the inversion, most basis vectors in \([G_1 \ G_2 \ \cdots \ G_K \ \cdots \ G_K]\) are essentially redundant. The aim of the sparse reflectivity inversion is to seek the useful basis vectors from \([G_1 \ G_2 \ \cdots \ G_K \ \cdots \ G_K]\) and solve \( r_j \) corresponding to the useful \( G_k \).

### 2.2 Sparse Bayesian learning reflectivity inversion

A SBL model (Tipping 2001) is proposed for solving the regression and classification problem. One advantage of the model is that it allows the use of arbitrary kernels. The basis vector \( G_k \) in equation (8) can be regarded as the kernel. Therefore, spectral reflectivity inversion can be combined with SBL. In this section, we utilize an inversion framework to introduce the SBL-based spectral reflectivity inversion.

Assuming that the noise \( n \) is approximated as uncorrelated zero-mean Gaussian noise, with variance \( \sigma^2 \), namely \( n \sim N(0, \sigma^2) \), the likelihood of the observed data is expressed as (Ulrych, Sacchi and Woodburg 2001)
\[ p(d \mid m, \sigma^2) = (2\pi)^{-M/2} |E|^{1/2} \exp \left[ -\frac{1}{2} (d - Gm)^T E (d - Gm) \right] \]  
(9)

where \( E = \sigma^{-2} I \), \(|\cdot|\) represents the determinant of a matrix and the superscript \( H \) represents the transpose conjugate. Maximizing the above likelihood is a simple quadratic optimization problem, the solution of which is returned to equation (4) or (5).

To obtain a solution driven by a priori geologic assumption, an automatic relevance determination Gaussian prior (Tipping 2000) of the reflectivity series is introduced:

\[ p(m \mid h) = \prod_{k=1}^{K} \mathcal{N}(r_k \mid 0, b_k^{-1}) \]  
(10)

where \( h = [h_1, h_2, \ldots, h_k]^T \) is a vector consisting in \( K \) independent hyper-parameters and \( \mathcal{N}(r_k \mid 0, b_k^{-1}) \) denotes that every reflectivity \( r_k \) obeys a zero-mean Gaussian distribution, with a different variance \( b_k^{-1} \). This is a little different from the popular zero-mean Gaussian distribution, where all reflectivity samples have a common variance.

Using equation (10) as a prior is to promote the sparsity of reflectivity series. To be specific, when \( b_k = \infty \), the corresponding \( r_k = 0 \). A more detailed discussion on how to encourage sparsity can be found in the literature (e.g., Tipping 2001; Wipf and Rao 2004).

According to Bayesian rule (Tarantola 2005; Yuan, Wang and Li 2012), the posterior probability of \( m \) is derived by combining the likelihood of the data \( p(d \mid m, \sigma^2) \) and the prior of the reflectivity series \( p(m \mid h) \) with the normalizing term \( p(d \mid h, \sigma^2) \):

\[ p(m \mid d, h, \sigma^2) = p(d \mid m, \sigma^2) p(m \mid h) / p(d \mid h, \sigma^2) \]  
(11)

where

\[ \Sigma = (H + \sigma^{-2} G H G)^{-1} \]  
(12)

\[ \mu = \sigma^{-2} \Sigma G d \]

and \( \Sigma \) denotes the covariance, \( \mu \) denotes the mean of the posterior probability of \( m \), regarded as the inverted reflectivity series and \( H = \text{diag}(b_1, b_2, \ldots, b_k) \) is a diagonal matrix. As equation (12) shows, \( \mu \) is dependent on the hyper-parameters \( h \) and the noise variance \( \sigma^2 \). To obtain \( \mu \), we first need to calculate \( h \). In this paper, \( h \) (and \( \sigma^2 \) if necessary) is estimated by using type-II maximum likelihood, in which the marginal likelihood is maximized and the marginal likelihood can be calculated by

\[ p(d \mid h, \sigma^2) = \int p(d \mid m, \sigma^2) p(m \mid h) dm \]  
(13)

where \( Q = \sigma^2 I + G H G^H \). Next \( \mu \) can be updated by substituting the calculated \( h \) into equation (12). We then repeatedly update \( h \) and \( \mu \) until a convergence condition is met. The final updated \( \mu \) is the inverted reflectivity series.

2.3 Marginal likelihood maximization

For SBL reflectivity inversion, a key step is the maximization of the marginal likelihood (equation (13)). In this section, we analyse several important properties of the marginal likelihood and introduce a maximization method. For more details, we refer the reader to the literature of Faul and Tipping (2002).

Considering the dependence of \( p(d \mid h, \sigma^2) \) on a single hyper-parameter \( b_{ij} \), \( j \in \{1, 2, \ldots, K\} \), \( Q \) in equation (13) can be decomposed as

\[ Q = \sigma^2 I + \sum_{b_{ij}} b_{ij}^{-1} G_i G_j^H + b_{ij}^{-1} G_i G_j^H = Q_{-j} + b_{ij}^{-1} G_i G_j^H, \]  
(14)

where \( Q_{-j} \) is the \( Q \) with the \( j \)-th basis vector removed. Substituting equation (14) into the logarithm of the marginal likelihood (equation (13)) and defining \( \log \, p(d \mid h, \sigma^2) = L(h), L(h) \) can be written as

\[ L(h) = -\frac{1}{2} \left[ M \log(2\pi) + \log |Q_{-j}| + d^H Q_{-j}^{-1} d \right. \]  
(15)

\[ - \log b_{ij} + \log \left( b_{ij} + G_i^H Q_{-j}^{-1} G_j \right) \]  
\[ - \left( G_i^H Q_{-j}^{-1} d \right)^2 / \left( b_{ij} + G_i^H Q_{-j}^{-1} G_j \right) \]  
\[ = L(h_{-j}) + l(h_j), \]

where \( L(h_{-j}) = -\frac{1}{2} \left[ M \log(2\pi) + \log |Q_{-j}| + d^H Q_{-j}^{-1} d \right] \) is a function independent of \( b_j \), and \( l(h_j) = \frac{1}{2} [\log b_{ij} - \log(b_{ij} + s_j + q_j^2/(b_j + s_j))] \) is a function related to \( b_j \) with \( s_j = G_i^H Q_{-j}^{-1} G_j \) and \( q_j = G_i^H Q_{-j}^{-1} d \).

By analysing the first-order and second-order derivatives of \( l(h_j) \), \( L(h) \) has a unique maximum with respect to \( h_j \) in the following two cases:

\[ q_j^2 > s_j, \quad b_j = \frac{s_j^2}{q_j^2 - s_j} \]  
(16)
and if
\[ q_j^2 \leq s_j, \quad b_j = \infty. \] (17)

The two cases imply that 1) if \( G_j \) is in the model (i.e., \( b_j < \infty \)) but \( q_j^2 \leq s_j \), \( G_j \) may be deleted (i.e., \( b_j \) is set as \( \infty \)); 2) if \( G_j \) is excluded from the model (i.e., \( b_j = \infty \)) and \( q_j^2 > s_j \), \( G_j \) may be added (i.e., \( b_j \) is set as some optimal finite value). In addition, \( b_j \) can be re-estimated if equation (16) applies for basis vectors already in the model.

These properties enable a principled and efficient sequential addition, deletion and re-estimation of basis vector \( G_j \) to monotonically increase the marginal likelihood objective function.

### 3 EXAMPLES

For convenience, we use SSBLRI as a short for SBL-based spectral reflectivity inversion or spectral sparse Bayesian learning reflectivity inversion. In this section, several examples are utilized to test the performance of the method.

#### 3.1 1D synthetic data example

We designed a sparse reflectivity series (the leftmost trace in Fig. 3) consisting in 701 reflectivity values with a sample interval of 1 ms, including 21 non-zero spikes and 680 zero values. The time positions and amplitudes of the 21 non-zero reflectivity spikes are shown in the 1st column of the left panel and the 1st column of the right panel in Table 1, respectively. In order to test how well the method estimates weak reflection, we set the minimum amplitude of the non-zero spikes as 0.03, at a time position of 470 ms. In order to test the resolution of the method, we set the vertical two-way traveltime of the thinnest bed as 8 ms, from 172–180 ms. By convolving a 30 Hz Ricker wavelet with the designed reflectivity series, we generate a synthetic trace (the black continuous line in the 82nd trace of Fig. 1) with a sample interval of 1 ms.

For the purpose of illustrating the working of the SSBLRI method, we first illustrate how the inverted reflectivity series evolves with the iterations in Fig. 1. In this example, we set the initial noise variance \( \sigma^2 \) as 0.009 and select the 5–70 Hz frequency band with a sample interval of 1 Hz to implement the inversion. As the figure shows, after the 1st iteration, there is only a spike with the largest amplitude resolved. As iterations continue, the spikes with larger amplitude are first resolved and then those with smaller amplitudes follow. In addition, in the first 21 iterations, each iteration only resolves one spike and the 21st iteration resolves the smallest one. The 2nd column of the left panel and that of the right panel in Table 1 are time positions and amplitudes of the inverted non-zero spikes during the 21st iteration, respectively. In the 22nd and subsequent iterations, the inverted non-zero spikes are re-updated by an addition, deletion or re-estimation operator (see also Table 1). The 3rd column of the left panel and that of the right panel in Table 1 are the finally inverted non-zero reflectivity spikes, also shown in the 82nd trace of Fig. 1. Compared with the seismic trace (the black continuous line in the 82nd trace of Fig. 1), we observe that the retrieved reflectivity spikes at 172 ms, 180 ms, 470 ms, 570 ms and 580 ms are not consistent with the seismic peaks or troughs. That is to say, directly picking peaks or troughs of seismic events to interpret the top or bottom of layers is probably not accurate, especially for thin layers. However, SSBLRI accurately delineates the layers. Besides the time positions, the amplitudes of inverted reflectivity spikes are almost consistent with the true model.

<table>
<thead>
<tr>
<th>Time position True</th>
<th>21st estimated</th>
<th>Finally inverted</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ms)</td>
<td>True</td>
<td>21st estimated</td>
</tr>
<tr>
<td>40</td>
<td>0.0750</td>
<td>0.0744</td>
</tr>
<tr>
<td>60</td>
<td>0.1100</td>
<td>0.1084</td>
</tr>
<tr>
<td>97</td>
<td>−0.0400</td>
<td>−0.0346</td>
</tr>
<tr>
<td>138</td>
<td>0.1000</td>
<td>0.1005</td>
</tr>
<tr>
<td>172</td>
<td>0.1300</td>
<td>0.1232</td>
</tr>
<tr>
<td>180</td>
<td>−0.0900</td>
<td>−0.0802</td>
</tr>
<tr>
<td>223</td>
<td>0.0850</td>
<td>0.0827</td>
</tr>
<tr>
<td>261</td>
<td>0.1400</td>
<td>0.1403</td>
</tr>
<tr>
<td>290</td>
<td>−0.0500</td>
<td>−0.0467</td>
</tr>
<tr>
<td>337</td>
<td>0.0800</td>
<td>0.0734</td>
</tr>
<tr>
<td>350</td>
<td>0.0800</td>
<td>0.0706</td>
</tr>
<tr>
<td>390</td>
<td>−0.1000</td>
<td>−0.1006</td>
</tr>
<tr>
<td>419</td>
<td>0.1300</td>
<td>0.1273</td>
</tr>
<tr>
<td>470</td>
<td>0.0300</td>
<td>0.0258</td>
</tr>
<tr>
<td>480</td>
<td>−0.0950</td>
<td>−0.0958</td>
</tr>
<tr>
<td>530</td>
<td>0.0500</td>
<td>0.0479</td>
</tr>
<tr>
<td>570</td>
<td>−0.0700</td>
<td>−0.0694</td>
</tr>
<tr>
<td>580</td>
<td>0.0700</td>
<td>0.0669</td>
</tr>
<tr>
<td>610</td>
<td>−0.2000</td>
<td>−0.1996</td>
</tr>
<tr>
<td>640</td>
<td>0.1500</td>
<td>0.1494</td>
</tr>
<tr>
<td>660</td>
<td>0.0600</td>
<td>0.0577</td>
</tr>
</tbody>
</table>
We then undertook a sensitivity study of the method with regards to the accuracy of the seismic wavelet. For convenience, we assume that the estimated wavelet only has a constant phase error compared with the true wavelet, which is denoted by the arrow in Fig. 2. All the other wavelets in Fig. 2 have a constant phase error with respect to the true wavelet. The inverted reflectivity spikes derived by using every wavelet in Fig. 2 are shown in Fig. 3. It can be observed that if the phase error between the estimated wavelet and the true wavelet is not more than 30°, the inverted result is acceptable. In particular, when the error is not more than 10°, the inverted result is almost the same as the true model. However, when the error is more than 30°, not only are false reflectivity values introduced into the series but also the true reflectivity values are distorted. That is to say, the accuracy of the wavelet is an important factor for the SSBLRI method.

3.2 2D synthetic data example

To test how well the SSBLRI method can identify thin beds from a noisy data set, we generate a wedge model (Fig. 4a). The reflectivity of the top and bottom of the model are set as 0.1 and −0.1, respectively. In the 1st trace (CDP number = 1), the vertical two-way traveltime between the top and bottom is 2 ms. As the CDP number increases, the two-way traveltime increases with a step of 2 ms. The long blue lines in Fig. 4(a) denote the top and bottom interfaces and the short black lines denote the reflectivity. By convolving the reflectivity with a 30 Hz Ricker wavelet, we obtain synthetic seismic data (Fig. 4c) with a sample interval of 1 ms. According to Chung and Lawton (1995), the tuning thickness of this model is approximately 13 ms.

Figure 4(d) is the amplitude spectrum of the true reflectivity model with a sample interval of 1 Hz. In the figure, every trace is a harmonic wave whose period is the inverse of the time thickness of the layer in this trace. This is the key idea of using spectral decomposition (Partyka, Gridley and Lopez 1999) to determine the thickness of a thin bed. In order to test whether SSBLRI is robust to noise, 20% (S/N = 5:1, defined as the ratio of the complex spectrum energy of the true reflectivity model to that of the noise in this paper) Gaussian noise is added to the complex spectrum of the true reflectivity model within the frequency band 10–100 Hz. As the amplitude spectrum in Fig. 4(e) shows, due to the noise, there are no obvious spectrum peaks or troughs to help determine the tuning thickness. Even
Figure 4 SSBLRI used for a 2D synthetic data example. (a) 2D wedge reflectivity model, (b) inverted reflectivity, (c) noise-free synthetic seismic data, (d) noise-free amplitude spectrum of the true reflectivity model, (e) noisy amplitude spectrum of the true reflectivity model and (f) amplitude spectrum of the inverted reflectivity series. For the first six traces of the model, the time thickness of the two non-zero spikes is smaller than the tuning thickness. The top and bottom interfaces of the 3rd to 6th trace are accurately resolved but that of the 1st and 2nd traces is not. The trace-by-trace inversion exhibits perfect lateral continuity.
if the time interval of the two non-zero spikes is relatively large, in, for example, the 20th trace, it is still difficult to find the clear shape of the harmonic wave.

We set $\sigma^2$ for every trace as 0.03 and undertake a trace-by-trace reflectivity inversion. The inverted result is shown in Fig. 4(b). To quantitatively evaluate the inverted result, we define the relative error as $\sum_{k=1}^{K} (\hat{r}_k - r_k)^2 / \sum_{k=1}^{K} r_k^2$, shown in Fig. 5. As Figs 4(b) and 5 show, the top and bottom interfaces of the 3rd to 20th trace are perfectly identified. For the 1st and 2nd trace, the relative error mainly results from the inaccurate inversion of the time positions of the two non-zero spikes. For the 3rd to 20th trace, the small relative error mainly results from inaccurate inversion of the amplitudes of the spikes, partially because of the influence of noise. Although the reflectivity model is inverted trace-by-trace and there are no constraints imposed on the time positions and number of layers, the lateral continuity of the inverted result is preserved well and no visible false reflectivity spikes are introduced. Figure 4(f) is the amplitude spectrum of the inverted reflectivity model. Compared with Fig. 4(e), every trace in Fig. 4(f) is smoother and almost identical to the amplitude spectrum of the true reflectivity model (Fig. 4d).

3.3 3D physical modelling data example

A 3D physical model (Fig. 6a), designed according to a field geology structure, was also used to test the performance of SSBRLRI. A 2D section of the model perpendicular to the structural trend is shown in Fig. 6(b). Note that the geology profile is converted from the physical scale to the seismic scale by using the proportion 1:5000.

A seismic experiment was modelled in a water tank using a multi-channel acquisition system. The sources are ultrasonic wave transducers with a peak frequency of 260 kHz. After converting the physical modelling scale to a typical seismic exploration scale, the dominant frequency of the seismic source is equivalent to 26 Hz. We processed the data using spherical spreading compensation, noise attenuation, deconvolution, multiple suppression, migration and stack and finally obtained a 3D stacked data set. Figure 7(a) shows an in-line section and a cross-line section extracted from the 3D stacked data and Fig. 7(c) shows an in-line section and a time slice. A seismic wavelet (Fig. 8) is first estimated based on a second-order cumulant method. Then SSBRLRI is undertaken by using parallel computation. In this example, the initial noise variance for every trace is set as $0.5 \times \text{var}(d)$, where $\text{var}(\cdot)$ denotes variance and the selected frequency band is 5–70 Hz with a sample interval of 0.5 Hz. Figure 7(b,d) are the inverted results for Fig. 7(a,c), respectively. In Fig. 7b, reflectivity spikes denoted by the left and right arrows delineate the CBB, those denoted by the middle arrow delineate S1, and those denoted by an oval delineate S3. In Fig. 7d, reflectivity spikes denoted by the two arrows delineate the CBB, those denoted by the upper oval delineate S1, and those denoted by the lower oval delineate S3. As the figures show, the shape of S1 and S2 are relatively accurately characterized. The position of the CBB is
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Figure 7 SSBLRI used for a 3D physical modelling data set. (a) An in-line section and a cross-line section extracted from the 3D stacked seismic data, (b) inverted result for (a), (c) an in-line section and a time slice extracted from the 3D stacked seismic data and (d) inverted result for (c). The shape of S1 and S2 and the position of CBB are relatively accurately inverted. For S3 with about 8 m average thickness, the inverted reflectivity series of the first 120 traces is very close to the true geology model.

almost accurately inverted with good lateral continuity. For S3 with about 8 m average thickness, the inverted reflectivity series of the first 120 traces is very close to the true model. However, some additional information is introduced into the inverted reflectivity series (Figs 6b, 7b and 7d), probably because 1) seismic data processing is not accurate, 2) 3D stacked data cannot be strictly described by the convolution model, 3) the estimated wavelet is not accurate and 4) there is coherent noise in the stacked data.

3.4 3D field data example

To further test the potential of SSBLRI, the method was applied to a 3D field post-stack seismic data set from the west

Figure 8 Estimated wavelet based on a second-order cumulant method.
of China, that consists in 556 × 801 traces with a time sample interval of 2 ms. An in-line section and a cross-line section are displayed in Fig. 9(a,c), respectively.

A mixed-phase wavelet, shown in Fig. 10, is first estimated based on a bispectral wavelet estimation method (Yu et al. 2011). At this point, SSBLRI is implemented by using parallel computation. In this example, the initial noise variance for every trace is set as \(0.02 \times \text{var}(d)\) and the selected frequency band is 5–65 Hz with a sample interval of 0.5 Hz. Figure 9(b,d) are the inverted results for Fig. 9(a,c), respectively. As Fig. 9 shows, compared with the original seismic data, the inverted reflectivity has higher resolution with more details (marked with ovals). Furthermore, the relative amplitude and lateral continuity of the reflectivity are preserved well. Some weak reflections in the seismic data are emphasized.

CONCLUSIONS

The spectral sparse Bayesian learning reflectivity inversion method we present can be used to obtain a reliable, sparse reflectivity series with sparsity assumption about the reflectivity. The method inverts for reflectivity spikes by
Sequentially adding, deleting or re-estimating hyper-parameters, without any \textit{a priori} constraint about the sparse reflectivity series, such as the number, time positions and/or amplitudes of non-zero spikes. In general, the spikes with large amplitude are first resolved and, following this, spikes with smaller amplitudes are resolved.

SSBLRI can be used to identify thin beds below the tuning thickness. Although the reflectivity is inverted trace-by-trace, the lateral continuity of the inverted result is preserved well, as shown in Figs 4(b), 7 and 9. For the reflectivity inversion of 3D multi-trace data, parallel computation can be readily adopted to enhance the computational efficiency of the method. The accuracy of the method is dependent on the accuracy of seismic wavelet estimation but the method does not require that the wavelet is minimum phase.

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