

# Establishment of a multi-cycle generalized Weng model and its application in forecasts of global oil supply

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**Abstract** Low oil prices under the influence of economic structure transformation and slow economic growth have hit the existing markets of traditional big oil suppliers and upgraded the conflict of oil production capacity and interest between OPEC producers and other big oil supplier countries such as the USA and Russia. Forecasting global oil production is significant for all countries for energy strategy planning, although many past forecasts have later been proved to be very seriously incorrect. In this paper, the original generalized Weng model is expanded to a multi-cycle generalized Weng model to better reflect the multi-cycle phenomena caused by political, economic and technological factors. This is used to forecast global oil production based on parameter selection from a large sample, depletion rate of remaining resources, constraints on oil reserves and cycle number determination. This research suggests that the world will reach its peak oil production in 2022, at about  $4340 \times 10^6$  tonnes. China needs to plan for oil import diversity, a domestic oil production structure based on the supply pattern of large oil suppliers worldwide and the oil demand for China's own development.

**Keywords** Oil production · Multi-cycle · Generalized Weng model · Energy strategy

## 1 Introduction

The US shale revolution has rapidly increased its oil and gas supply. Meanwhile, OPEC chose to maintain production to protect its market. The oil production of OPEC in 2015 was  $3160 \times 10^4$  bbl/d, which had increased by 2.7% compared to that in 2014 (OPEC 2016; EIA 2016). The slowdown of world economic growth and transformation of economic structures in many countries have intensified the production contest among OPEC and other large oil suppliers like Russia and the USA. Under the dual effects of supply increase and demand decline, oil price fell continuously and sharply, which had serious influences on the investment and production capacity construction of oil resources and new energy resources. Some shale gas suppliers have withdrawn from the market due to high production costs. What is more, the special requirements of oil exploitation concerning geological conditions and construction make this exit irreversible in the long term.

The previous supply-dominated oil market has gradually turned into a demand-oriented situation. Oil market imbalance, which is manifested as the rapid decline of oil price, affects the short- and long-term production decisions of oil suppliers. However, huge differences in some inherent historical factors in different areas, such as production costs, resource conditions and stakes, can lead to completely different final supply decisions (Apergis et al. 2016). The global oil production trend will finally affect the strategies of various stakeholders.

Using the method considering key production constraints to undertake quantitative research into global oil supply volumes and provide information for national energy strategy, has become the focus and the difficulty in the present study. Since the shale revolution, a large number of studies have focused on unconventional oil and

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gas production. The oil market, which has experienced great changes, is still unstable. It is now necessary to analyze the long-term oil supply trends of the global market.

In terms of existing model theory, most models are established with lack of consideration of the actual characteristics of oil and gas production. Among them, the existing generalized Weng model applications addressing national oil and gas production still stay on the stage of original model use and simple piecewise curve fitting, rather than extending the whole model to multi-cycle mode to fully reflect production trends. Resource depletion constraints have not used through specific functions to avoid unrealistic production growth forecasts which may occur in results. In terms of model implications, much of the existing research focuses on single fields or single countries, which does not reflect the global supply pattern, so it is not relevant to allow oil-consuming nations to develop strategic plans. Studies are rarely focused on international production, at the same time there is a lack of analysis combining future supply and demand situations, geographical features and development appeals of oil suppliers and consumers. In view of the defects above, this paper implements improvements in the aspects of objective function selection, production calculation, internal and external constraints and the frequency of multi-cycle fitting to model future world oil supplies.

## 2 Methodology

### 2.1 Existing oil production forecast models

The current oil prediction methods can be mainly divided into three categories: curve-fitting models, which are based on historical production data. These include the Hubbert, Gaussian and Logistic models (Reynolds 2014; Saraiva et al. 2014; Brandt 2007); system simulation methods, which are based on causal relationship of factors, such as the system dynamics method (Tao and Li 2007; Tang et al. 2010; Hosseini and Shakouri 2016); and econometric models based on economic theory (Kaufmann 1991; Pindyk and Rubinfeld 1998). The most widely used method is the curve-fitting model (Gallagher 2011; Sorrell and Speirs 2010; Nashawi et al. 2010; Ebrahimi and Ghasabani 2015).

Hubbert was the first scholar to use forward curve fitting. Hubbert pointed out bell-shaped curve regularity of fossil energy development (Hubbert 1949). In 1956, Hubbert used a hand-drawn bell-shaped curve to forecast oil production in the 48 contiguous states of the USA. According to his prediction, the US oil production would peak in the early 1970s and then decrease (Hubbert 1956). This prediction was confirmed by the actual oil production.

Because of this successful prediction and the social concern about oil shortage, using bell-shaped curves to predict oil production has become especially popular. More and more scholars have begun to join in the forecasting of oil production. The forecasting method used by Hubbert has been adopted by more and more people and is named the “Hubbert model.” Although many scholars used this method, Hubbert had not given the specific formula of the method and its derivation. Until 1982, for the first time, Hubbert published the full formula and derivation process of the Hubbert model (Hubbert 1982). Since then, Hubbert model has been widely used.

Although the Hubbert model is the most widely used method in curve fitting, the model is not perfect. For instance, the model has poor accuracy when it is applied in the regions which have multiple oil production peaks. Therefore, many scholars began to improve the model. Current improvements mainly include two categories: Firstly, additional production cycles were added into model to fit multiple historical production peaks and improve the prediction accuracy, which is called the multi-cycle model; second, the Generalized Hubbert model was established by extending the typical Hubbert model. Wang et al. (2011) pointed out that the multi-cycle model is the most widely used model.

After many modifications, the forecasting accuracy of Hubbert’s model has been improved significantly. Even so, many inherent problems remain unresolved. The curve shape of the Hubbert model is completely symmetrical. However, the reality is that in many oilfields, the production grows fast at the beginning and then declines slowly after reaching the peak. It is mainly because many measures are always taken to prevent rapid decline of oil production, such as improving recovery efficiency. It means that the production curve shapes of many oilfields are not completely symmetrical. Brandt (2007) analyzed 67 oil producing countries which have passed the production peak and found that most of these production curves follow a positive skewness distribution.

The generalized Weng model is the most widely used oil production prediction method in China. The curve shape of generalized Weng model is positive skewness. Wang et al. (2011) established a multi-cycle generalized Weng model on the basis of the generalized Weng model and compared it with the multi-cycle Hubbert model. This shows that the forecasting accuracy of multi-cycle generalized Weng model is better than that of the multi-cycle Hubbert model.

However, both the multi-cycle generalized Weng model and the multi-cycle Hubbert model lack a quantitative basis for choosing the number of cycles. Generally, the fitting effect of models would be better if the production cycles are increased. But meanwhile, excessive production cycles may cause overfitting. Overfitting could reduce the

forecasting function of the model. Therefore, how to determine the optimal number of production cycle is very important. Wang and Feng (2016) proposed a quantitative method to quantify production cycle numbers, namely the  $F$  test. But this method has not been applied to multi-cycle generalized Weng models. In addition, many scholars have pointed out that the depletion rate of residual resources would also have significant impact on prediction. Therefore, the depletion rate of residual resource was suggested to be added into the model as a constraint parameter (Wang et al. 2013; Wang and Feng 2016).

Based on the multi-cycle generalized Weng model proposed by Wang et al. (2011), this study will establish a new multi-cycle generalized Weng model by adding the  $F$  test and residual resource depletion rate and apply this new model to forecast future global oil production.

## 2.2 Traditional Weng model and its characteristics

Among many oil production prediction methods, the Weng model takes into account the life-cycle characteristics of non-renewable resources and that “for many life limited systems, such as non-renewable resources, their whole life process can be imaged as a Poisson distribution probability function” (Weng 1984). This method can improve the measuring accuracy due to its full reflection of the known oil and gas resources in a short time. On this basis, Chen (1996) further derived a generalized Weng model which can be used to predict oil field production, final recoverable reserves and peak production based on a gamma distribution. The prediction model is as follows:

$$Q = at^b e^{-(t/c)} \quad (1)$$

$$a = \frac{N_R}{c^{b+1} \Gamma(b+1)} \quad (2)$$

Take logarithm for both sides:

$$\log \frac{Q}{t^b} = \log a - \frac{1}{2.303c} t \quad (3)$$

Let:

$$A = \log a, \quad B = 1/2.303c \quad (4)$$

Then:

$$\log \frac{Q}{t^b} = A - Bt \quad (5)$$

where  $Q$  represents the production;  $t$  represents relative development time of the oil field;  $a$ ,  $b$ ,  $c$  are unknown parameters;  $N_R$  represents recoverable reserves of the oil-field. The simplified Eq. (5) can be solved by using a linear differential method. In particular, first, different values of  $\log(Q/t^b)$  can be obtained by plugging into different  $b$  values. Second, the correlation coefficient between

$\log(Q/t^b)$  and  $t$  can be obtained, select the  $b$  value which maximizes the correlation coefficient to fit the straight line represented by Eq. (5), and then the two values of  $A$  and  $B$  can be obtained. Finally, the two values  $a$  and  $c$  can be obtained, and Eq. (1) is identified.

## 2.3 Establishment of a multi-cycle generalized Weng model

The oil production at the regional level is affected by many factors such as politics, economy and technology. The historical yield curve of many regions showed multi-cycle phenomena. A single generalized Weng model cannot accurately describe this characteristic and causes large deviation in production estimation. This paper expands the single generalized Weng model to a multi-cycle generalized Weng model, which is established through stages as follows:

$$q(t) = \frac{\text{URR}}{c^{b+1} \Gamma(b+1)} t^b e^{-(t/c)} \quad (6)$$

where  $q(t)$  represents production;  $b$  and  $c$  are unknown parameters.

In terms of goodness-of-fit tests, most scholars like Chen and Hu (1996) adopt a decision coefficient as a measure gauge. If the decision coefficient is close to 1, the fitting effect is better. But the determination coefficient represents the interpretation of the independent variable on the dependent variable; if the production fluctuation is large, even if the determination coefficient value is high, the gap between predicted values and real values may not be minimized. Root-mean-square error (RMSE) directly measures the deviation between predicted values and real values. The prediction goal is to minimize the gap between predicted values and real values. So in this paper, RMSE is used instead of  $R^2$  to evaluate the predictive ability of model; RMSE is expressed as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (q_{\text{act}} - q_{\text{for}})^2}{n}} \quad (7)$$

where  $n$  represents the number of the empirical data,  $q_{\text{act}}$  represents actual historical production,  $q_{\text{for}}$  represents forecast production, the target of the model is minimizing RMSE.

The analysis of constraints is as follows: external URR (ultimate recoverable resources) are used to constrain production. The  $F$  test is used to determine the number of production cycles.

In many traditional oil production forecasting models, URR is usually regarded as an internal variable, together with oil production, becoming the production variable of

the prediction model. The disadvantage of this approach is that URR cannot constrain production. It is likely to overestimate or underestimate future oil production. This paper uses URR as an external variable to constrain production. The value of URR can be obtained by summing cumulative production and reserves. This constraint is expressed in the following equation:

$$URR_{en} = URR_{ex} \tag{8}$$

The left side is internal URR, and the right side is external URR.

*F* test is established as follows.

First, the variance of sample sequence can be obtained from Eq. (7).

$$S^2 = \frac{\sum_{i=1}^n (q_{act} - q_{for})^2}{n - m - 1} = \frac{RMSE^2 \times n}{n - m - 1} \tag{9}$$

where *m* represents the number of unknown parameters in Eq. (9). *n* - *m* - 1 represents the degrees of freedom. Then in terms of the prediction results in two groups (one group is established before an additional cycle is added, and another group is established after an additional cycle is added); then the *F* statistic is established as follows:

$$F_{value} = \frac{S_1^2}{S_2^2} = \frac{\frac{RMSE_1^2 \times n}{n - m_1 - 1}}{\frac{RMSE_2^2 \times n}{n - m_2 - 1}} = \frac{RMSE_1^2 n - m_2 - 1}{RMSE_2^2 n - m_1 - 1} \tag{10}$$

where *RMSE*<sub>1</sub> and *RMSE*<sub>2</sub> represent the mean square root before and after an additional production cycle is added, respectively. In general, *RMSE*<sub>1</sub> > *RMSE*<sub>2</sub>; *m*<sub>1</sub> and *m*<sub>2</sub> represent the number of free variables in the model before and after an additional production cycle is added, respectively. In general, *m*<sub>1</sub> < *m*<sub>2</sub>; *n* represents the number of the empirical data.

A production cycle can be added only when the following conditions are met:

$$F_{value} > F_{\alpha}(n - m_1 - 1, n - m_2 - 1) \tag{11}$$

where  $\alpha$  represents significance level, whose value is 0.01 in this paper.

The significance of the *F* test is that a new production cycle is allowed only when it can significantly improve the goodness of fit.

In reality, under the influence of economy, technology and other factors, the remaining resource depletion rate cannot grow without limit. Further, extremely high depletion rates mean destructive exploitation of underground resources, which is unfavorable for long-term development. Therefore, in actual production, the residual resource depletion rate has a maximum ceiling. The residual resource depletion rate is expressed as follows:

$$d(t) = \frac{q(t)}{URR - Q(t)} \tag{12}$$

where *d*(*t*) represents the residual resource depletion rate, *q*(*t*) represents annual production, and *Q*(*t*) represents cumulative production.

Above all, the multi-cycle generalized Weng model can be expressed as follows:

$$\begin{cases} \text{Min RMSE} = \sqrt{\frac{\sum_{i=1}^n (q_{act} - q_{for})^2}{n}} \\ \left. \begin{aligned} q(t) &= \frac{URR}{c^{b+1}\Gamma(b+1)} t^b e^{-(t/c)} \\ Q(t) &= \sum_{i=1}^k q(t)_i \\ \text{st. } \begin{cases} URR_{en} &= URR_{ex} \\ F_{value} &> F_{\alpha}(n - m_1 - 1, n - m_2 - 1) \\ d(t) &= \frac{q(t)}{URR_{en} - Q(t)} \leq d_{max} \\ b &> 0, c > 0 \end{cases} \end{aligned} \right\} \tag{13} \end{cases}$$

where *Q*(*t*) is the annual forecast production, whose value is the summation of the forecast production of all cycles. *d*<sub>max</sub> represents the maximum residual resource depletion rate which is extracted by combining the existing research literature with the investigation into the current oil production situation. This model is solved by using Excel VBA programming.

### 3 Application of a multi-cycle generalized Weng model in forecasts of global oil supply

#### 3.1 Current situation of global oil supply

The world’s main oil sources are OPEC and some other traditional large oil suppliers, such as Russia and the USA. Venezuela, Saudi Arabia, Iran and Iraq have abundant oil reserves, with more than 2 × 10<sup>10</sup> tonnes for each country. The production gap between OPEC and non-OPEC’s total oil production is less than 5%. The reserve-production ratios of Venezuela, Libya, Iran or Iraq are more than 100. The overall OPEC reserve-production ratio is 91, which proves that OPEC has strong oil supply potential under the current oil production situation.

At the same time, the shale oil revolution has significantly boosted the traditional oil market in recent years, and changes have taken place in oil market patterns. However, the rapid price fall not only made the oil market cool down, but also curtailed the unconventional oil and gas revolution which had just arisen. The global oil market has entered a stable phase recently after huge short-term fluctuations.

**Table 1** Value of parameters in model

Index	URR <sub>en</sub>	RMSE	Number of years	Number of production cycles
Value	398,500 × 10 <sup>6</sup> tonnes	114	50	5

Root-mean-square error (RMSE) measures the deviation between predicted values and real values

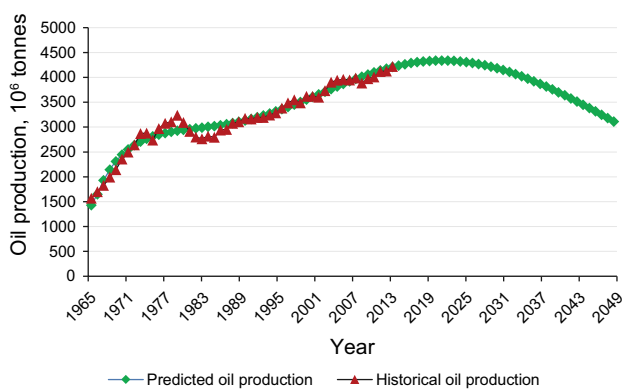
Oil price decline due to global oil being excessively supplied has resulted in the cessation of drilling in a large number of oilfields. Oil stocks continued to decline, which reduced the global oil surplus. But the oversupply situation still exists and the status of many traditional oil-rich countries is being challenged. Along with the conventional and unconventional oil production tending to be stable, what pattern will global oil supply evolve into? This has profound influence on the main oil suppliers and consumers who have just experienced sharp fluctuations of oil price. This paper forecasts the global oil supply using the multi-cycle generalized Weng model.

### 3.2 Data

In this paper, the oil data from 1965 to 2014 have been chosen for analysis; the data on annual oil production, proven oil reserves and relative exploitation time (the base year is 1965) are obtained from the BP Statistical Review of World Energy 2015.

### 3.3 Results

The value of some key parameters in the multi-cycle generalized Weng model which is applied to forecast global oil supply is listed as follows (Table 1).

**Fig. 1** Prediction of global oil production**Table 2** Peak time and peak yield, 10<sup>6</sup> tonnes

Peak time	Peak yield	Yield in 2020	Yield in 2030	Yield in 2040	Yield in 2050
2022	4340	4330	4215	3760	3110

The analysis and prediction on global oil production are carried out based on the multi-cycle generalized Weng model established above, as shown in Fig. 1.

Large fluctuations in oil production happened in the 1970s and 1980s, mainly caused by turmoil in the Middle East. This multi-cycle model can well reflect the multi-modal phenomena of oil supply. Before 2022, oil supply will slowly rise on the basis of status quo; after the peak, it will continue to drop.

So far, Saudi Arabia has occupied the main position in oil supply for a long time. Meanwhile, the oil supply of Persian Gulf is still the focus of the world. In the future, global oil supply and social situations will be more closely linked in this region due to resource depletion and the global competition for energy. Instability in this region will rapidly affect global development through energy chains.

The oil production of some countries is going to change significantly. On the one hand, China should make corresponding preparations in advance and expand diverse oil import channels; On the other hand, strategic oil cooperation with Africa is still a key support for China's economic and social development in the short term. In addition to cooperation with Nigeria, cooperation with Sudan, Congo and other countries in the field of energy must be strengthened to achieve win-win situations and energy security.

Table 2 lists the various peak time and the productions in four time points of the world.

The world's peak time is estimated to happen in 2022 from this research, and it is close to the results of Tang et al. (2009) and Shell (2011).

## 4 Conclusion

To overcome the shortcomings in existing oil production forecast models, this paper establishes a multi-cycle generalized Weng model and predicts global oil production based on data from the BP Statistical Review of World Energy, 2015. This model includes parameter selection from a large sample, depletion rate of remaining resources, constraint of oil reserves and cycle number determination,

not only to better fit curves but to strengthen the forecast capacity of model.

In the process of model establishment and application, it is found that the number of model cycles can dramatically affect the prediction outcome. So, an appropriate number of cycles determined by the fitting error can effectively avoid excessive fitting, promoting model prediction reliability. The residual resource depletion rate can effectively avoid unrealistic production changes in many models; recoverable reserves will have a significant impact on future oil supply. At present, the global oil supply exceeds demand, however, its peak is going to be reached following China's "13th Five-Year plan," and then oil supply is predicted to decline. Therefore, during this period, China should accelerate the conventional and unconventional oil exploration and imports to ensure the future oil demand can be satisfied.

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