



Multifractal characteristics of shale and tight sandstone pore structures with nitrogen adsorption and nuclear magnetic resonance

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Abstract

Based on the experiments of nitrogen gas adsorption (N_2 GA) and nuclear magnetic resonance (NMR), the multifractal characteristics of pore structures in shale and tight sandstone from the Chang 7 member of Triassic Yanchang Formation in Ordos Basin, NW China, are investigated. The multifractal spectra obtained from N_2 GA and NMR are analyzed with pore throat structure parameters. The results show that the pore size distributions obtained from N_2 GA and NMR are different, and the obtained multifractal characteristics vary from each other. The specific surface and total pore volume obtained by N_2 GA experiment have correlations with multifractal characteristics. For the core samples with the similar specific surface, the value of the deviation of multifractal spectra R_d increases with the increase in the proportion of large pores. When the proportion of macropores is small, the R_d value will increase with the increase in specific surface. The multifractal characteristics of pore structures are influenced by specific surface area, average pore size and adsorption volume measured from N_2 GA experiment. The multifractal characteristic parameters of tight sandstone measured from NMR spectra are larger than those of shale, which may be caused by the differences in pore size distribution and porosity of shale and tight sandstone.

Keywords Shale · Tight sandstone · Nitrogen adsorption · Nuclear magnetic resonance · Multifractal · Pore structure

List of symbols

D_q	Generalized fractal dimension
D_1	The generalized fractal dimension when $q=1$
$f(\alpha)$	Multifractal spectra
$f(\alpha)_{\max}$	The maximum values of $f(\alpha)$
$f(\alpha)_{\min}$	The minimum values of $f(\alpha)$
k	Positive integer
$N(\varepsilon)$	The number of sections divided
$N_a(\varepsilon)$	The number of boxes with singularity index between α and $\alpha + d\alpha$
$P_i(\varepsilon)$	Probability distribution function
q	Exponent
R_d	Deviation of the multifractal spectra
α	Singularity index
$\alpha_0(q)$	Singularity index corresponding to $f(\alpha)_{\max}$
$\alpha(q^+)$	The smallest value of $\alpha_0(q)$ when $q > 0$

$\alpha(q^-)$	The largest value of $\alpha_0(q)$ when $q < 0$
$\alpha_1(q)$	Width to the left of the multifractal spectra
$\alpha_2(q)$	Width to the right of the multifractal spectra
$\Delta\alpha(q)$	The width of the multifractal spectra
$\chi(q, \varepsilon)$	Partition function
ε	The scale of box
$\mu_i(\varepsilon)$	The pore volume for the <i>i</i> th interval
$\tau(q)$	Mass exponent

1 Introduction

Shale oil resources are abundant in China and it is another hot spot in current exploration and development (Hu et al. 2017). The Chang 7 member of the Yanchang Formation in the Ordos Basin of China is rich in unconventional petroleum resources. In particular, the exploration and development of shale oil in Ordos Basin has made important progress (Yang et al. 2016, 2019a, b). The Chang 7 member has the characteristics of continuous development of organic shale and sandstone, and therefore studying the pore structure characteristics of shale and tight sandstone is of great significance for the effective development of the Chang 7

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shale oil resources in the Yanchang Formation of the Ordos Basin, China (Er et al. 2016; Yin et al. 2020).

At present, there are several experimental methods for characterizing shale and tight sandstone pore structures, including thin section analysis, scanning electron microscopy (SEM), high-pressure mercury intrusion (HPMI) (Lai et al. 2018; Wang et al. 2019), N₂GA (Chen et al. 2017; Li et al. 2019; Singh 2016; Singh and Cai 2018), NMR (Shao et al. 2017; Zhao et al. 2020), micro/nano CT scanning (Peng et al. 2011; Yang et al. 2019) and so on. In 1980s Mandelbrot proposed the fractal theory and then it has been widely used in many fields (Mandelbrot 1983), and many scholars have studied the characteristics of pore structures in sedimentary rocks combining fractal theory with various experimental methods (Li et al. 2017; Wang et al. 2018). For example, Lai et al. (2016) used HPMI and NMR to study the pore structures of tight sandstone, and found that micropores play an important role in the heterogeneity of reservoir rocks and they can be quantitatively characterized by fractal dimension.

As the one of the main tight oil/gas reservoirs in China, the pore structures of tight sandstones in the Ordos Basin have been widely reported (Du and Shi 2019; Li et al. 2016; Wang et al. 2020). The results show that the fractal dimension can effectively characterize the complexity of the pore structure, and the pore structure complexity and heterogeneity increase with the increase in fractal dimension (Wang et al. 2018; Lyu et al. 2017). Li et al. (2017) studied the pore structures of Chang 7 tight sandstones using HPMI combined with fractal theory, and the value of calculated fractal dimensions changes from 2.2520 to 2.7875. Ju et al. (2019) studied the pore structure and fractal characteristics of Chang 7 shale were combined with N₂GA, and found that the fractal dimensions increase with the increase in organic matter and clay mineral content.

Although the fractal dimension can well represent the complexity of core pore structure, as the pores are not continuously distributed, the single value of the fractal dimension cannot comprehensively characterize the whole pore size distribution. As an extension of the fractal dimension, the multifractal dimension can get the whole properties and local information by analyzing the fluctuation of its probability density (Ferreiro et al. 2010; Liu and Ostadhassan 2019; Liu et al. 2018; Posadas et al. 2009; Xie et al. 2010; Yu et al. 2018; Zhu et al. 2014). Jouini et al. (2011) studied the multifractal characteristics of carbonate rocks based on SEM, and the influence of image magnification on the multifractal characteristics of pore structures was analyzed. Ge et al. (2015) found that the multifractal characteristic parameters of NMR spectra of fine sandstone has strong correlations with the values of $T_{2\text{cutoff}}$, and the formula for predicting $T_{2\text{cutoff}}$ based on multifractal parameters was developed. Jiang et al. (2018) studied multifractal characteristics of tight sandstone pore structures with

mercury intrusion experiments, and found that the multifractal characteristics have a good correlation with tight sandstone permeability and porosity. Liu et al. (2018) used the NMR T_2 spectra to analyze the multifractal characteristics of tight sandstone pore structures, and found that the multifractal characteristics of different types of tight sandstones are different and the obtained multifractal parameters can be used for characterize the heterogeneity of pore structures.

In this paper, the multifractal characteristics of the pore structures of shale and tight sandstone core samples from the same exploration well located in the Chang 7 Member of the Ordos Basin are studied based on N₂GA and NMR experiments. The correlations between multifractal parameters and pore structure parameters of shales and tight sandstones are revealed. Meanwhile, the differences of multifractal characteristics of shales and tight sandstones obtained from N₂GA and NMR are compared and discussed.

2 Multifractal theory

To obtain the multifractal characteristic of pore structures, the intervals of pore size are equally divided into the intervals with the length of ε , and $\varepsilon=L \times 2^{-k}$, where L is the overall range of pore size and k is a positive integer. Therefore, when the scale is ε , the number of sections divided is 2^k , i.e., $N(\varepsilon) = 2^k$. Therefore, the probability of each interval can be expressed as (Ferreiro et al. 2009):

$$P_i(\varepsilon) = \frac{\mu_i(\varepsilon)}{\sum_{i=1}^{N(\varepsilon)} \mu_i(\varepsilon)} \quad (1)$$

where $\mu_i(\varepsilon)$ is the pore volume for the i th interval. When NMR spectra are used to calculate the multifractal parameters, $\mu_i(\varepsilon)$ is the incremental saturation for the i th interval of T_2 . For the N₂GA test, $\mu_i(\varepsilon)$ is the pore volume of adsorbed nitrogen for the i th interval of relative pressure.

According to the multifractal theory, the probability distribution function $P_i(\varepsilon)$ is in a power exponential relationship with the scale ε (Vázquez et al. 2008):

$$P_i(\varepsilon) \propto \varepsilon^{\alpha_i} \quad (2)$$

where α_i is the singularity index or named as the coarse Holder exponent.

The number of boxes with singularity index between α and $\alpha + d\alpha$ is $N_\alpha(\varepsilon)$, which also has a power exponential relationship with the scale ε (Vázquez et al. 2008):

$$N_\alpha(\varepsilon) \propto \varepsilon^{-f(\alpha)} \quad (3)$$

where $f(\alpha)$ is the multifractal spectra, representing the exponential relationship between the similar singularity index and the number of boxes.

There are many methods of calculating multifractal spectra (Lopes and Betrouni 2009), such as the direct method and the indirect method (the Legendre transformation). The principles of these two different methods are briefly introduced as follows.

2.1 Direct calculation method

In 1989, Chhabra and Jensen proposed a direct method to calculate multifractal spectrum, which has been widely used (Chhabra and Roderick 1989; Cuevas 2003). The value of $f(\alpha)$ and α can be calculated with the following formula

$$\alpha = \lim_{\epsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\epsilon)} I_i(q, \epsilon) \times \log P_i(\epsilon)}{\log(\epsilon)} \tag{4}$$

$$f(\alpha) = \lim_{\epsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\epsilon)} I_i(q, \epsilon) \times \log I_i(\epsilon)}{\log(\epsilon)} \tag{5}$$

where the parameter $I_i(q, \epsilon)$ is defined as:

$$I_i(q, \epsilon) = \frac{P_i(\epsilon)^q}{\sum_{i=1}^{N(\epsilon)} P_i(\epsilon)^q} \tag{6}$$

where q is an exponent with a range of $[-10, 10]$.

2.2 Indirect calculation method

The indirect calculation method uses the relationship between the generalized fractal dimension D_q and the exponent q to determine the multifractal characteristic, and then the fractal dimension spectra can be calculated (Falconer 2004; Meakin 1990). The generalized fractal dimension can be calculated by its definition (Rényi 1955):

$$D_q = \frac{1}{q-1} \lim_{\delta \rightarrow 0} \frac{\ln \chi(q, \epsilon)}{\ln \epsilon} \quad (q \neq 1) \tag{7}$$

where $\chi(q, \epsilon)$ is a partition function, which is defined as:

$$\chi(q, \epsilon) = \sum_{i=1}^{N(\epsilon)} P_i^q(\epsilon) \propto \epsilon^{\tau(q)} \tag{8}$$

when $q = 1$, the following formula can be used for calculating the generalized fractal dimension D_1 , which is also called the information dimension:

$$D_1 = \lim_{\epsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\epsilon)} P_i(\epsilon) \log P_i^q(\epsilon)}{\log(\epsilon)} \tag{9}$$

$\tau(q)$ is the mass exponent and it can be processed by the following equation:

$$\tau(q) = \lim_{\epsilon \rightarrow 0} \frac{\log \chi(q, \epsilon)}{\log \epsilon} \tag{10}$$

Then combine Eq. (7) and Eq. (10), D_q can be simplified as (Halsey et al. 1986):

$$D_q = \frac{\tau(q)}{q-1} \tag{11}$$

Through the Legendre transformation (Halsey et al. 1986), the following equation can be derived as:

$$\alpha(q) = \frac{d\tau(q)}{dq} \tag{12}$$

$$f(\alpha) = q\alpha(q) - \tau(q) \tag{13}$$

Previous studies have found that the direct method can calculate the multifractal spectrum more simply and there may be some mistakes in the calculation of multifractal spectrum using Legendre transformation (Halsey et al. 1986). Comparing the two methods mentioned above, it can be found that the generalized fractal dimension and mass exponent can be obtained using the indirect calculation method, which can be used to judge whether the core pore space has multifractal features. Therefore, the direct calculation method is used to obtain the multifractal spectrum and the indirect calculation method is used to compute the generalized fractal dimension and mass exponent in this study.

To characterize the multifractal spectra, the parameters of $\Delta\alpha(q)$, $f(\alpha)_{\max}$, $f(\alpha)_{\min}$, $\alpha_0(q)$, $\alpha_1(q)$, $\alpha_2(q)$, R_d and $\Delta\alpha(q)$ are defined as Eqs. (14–17). (Ferreiro et al. 2009, 2010). $\Delta\alpha(q)$ is the difference between the maximum and the minimum values of $\alpha(q)$, representing the width of the multifractal spectra and indicating the complexity of the spatial distribution of the pores. The larger the value of $\Delta\alpha(q)$ is, the larger the internal difference and more uneven distribution of pore space will be. $f(\alpha)_{\max}$ and $f(\alpha)_{\min}$ are the maximum and minimum values of $f(\alpha)$, respectively. $\alpha_0(q)$ is the singularity index corresponding to $f(\alpha)_{\max}$, and the large value of $\alpha_0(q)$ indicates the high heterogeneity of pore volume distribution.

$$\alpha_1(q) = \alpha_0(q) - \alpha(q^+) \tag{14}$$

$$\alpha_2(q) = \alpha(q^-) - \alpha_0(q) \tag{15}$$

$$R_d = \alpha_1(q) - \alpha_2(q) \tag{16}$$

$$\Delta\alpha(q) = \alpha_1(q) + \alpha_2(q) \tag{17}$$

with $\alpha_0(q)$ as the boundary, $\alpha_1(q)$ is the width to the left of the multifractal spectra, and $\alpha(q^+)$ is the smallest value of $\alpha(q)$ when $q > 0$; $\alpha_2(q)$ is the width of the right side of

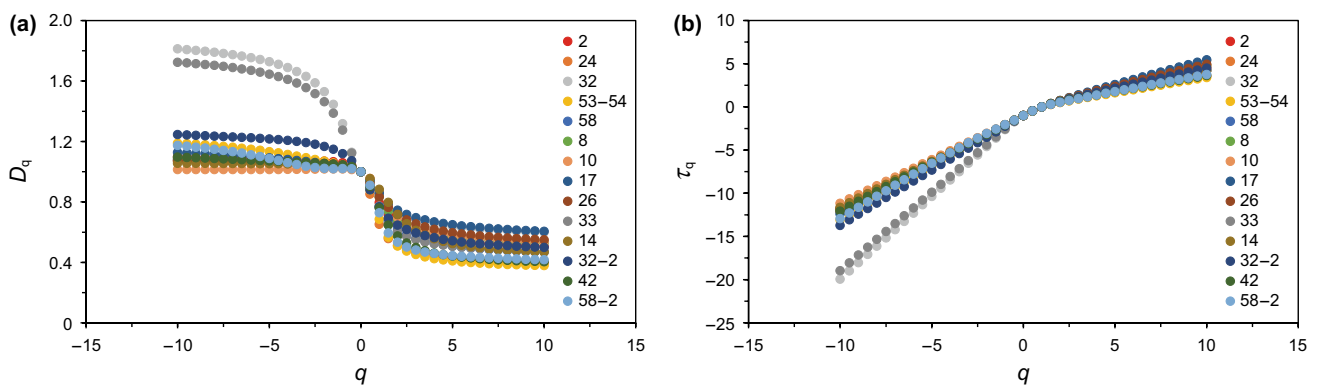


Fig. 3 Variation curve of generalized fractal dimension D_q and mass exponent τ_q with index q (a is the variation curve of generalized fractal dimension; b is the variation curve of mass exponent)

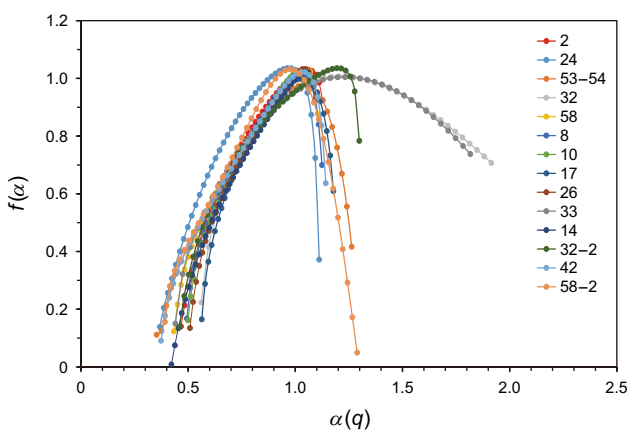


Fig. 4 Multifractal spectra of shale and tight sandstone pore structures obtained from N_2 GA

concentration of the pore size distribution, and its value is between 1.01 and 1.22. The $\alpha_0(q)$ of core 32–2 is 1.22 and is the maximum in all core samples, which means the fluctuation of pore size distribution of this core sample is largest. The deviation of the multifractal spectra R_d is between -0.2 and 0.68 . The core 32 is shale and its value of R_d is less than 0 , meaning the multifractal spectra are right-skewed, and the low-value information has a greater impact on multifractal spectra. Except the core 32, the multifractal spectra of the other core samples are all left-skewed, indicating the high-value information has a great influence on multifractal spectra. It can be found from the chart that the pore size distribution of the core 32 is the most complicated, but the core 10 which is shale has a small $\Delta\alpha(q)$ and $\alpha_0(q)$, indicating its pore size distribution is more concentrated and evenly distributed. Their pore size distributions measured by BJH method are shown in Fig. 5.

Table 3 Multifractal parameters calculated from N_2 GA

Core no.	$\Delta\alpha(q)$	$f(\alpha)_{\max}$	$f(\alpha)_{\min}$	$\alpha_0(q)$	$\alpha_1(q)$	$\alpha_2(q)$	R_d
2	0.66	1.03	0.12	1.09	0.62	0.04	0.58
8	0.63	1.01	0.17	1.06	0.57	0.06	0.50
10	0.55	1.03	0.16	1.01	0.51	0.04	0.47
17	0.61	1.00	0.18	1.08	0.51	0.10	0.42
26	0.60	1.03	0.13	1.07	0.56	0.04	0.52
32	1.40	1.00	0.22	1.16	0.60	0.80	-0.20
33	1.40	1.00	0.15	1.21	0.77	0.63	0.14
58	0.68	1.02	0.12	1.07	0.63	0.05	0.58
32-2	0.84	1.00	0.10	1.22	0.76	0.08	0.68
58-2	0.90	1.02	0.03	1.02	0.63	0.27	0.36
53-54	0.91	1.01	0.11	1.09	0.74	0.18	0.56
42	0.77	1.02	0.08	1.07	0.70	0.07	0.63
24	0.78	1.03	0.12	1.04	0.68	0.10	0.57

distributions are extremely uneven. The $\alpha_0(q)$ indicates the

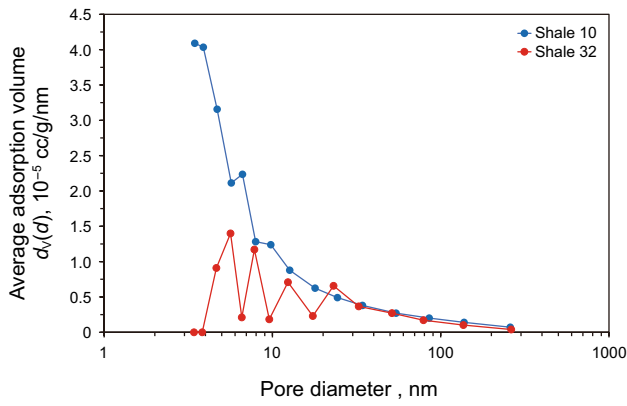


Fig. 5 Pore size distributions of the shale samples 10 and 32 obtained from N₂GA tests

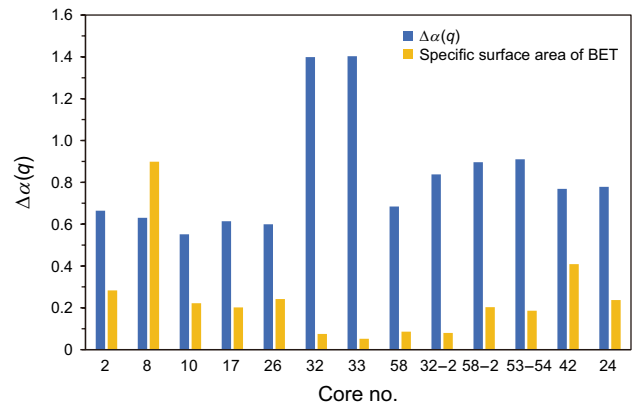


Fig. 7 Comparison of $\Delta\alpha$ value with the specific surface area measured from N₂GA experiment

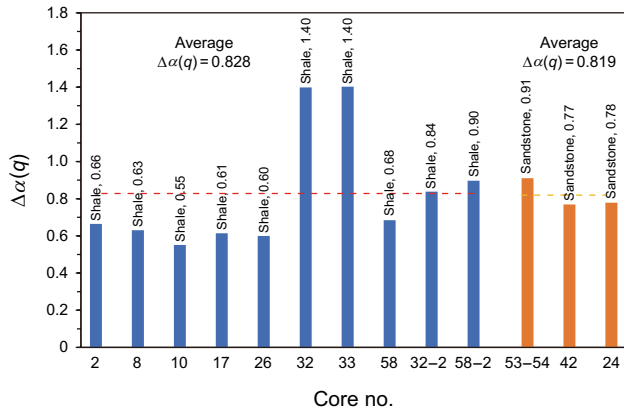


Fig. 6 Comparison of $\Delta\alpha$ between shale and sandstone core samples

As shown in Figs. 3 and 4, the mass exponent diagram and the multifractal spectra of shale 32 and shale 33 are significantly different from other core samples. When $q < 0$, the generalized fractal dimension curve and mass exponent curve of these two core samples are clearly separated from other core samples. The $\Delta\alpha(q)$ values of the cores 32 and 33 are 1.40, nearly twice the value of other core samples, as shown in Fig. 6. The higher the value of $\Delta\alpha(q)$ is, the more complex the pore size distribution inside the core sample will be, and the internal difference of the pore space will become greater. Therefore, the multifractal characteristics of cores 32 and 33 are discussed in detail. The study shows that the $\Delta\alpha(q)$ value decreases with the increase in the specific surface area. Corresponding to the higher values $\Delta\alpha$ of the cores 32 and 33, their specific surface areas are significantly smaller than other core samples. Comparing the specific surface area, total pore volume, and R_d value of each core sample, it is found that the total pore volumes of cores 32 and 33 are the smallest, as shown in Fig. 7. Therefore,

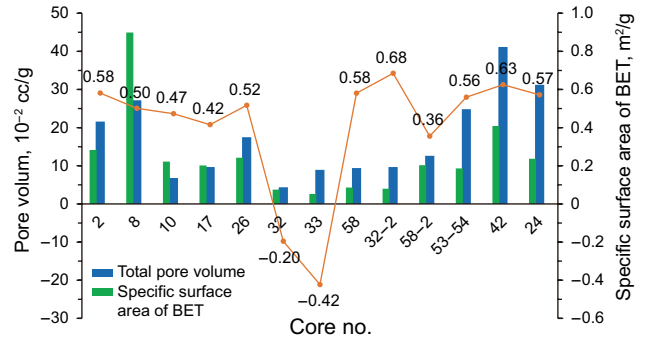


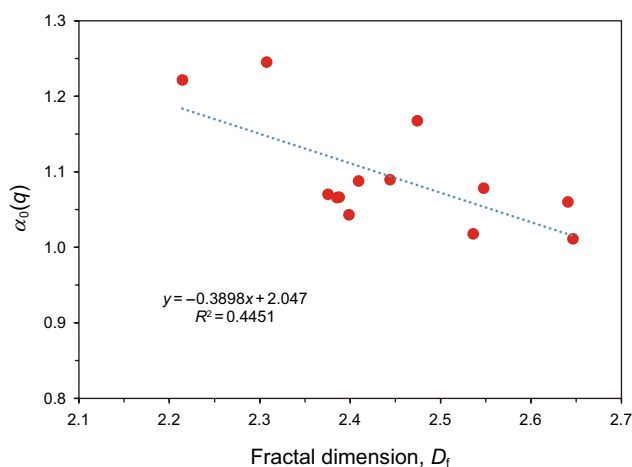
Fig. 8 Comparison of specific surface area, total pore volume and R_d

the values of $\Delta\alpha(q)$ have relations with the total pore volume and specific surface area.

Figure 8 shows the correlations between the specific surface area, total pore volume, and R_d . For shales, it is usually believed that the pores with the radius less than 2 nm are micropores, and the pores with the radius between 2 and 50 nm are mesopores, and the pores with the radius greater than 50 nm are macropores (Loucks et al. 2012). As the minimum pore radius measured by the N₂GA experiment is about 3 nm, the measured pores are mainly mesopores and macropores. Table 4 shows that the pores of the cores 32 and 33 are mainly mesopores, and their proportions of the large pores are low, 10% and 6%, respectively. The R_d values of the cores 32 and 33 are negative, indicating the low-value information of pore sizes has a great influence on multifractal spectra, which is consistent with the pore radius distribution. The R_d values of the cores 58 and 32–2 are positive and large, indicating that the high-value information has great influence on multifractal spectra, and it is consistent with that their proportions of macropores are large, around 30%. The proportions of macropores, R_d values, and specific surface areas of all core samples are compared. It can be found

Table 4 The proportion of macropores, R_d , and specific surface area of core samples

Core type	Core no.	R_d value	Proportion of macropores, %	Specific surface area, m^2/g
Cores with small specific surface area	32	-0.2	18.04	0.075
	33	-0.42	5.4	0.052
	58	0.58	29.95	0.086
	32-2	0.68	31.94	0.08
Cores with small proportion of macropores	2	0.58	8.26	0.283
	8	0.5	5.01	0.899
	10	0.47	3.23	0.222
	17	0.42	5.95	0.202
	26	0.52	12.54	0.242
	58-2	0.36	14.23	0.204


Fig. 9 The correlation between the fractal dimension calculated from FHH model and multifractal parameters $\alpha_0(q)$

that when the specific surface areas are similar, the larger the proportion of macropores is, the larger the R_d value will be. When the proportion of macropores is small, R_d value increases as the specific surface area increases.

The conventional fractal dimensions of these core samples have been calculated with Frenkel–Halsey–Hill (FHH) model based on N_2 GA (Li et al. 2019). Comparing the fractal dimension with the parameters of multifractal characteristics, it is found that the fractal dimension calculated from FHH model has a negative linear correlation with $\alpha_0(q)$, as shown in Fig. 9. As $\alpha_0(q)$ reflects the fluctuation of pore size distributions, the larger the $\alpha_0(q)$ value is, the stronger the heterogeneity of pore size distribution is, and the proportion of macropores also increases.

Figure 10 shows the correlations between the multifractal parameters and the parameters of core samples calculated

from N_2 GA. It can be found that as the specific surface area increases, the width of the multifractal spectra $\Delta\alpha(q)$ (Fig. 10a), the maximum singularity index $\alpha_0(q)$ (Fig. 10b), and the width of the right side of the multifractal spectra $\alpha_2(q)$ (Fig. 10c) all decrease, which means the complexity of pore space distribution decreases, and the pore size distribution becomes more concentrated, and the influence of low-value information on the multifractal spectra decreases. As shown in Fig. 10d, the average pore radius has a good correlation with $\alpha_1(q)$. As the pore radius increases, $\alpha_1(q)$ increases, indicating the increase in the influence of the high-value information on multifractal spectra. Figure 10e shows that as the adsorption pore volume increases, R_d increases logarithmically, and the influence of high-value information on multifractal spectra increases. Figure 10f depicts that the correlation between $\alpha_2(q)$ and adsorption pore volume is weak. As the adsorption pore volume increases, $\alpha_2(q)$ decreases logarithmically, and the influence of low-value information on multifractal spectra decreases.

5 Multifractal characteristics of pore structures with NMR

The multifractal spectra obtained using the NMR spectra are shown in Fig. 11. The parameters of the multifractal spectra are shown in Table 5. It can be seen that the multifractal spectra of the three plunger sandstones are right-skewed, and the R_d value ranges from -0.42 to -0.09, indicating the low-value information has a great influence on the multifractal spectra of NMR. As there are segmentations in NMR spectra of shale samples, the calculated multifractal spectra do not have obvious multifractal characteristics. The R_d value of sandstone core 53 and other shale samples varies from -0.12 to 0.13, and their multifractal spectra are closer to symmetrical bells.

Table 5 shows that the values of $\Delta\alpha(q)$, $\alpha_1(q)$ and $\alpha_2(q)$ of sandstone are larger than those of shale. It means that compared with shale, tight sandstone has a wider pore size distribution, and more low-value information and high-value information. According to Liu et al. (2018), the clay mineral composition has a certain influence on the multifractal characteristics. As shown in Table 5, the mineral compositions of the tight sandstones and shales have been measured with X-ray diffraction analysis (XRD) and introduced in the previous study (Li et al. 2019). It can be found that there are significant differences in the clay content and quartz content of shale and sandstone. The clay content for shales ranges from 24.8 to 35.0%, while that of tight sandstones changes from 13.3 to 19.8%. The shale quartz content is between 23.3 and 37.2%, and less than that in sandstones, which varies between 60.9 and 61.3%. The differences in clay content and quartz content lead to more small pores in shale, and

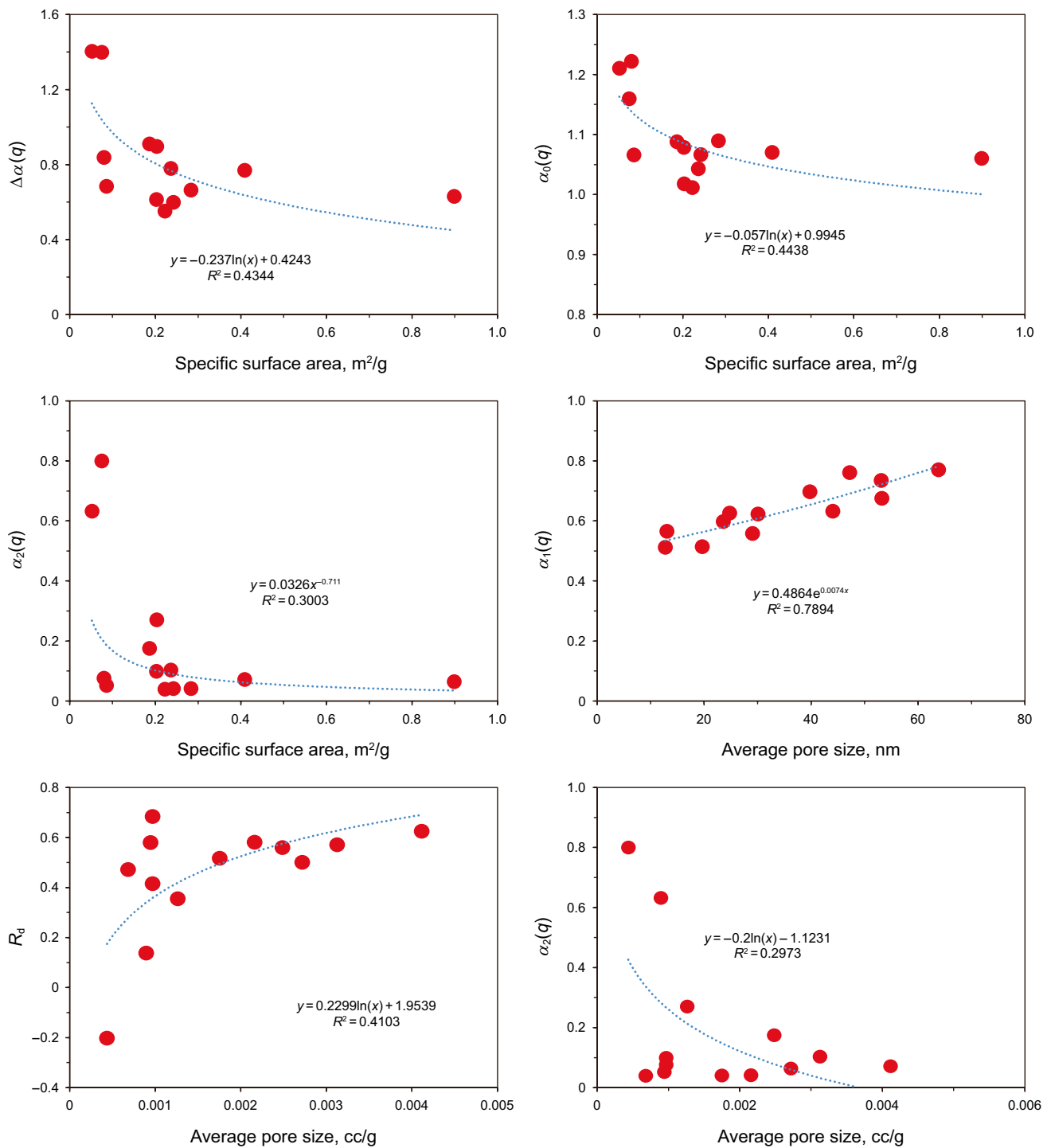


Fig. 10 Relationships between parameters of core samples and the multifractal parameters obtained by N₂GA (a, b, c is the relation curve between specific surface area and the width of the multifractal spectra $\Delta\alpha(q)$, the maximum singularity index $\alpha_0(q)$, and the width of the right side of the multifractal spectra $\alpha_2(q)$, respectively; d, e, f is the relation curve between average pore size and the width of the left-skewed of the multifractal spectra $\alpha_1(q)$, the value of R_d and the width of the right-skewed of the multifractal spectra $\alpha_2(q)$, respectively)

some larger pores in sandstones. Therefore, the $\Delta\alpha$ value of tight sandstone is higher and its heterogeneity of pore size distribution is stronger.

Comparing NMR multifractal spectra and N₂GA multifractal spectra of the sandstone cores 24 and 53–54, it can be found that the multifractal spectra obtained from NMR is wider than those of N₂GA, and the maximum singularity

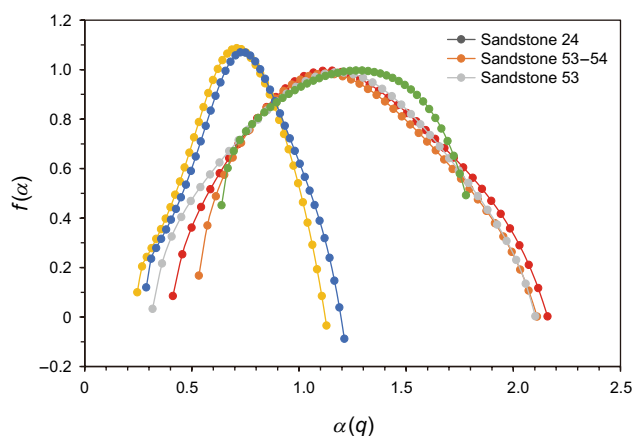


Fig. 11 The multifractal spectra calculated from NMR spectra

index of NMR multifractal spectra is larger, indicating that the pore size distribution obtained by NMR are more complicated and wider. The reason is that NMR can obtain the full-size distribution of pore sizes, from nanoscale pores to microscale pores. However, N₂GA cannot characterize the microscale pores in tight sandstone, and the pore information obtained by N₂GA only contains the nanoscale pores. In addition, the deviation direction of the NMR multifractal spectra is opposite to that of N₂GA multifractal spectra. The NMR multifractal spectra are right-skewed, meaning the low-value information has a great influence on it, but the N₂GA multifractal spectra are left-skewed, indicating it is more affected by high-value information.

6 Conclusions

The multifractal characteristics of shale and tight sandstone pore structures from the Chang 7 member of the Triassic Yanchang Formation in the Ordos Basin, NW China are studied with N₂GA and NMR experiments. The following conclusions can be drawn:

- The multifractal parameters $\Delta\alpha(q)$ and $\alpha_0(q)$ can reflect the characteristics of pore size distributions in shale and tight sandstone, and can be used to characterize the complexity of pore size distribution. The increases of $\Delta\alpha(q)$ and $\alpha_0(q)$ indicate the pore size distributions of shale and tight sandstone become more complicated.
- The specific surface area and total pore volume of shale and tight sandstone are correlated with multifractal characteristics. When the specific surface area is similar, the larger the proportion of macropores is, the larger the R_d value, and the greater the effect of high-value information on multifractal characteristics will be. For the core samples with the small proportion of macropores, the value of R_d increases with the increase in the specific surface area.
- The specific surface area is negatively correlated with the width of the multifractal spectra $\Delta\alpha(q)$ and the maximum singularity index $\alpha_0(q)$. $\alpha_1(q)$ increases as pore size increases, and the influence of high-value information on the multifractal spectra increases. With the increase in total pore volume, R_d increases logarithmically and the influence of the high-value information on multifractal spectra will increase.
- The multifractal parameters obtained from NMR and N₂GA tests are different from each other. The values of R_d measured from NMR spectra are negative, contrary to the results obtained from N₂GA tests, which might be caused by the different pore size ranges characterized by these two different experiments. As tight sandstone usually has wider pore size distribution and higher porosity than shale, the values of $\Delta\alpha(q)$, $\alpha_1(q)$ and $\alpha_2(q)$ of tight sandstone measured from NMR spectra are greater compared with those of shale.

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Table 5 The multifractal parameters calculated from NMR spectra

Core no.	$\Delta\alpha(q)$	$f(\alpha)_{\max}$	$f(q)_{\min}$	$\alpha_0(q)$	$\alpha_1(q)$	$\alpha_2(q)$	R_d
Sandstone 24	1.75	1.00	0.00	1.11	0.70	1.05	-0.36
Sandstone 53-54	1.58	1.00	0.00	1.11	0.58	1.00	-0.42
Sandstone 53	1.79	1.00	0.00	1.17	0.85	0.94	-0.09
Shale 32	1.06	1.09	-0.04	0.74	0.49	0.57	-0.08
Shale 58	0.98	1.09	-0.09	0.72	0.43	0.55	-0.12
Shale 1	1.14	1.00	0.45	1.28	0.64	0.50	0.13

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