Petroleum Science 19 (2022) 2663–2674

Contents lists available at ScienceDirect

Petroleum Science

journal homepage: www.keaipublishing.com/en/journals/petroleum-science

Original Paper

# An improved method for internal multiple elimination using the theory of virtual events

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ARTICLE INFO

*Article history:*
Received 29 September 2021
Received in revised form
20 March 2022
Accepted 29 May 2022
Available online 3 June 2022

Edited by Jie Hao

*Keywords:*
Internal multiple
Convolution factor
Elimination
Virtual events

ABSTRACT

Compared with first-order surface-related multiples from marine data, the onshore internal multiples are weaker and are always combined with a hazy and occasionally strong interference pattern. It is usually difficult to discriminate these events from complex targets and highly scattering overburdens, especially when the primary energy from deep layers is weaker than that from shallow layers. The internal multiple elimination is even more challenging due to the fact that the velocity and energy difference between primary reflections and internal multiples is tiny. In this study, we propose an improved method which formulates the elimination of the internal multiples as an optimization problem and develops a convolution factor *T*. The generated internal multiples at all interfaces are obtained using the convolution factor *T* through iterative inversion of the initial multiple model. The predicted internal multiples are removed from seismic data through subtraction. Finally, several synthetic experiments are conducted to validate the effectiveness of our approach. The results of our study indicate that compared with the traditional virtual events method, the improved method simplifies the multiple prediction process in which internal multiples generated from each interface are built through iterative inversion, thus reducing the calculation cost, improving the accuracy, and enhancing the adaptability of field data.


## 1. Introduction

Multiples refer to seismic waves that undergo multiple downward reflections. If they are not appropriately removed, erroneous seismic phases may appear in seismic images. During seismic data processing, the filtering approaches based on normal moveout and periodicity difference between primary reflections and multiples (Robinson, 1957; Hampson, 1986; Foster and Mosher, 1992; Shi and Wang, 2012; Sun et al., 2019), as well as the prediction subtraction method based on wave equation (Verschuur et al., 1992; Berkhout and Verschuur, 1997; Shi et al., 2010; Ypma and Verschuur, 2013; Lopez and Verschuur, 2015; Vrolijl et al., 2017; Zhang et al., 2020) are developed to eliminate surface-related multiples. However, when the normal moveout and periodicity difference between primary reflections and internal multiples are small, especially for the internal multiples generated from complex geological settings, these methods may not work effectively. Furthermore, the removal of the internal multiples may suffer from greater difficulties and challenges due to the fact that internal multiples can be randomly generated by any reflector in the subsurface with unknown position and reflectivity. A large number of approaches are developed to utilize the prediction subtraction strategy based on wave equation to eliminate the internal multiples, and they can be roughly divided into three categories: the inverse scattering series (ISS) approach, the derivation approach of surface-related multiple elimination (SRME), and the Marchenko multiple elimination (MME) method.

Weglein et al. (1997) proposed the ISS method which can accurately predict internal multiples. The approach can also predict given-order internal multiples each time without prior information of the subsurface, which is optimal for predicting internal multiples

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https://doi.org/10.1016/j.petsci.2022.05.021
1995-8226/

in the absence of structural information or subsurface velocity when other primary reflections or multiple discriminators are unavailable. For the purpose of tackling the problem of expensive computation, Jin et al. (2008) improved the 1D and 1.5D internal multiple prediction to estimate the calculation cost. Fu and Weglein (2014) applied the 1.5D elastic ISS internal multiple elimination approach on on-shore field data. Although the method has been employed in some previous investigations, it is often limited by the problem of static parameters that could not deal with the interface of scattering waves with long lateral extension. In order to address this problem, Innanen (2017) proposed ISS nonstationary parameters based on 1D and 1.5D versions. However, the defect of large computational cost still exists. Thus, it cannot be widely applied to 2D or 3D prestack data and it is difficult to predict internal multiples with far offsets.

Jakubowicz (1998) proposed a method to construct internal multiples, which skillfully moved subsurface scattering point to the surface and has obtained excellent results in synthetic data; unfortunately, this method requires water-bottom-related primaries that should be identified and isolated. Berkhout and Verschuur (2005); Verschuur and Berkhout (2005) removed internal multiples with the common-focus-point (CFP) approach which is extended from surface-related multiple elimination (SRME). The CFP theory downward extrapolates seismic records to a subsurface interface and predicts the corresponding internal multiples. Therefore, velocity information is required to obtain the accurate focus operator for its implementation. Because of the approximate nature of predicted internal multiples, both ISS and CFP schemes need adaptive subtraction to achieve a multiple-attenuated data set. Based on the estimation of primaries by sparse inversion (EPSI), Ypma and Verschuur (2013) predicted wavelets and internal multiples simultaneously. Lin and Herrmann (2013) proposed a robust estimation of primaries by sparse inversion. This method is able to avoid the adaptive subtraction step and effectively protects primary reflections, as tested using field data (Czyczula-Raudjord et al., 2015). However, the wavelet estimation of this method is unstable and the calculation amount is large. Wang (2004) first proposed the concept of multiple prediction through inversion (MPI) and Wang (2007) further illustrated the theoretical advancements of MPI method which can eliminate internal multiples to some extent.

Recently, the Marchenko equation approach to eliminate internal multiples has been developed rapidly (Slob et al., 2014; Wapenaar et al., 2014; Meles et al., 2015; Thorbecke et al., 2017, 2021). Singh et al. (2015, 2017) added the surface-related multiple generation term to the Marchenko equation, indicating that surface-related multiples and internal multiples can be eliminated simultaneously. Filho et al. (2017) introduced the Marchenko internal multiple elimination method into elastic media. Zhang and Slob (2019) developed a scheme to retrieve primary reflections from seismic data by eliminating the free-surface and internal multiple reflections in one step, without model information and adaptive subtraction. The Marchenko multiple elimination method has also been applied in field data (Staring et al., 2018, 2021; Zhang and Slob, 2020). However, the Marchenko method requires high-density seismic data acquired by full sampling, and better results are usually obtained only when the shot spacing and trace spacing are less than 10 m, which is difficult to realize for field data due to the acquisition and computation cost.

In this study, we formulate the elimination of the internal multiples as an optimization problem and develop an iterative solver. Our approach combines the multiple predictions through inversion and the theory of virtual events to construct an initial multiple model, in which all internal multiples can be produced by iteration. There is no requirement for the high-density acquisition geometry in the proposed approach. Moreover, it can avoid the construction of internal multiples in each layer. As a result, it not only powerfully improves the calculation efficiency of internal multiple prediction and elimination but also reduces cumulative prediction-and-subtraction error, thus enhancing the adaptability of field data. Through several synthetic experiments, we have determined that the proposed method can effectively eliminate the internal multiples.

The paper is organized as follows. After the introduction, we briefly review the theory of virtual events and the inversion approach for the elimination of internal multiples. Then, we introduce the convolution factor $T$ for the virtual events approach. Finally, we utilize several synthetic experiments to validate the effectiveness of our approach and draw the corresponding conclusions.

## 2. Theory

### 2.1. The theory of virtual events

The theory of virtual events (VE) is distinguished by the ability of moving subsurface scattering point to surface, which allows us to construct internal multiples with surface seismic data. In addition, the method does not require any information of the subsurface or explicit information of specific interfaces that generates internal multiples. Ikelle (2006) and Ikelle et al. (2009) proposed an internal multiple elimination method using the virtual seismic events theory whereby virtual events can be obtained by primary reflections from different interfaces. It supposes that $P_0(x_S, x_R)$ is the primary reflection related to a specific interface in the subsurface. $P^-(x_S, x_R)$ is the primary reflection from other interfaces below the specific interface, suggesting that the reflection records associated with interfaces above the special interfaces are removed. The virtual events $P_V(x_S, x_R)$ can be constructed by the convolution of $P^-(x_S, x_R)$ and $P_0^H(x_S, x_R)$, the process of which is expressed as follows:

$$P_V(x_S, x_R) = \int P^-(x_S, x) P_0^H(x, x_R) dx \tag{1}$$

where $x_S$ and $x_R$ are the source and receiver positions of shot records, $x$ is the horizontal coordinate for the summation. Summation along the $x$ coordinate indicates that all ray path combinations are considered. “H” denotes the complex conjugation or time inversion, $P_0^H(x_S, x_R)$ is the time inversion of $P_0(x_S, x_R)$. The process of forming virtual events using the common shot point gather $P^-(x_S, x)$ of $P^-(x_S, x_R)$ and the common receiver gather $P_0^H(x, x_R)$ of $P_0^H(x_S, x_R)$ is shown in Fig. 1a and Fig. 2a.

Internal multiples can be constructed by the convolution of virtual events $P_V(x_S, x_R)$ related to the specific interface and primary reflection $P^-(x_S, x_R)$, which is expressed as:

$$M_I(x_S, x_R) = \int P_V(x_S, x) P^-(x, x_R) dx \tag{2}$$

where $M_I(x_S, x_R)$ is the internal multiples of all orders associated with the specific interface, the construction of $M_I(x_S, x_R)$ is shown in Figs. 1b and 2b, which will be used as the initial model of internal multiples in our method, and we obtain internal multiples generated at all interfaces through iteration. The detailed derivation is as follows. $P_V(x_S, x)$ is the common shot point gather and $P^-(x, x_R)$ is the common receiver gather.

In Fig. 1, internal multiples corresponding to interface A is constructed, where $P_0(x_S, x_R)$ represents the primary reflection from interface A, $P_0^H(x_S, x_R)$ is the time inversion of $P_0(x_S, x_R)$, and $P^-(x_S, x_R)$ is other primary reflection besides $P_0(x_S, x_R)$. At the initial

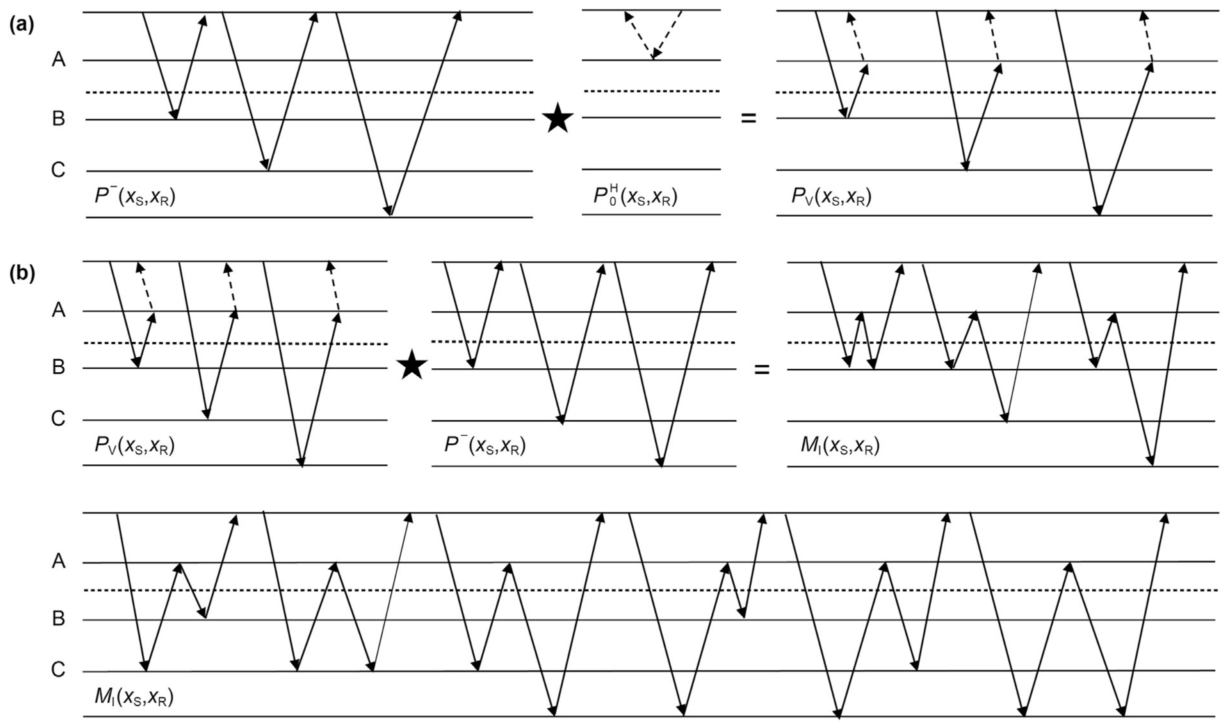


**Fig. 1.** Constructed internal multiples generated by interface A. (a) An illustration of constructing virtual events generated by interface A. (b) Constructing internal multiples associated with interface A using virtual events and primaries.

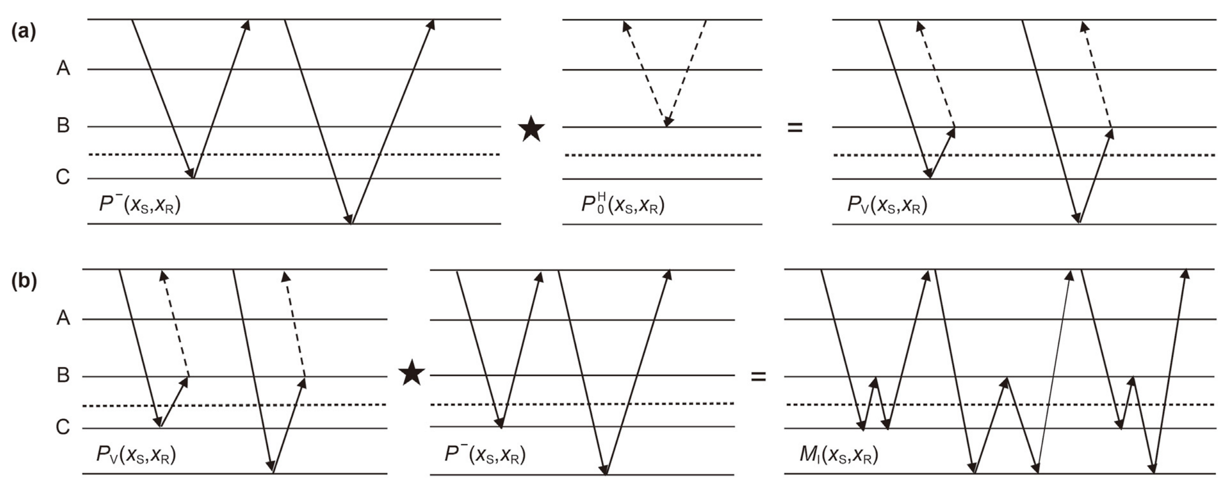


**Fig. 2.** Constructed internal multiples generated by interface B. (a) An illustration of the construction of virtual seismic data corresponding to interface B. (b) Constructing internal multiples associated with interface B using virtual events and primaries.

stage, the virtual events $P_V(x_S, x_R)$ related to interface A is constructed by the convolution of $P^{-}(x_S, x_R)$ and $P_0^H(x_S, x_R)$. Then, internal multiples $M_I(x_S, x_R)$ related to interface A is constructed by the convolution of virtual events $P_V(x_S, x_R)$ and the primary reflection $P^{-}(x_S, x_R)$. Based on the established virtual events, the internal multiples corresponding to only a specific interface can be constructed at once. In order to eliminate internal multiples from the next interface B, the same procedure must be implemented, as shown in Fig. 2, where $P_0(x_S, x_R)$ is the primary reflection from interface B, $P_0^H(x_S, x_R)$ is its time reversal and $P^{-}(x_S, x_R)$ is other primary reflection in addition to the primary reflection generated above interface B. Internal multiples $M_I(x_S, x_R)$ and the virtual events $P_V(x_S, x_R)$ are related to interface B. After the internal multiples of each layer are constructed, they will be subtracted from the original data successively using the adaptive subtraction method. For field data from complex subsurface structures, the limitations include not only the costly calculation expense, but also the difficulty in achieving accurate primary reflection extraction and the construction of multiples. These problems have greatly reduced the practicality of VE. In our study, we introduce the convolution factor $T$ to VE, the step of which can employ internal multiples $M_I(x_S, x_R)$ associated with a specific interface as the initial model to obtain all orders internal multiples of all interfaces through iteration.

 

### 2.2. Internal multiple elimination by inversion

Based on internal multiple elimination using the virtual events proposed by Ikelle (2006) and Ikelle et al. (2009), we provide an improved internal multiple elimination method assuming that seismic data after the elimination of internal multiples by the $n$th iteration can be obtained by the previous iteration results and the convolution factor. Assume that $\{P(z_{S_0},z_{R_0})\}^n$ and $\{P(z_{S_0},z_{R_0})\}^{n-1}$ respectively express multiple elimination results after the $n$th and ($n$-1)th iterations, and $T^{(n-1)}$ stands for the convolution factor, then:

$$\{P(z_{S_0},z_{R_0})\}^n = T^{(n-1)} \times \{P(z_{S_0},z_{R_0})\}^{n-1} \tag{3}$$

where $n = 0, 1, 2, \ldots N$ denotes the corresponding iteration times to obtain a more accurate multiple model, $N$ is the maximum iterations. $\{P(z_{S_0},z_{R_0})\}^n$ marks the seismic data after internal multiple elimination by the $n$th iteration in which the subscript $S_0$ and $R_0$ represent the information regarding the shot points and receiver points at surface, respectively. The convolution factor $T^{(n-1)}$ is similar to the surface operator $A(z_{S_k},z_{R_k})$ obtained using CFP. Therefore, internal multiples $\{M(z_{S_0},z_{R_0})\}^n$ for the $n$th iteration can be expressed as follows:

$$\begin{aligned}\{M(z_{S_0},z_{R_0})\}^n &= T^{(n-1)} \times \left(P(z_{S_0},z_{R_0}) - \{P(z_{S_0},z_{R_0})\}^{n-1}\right)\\ &= T^{(n-1)} \times \{M(z_{S_0},z_{R_0})\}^{n-1}\end{aligned} \tag{4}$$

where $P(z_{S_0},z_{R_0})$ contains all seismic measurements.

According to the CFP internal multiple elimination approach (Berkhout and Verschuur, 2005), multiple-eliminated seismic data related to $z \le z_k$ can be shown by the subtraction of multiple-eliminated seismic data related to $z \le z_{k-1}$ and internal multiples from $z_k$. Internal multiples from $z_k$ can also be written as the surface operator and the seismic data in which all related reflections are removed for $z \le z_k$ and $z \le z_{k-1}$ respectively. The procedure is expressed by:

$$\begin{aligned}&\{P(z_{S_0},z_{R_0})\}_k = \{P(z_{S_0},z_{R_0})\}_{k-1} - \{\delta M(z_{S_0},z_{R_0})\}_k\\ &= \{P(z_{S_0},z_{R_0})\}_{k-1}\\ &-\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k A(z_{S_k},z_{R_k})\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}\end{aligned} \tag{5}$$

where $k = 0, 1, 2, \ldots K$ marks the corresponding interfaces, and $K$ is the maximum number of interface. $\{P(z_{S_0},z_{R_0})\}_k$ and $\{P(z_{S_0},z_{R_0})\}_{k-1}$ denotes that all multiples related to $z \le z_k$ and $z \le z_{k-1}$ have been removed respectively. $\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\}_k$ and $\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\}_{k-1}$ represent the seismic data in which all related reflections containing primaries and multiples are removed for $z \le z_k$ and $z \le z_{k-1}$ respectively. Equation (5) also can be rewritten as:

$$\begin{aligned}&\left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_k = \left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_{k-1}\\ &-\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k A(z_{S_k},z_{R_k})\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}\end{aligned} \tag{6}$$

This formula indicates that the internal multiples related to the interface $z_k$ is given by:

$$\{\delta M(z_{S_0},z_{R_0})\}_k = \left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k A(z_{S_k},z_{R_k})\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1} \tag{7}$$

An iterative procedure is adopted by rewriting Equation (6) as

$$\begin{aligned}&\left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_k^n = \left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_{k-1} - \Lambda\\ &\times\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-1} A(z_{S_k},z_{R_k})^{n-1}\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}\end{aligned} \tag{8}$$

Namely

$$\begin{aligned}&\left\{\delta\tilde{M}(z_{S_0},z_{R_0})\right\}_k^n = \Lambda\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-1} A(z_{S_k},z_{R_k})^{n-1}\\ &\times\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}\end{aligned} \tag{9}$$

where $\{\delta\tilde{M}(z_{S_0},z_{R_0})\}_k^n$ is the multiple model related to interface $z_k$ for the $n$th iteration, $\Lambda$ is a shaping operator (Wang, 2004) and Equation (10)

$$\{\delta M(z_{S_0},z_{R_0})\}_k^n = \Lambda\left\{\delta\tilde{M}(z_{S_0},z_{R_0})\right\}_k^n \tag{10}$$

is the eliminated internal multiples related to interface $z_k$ after the $n$th iteration. According to Berkhout and Verschuur (1997), Equation (6) is rewritten as:

$$\begin{aligned}&\left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_k^n = \left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_{k-1}\\ &-\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-1} A(z_{S_k},z_{R_k})^n\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}\end{aligned} \tag{11}$$

The purpose of Equation (11) is to improve the accuracy of $\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\}_k$ through iteration. Namely, the accuracy of internal multiples will be closer to the actual one, and the surface operator $A(z_{S_k},z_{R_k})^n$ can be written as:

$$\begin{aligned}&A(z_{S_k},z_{R_k})^n = \left(\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_K^{n-1}\right)^{-1}\\ &\times\left(\left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_{k-1} - \left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_k^n\right)\\ &\times\left\{\overset{\Delta}{P}(z_{S_k},z_{R_0})\right\}_{k-1}^{-1}\end{aligned} \tag{12}$$

Substituting equation $A(z_{S_k},z_{R_k})^{n-1}$ for the ($n$-1)th iteration in Equation (9), we can obtain the following expressions:

$$\begin{aligned}&\{\delta M(z_{S_0},z_{R_0})\}_k^n = \left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-1}\left(\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-2}\right)^{-1}\\ &\times\left(\left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_{k-1} - \left\{\overset{\Delta}{P}(z_{S_0},z_{R_0})\right\}_k^{n-1}\right)\\ &= \left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-1}\left(\left\{\overset{\Delta}{P}(z_{S_0},z_{R_k})\right\}_k^{n-2}\right)^{-1}\\ &\times(\{P(z_{S_0},z_{R_k})\}_{k-1}) - \left(\{P(z_{S_0},z_{R_k})\}_k^{n-1}\right)\end{aligned} \tag{13}$$

For field data, the inverse of matrix can be calculated by the least square method (Berkhout, 2006), and thus Equation (13) can be established as:

$$\begin{aligned}\{\delta M(z_{S_0}, z_{R_0})\}_k^n &= \left\{\overset{\Delta}{P}(z_{S_0}, z_{R_k})\right\}_k^{n-1}\left(\left\{\overset{\Delta}{P}(z_{S_k}, z_{R_0})\right\}_{k-1}^{n-1}\right)^{\mathrm{H}} \\ &\times\left[\left\{\overset{\Delta}{P}(z_{S_k}, z_{R_0})\right\}_{k-1}^{n-2}\left(\left\{\overset{\Delta}{P}(z_{S_k}, z_{R_0})\right\}_{k-1}^{n-2}\right)^{\mathrm{H}}\right]^{-1} \\ &\times\left(\{P(z_{S_0}, z_{R_0})\}_{k-1} - \{P(z_{S_0}, z_{R_0})\}_k^{n-1}\right)\end{aligned} \tag{14}$$

If we consider all internal multiples, Equation (14) can be written as:

$$\begin{aligned}\{M(z_{S_0}, z_{R_0})\}^n &= \{P(z_{S_0}, z_{R_0})\}^{n-1}\left(\{P(z_{S_0}, z_{R_0})\}^{n-1}\right)^{\mathrm{H}} \\ &\times\left[\{P(z_{S_0}, z_{R_0})\}^{n-2}\left(\{P(z_{S_0}, z_{R_0})\}^{n-2}\right)^{\mathrm{H}}\right]^{-1} \\ &\times\left(P(z_{S_0}, z_{R_0}) - \{P(z_{S_0}, z_{R_0})\}^{n-1}\right)\end{aligned} \tag{15}$$

The convolution factor $T^{(n-1)}$ can be estimated as:

$$\begin{aligned}T^{(n-1)} &= \{P(z_{S_0}, z_{R_0})\}^{n-1}\left(\{P(z_{S_0}, z_{R_0})\}^{n-2}\right)^{\mathrm{H}} \\ &\times\left[\{P(z_{S_0}, z_{R_0})\}^{n-2}\left(\{P(z_{S_0}, z_{R_0})\}^{n-2}\right)^{\mathrm{H}}\right]^{-1}\end{aligned} \tag{16}$$

Subsequently, the expression for eliminating all internal multiples can be formulated as:

$$\{P(z_{S_0}, z_{R_0})\}^n = P(z_{S_0}, z_{R_0}) - \{M(z_{S_0}, z_{R_0})\}^n. \tag{17}$$

The above derivation steps indicate that the improved multiple elimination method depends on the convolution factor $T^{(n-1)}$ rather than the surface operator $A(z_{S_k}, z_{R_k})$. The convolution operator is replaced by the original data $P(z_{S_0}, z_{R_0})$ and the two previous multiple elimination results $\{P(z_{S_0}, z_{R_0})\}^n$ and $\{P(z_{S_0}, z_{R_0})\}^{n-1}$. In this way, the method implicitly considers spatial variation of the surface operator in multiple prediction, and realizes spatial variation of the internal operator in the continuous iterative updating. In addition, the predicted internal multiples are closer to the real value through iteration and the nonlinear problem is reduced for subsequent multiples subtraction.

From the above derivations and analyses, we can see that our approach is realized through the following steps. Firstly, internal multiples of a random interface generated by the virtual events method are used as the initial multiple model including internal multiples of all orders related to a certain interface. Secondly, the spatial variability of the convolution factor $T$ is used to obtain a more precise multiple model including all internal multiples by iteration. Finally, we obtain primaries without internal multiples utilizing subtraction. By contrast, the traditional virtual events method needs a step to construct the internal multiples generated from each layer, extremely complicating the calculation process. Our method realizes the fully data-driven internal multiple elimination and avoids the construction of internal multiples from each interface. The final results after multiple elimination is the subtraction output in the last iteration. It is not a result in which internal multiples have been eliminated gradually through iteration. Generally, after three times of iterations, we can obtain $\{P(z_{S_0}, z_{R_0})\}^n$ and $\{P(z_{S_0}, z_{R_0})\}^{n-1}$, which are adequate for the calculation of convolution factor $T$. The method not only reduces the computational complexity significantly, but also improves the application feasibility in field data.

## 3. Numerical experiments

In this section, we use a horizontal stratified model and a synclinal model to validate the effectiveness of the proposed method.

### 3.1. Horizontally layered example

Fig. 3 displays the horizontal stratified model that has a size of 2000 m × 1200 m. We synthesize surface seismic data using acoustic finite-difference modeling. Fig. 3 also shows the varied values in different layers for acoustic velocity, which are 1500, 2500, 2000, and 3000 m/s respectively. In addition, we add density parameters in forward modeling to generate well developed multiples. The varied values in different layers are 1500, 2500, 1000, 3000 kg/m$^3$. Absorbing boundary conditions are applied at all sides of the model. The time sampling interval is 4 ms, the seismic reflection record is 2.5 s, the grid size is 2.5 m, and the grid dimensions are 801 × 481. A total of 101 sources and 101 receivers are distributed at the top of the model with an interval of 20 m, and the explosive source is a Ricker wavelet with a central frequency of 25 Hz. All direct waves have been eliminated.

Fig. 4 shows one of the computed reflection responses with primaries and internal multiples. Primary 1, primary 2, and primary 3 refer to the primary reflections of three reflectors at 200 m, 600 m, and 800 m respectively, while the remaining events are internal multiples. In the figure, m1 expresses the first-order internal multiples related to interface A, m12 represents the second-order internal multiples related to interface A, m2 stands for the first-order internal multiples related to interface B, and m1+m2 is the second-order internal multiples which are generated by interface A and B, etc.

Fig. 5 exhibits the comparison of shot gathers with internal multiples, shot gathers without internal multiples associated with interface A, and shot gathers without internal multiples related to interface A and B, which eliminates multiples using the traditional virtual events method. Fig. 5a shows shot gather with internal multiples. It can be seen from Fig. 5b that most of the internal multiples produced by interface A have been effectively eliminated, but internal multiples with strong energy related to interface B still exist and need to be further constructed. Fig. 5c shows shot gathers after all multiples have been eliminated. Fig. 6 illustrates the predicted internal multiples reflected from interface A and interface B, respectively. Compared with internal multiples analyzed in Fig. 4, they show good consistency in travel time but differences in phase and amplitude. These differences can be matched and modified in the process of adaptive subtraction.

Fig. 7 displays the seismic record after one, two, and three iterations to eliminate internal multiples respectively, and the output

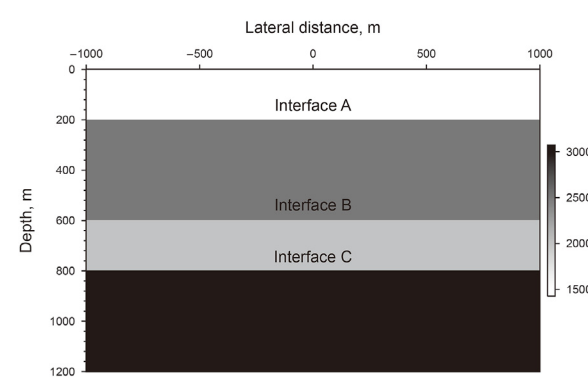


**Fig. 3.** Horizontal stratified velocity model.

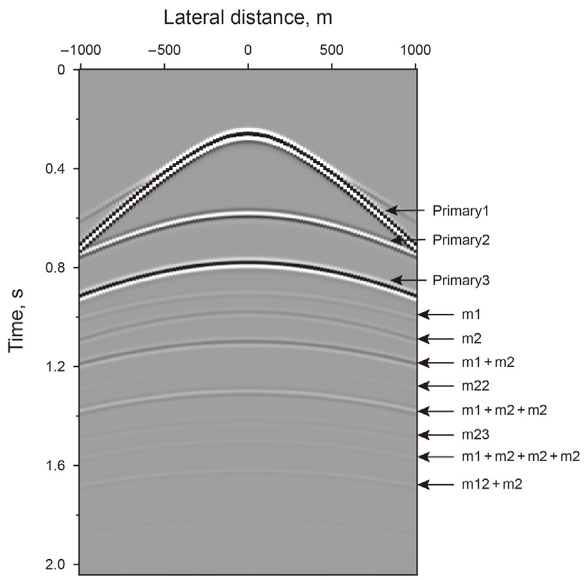


**Fig. 4.** Shot gathers with primaries and internal multiples.

of the third iteration produces a satisfactory result. Fig. 8 shows internal multiples with different iterations. The predicted internal multiples after two or three iterations are more complete, especially for the time windows of 1.0–1.2 s and 1.4–1.6 s, which are closer to the actual internal multiples in terms of phase and energy. Thus, after three times of iterations, the internal multiples are basically successfully predicted and the effect of elimination is perfect.

Fig. 9 shows the comparison of zero-offset traces from Fig. 5a, c, and 7c. The black line originates from the original shot gather shown in Fig. 5a, the red line is obtained from the shot gather with internal multiple elimination shown in Fig. 5c, and the green dashed line is produced by the shot gathers with internal multiple elimination shown in Fig. 7c. The graph suggests that compared with the traditional virtual events method, the improved method in our study can eliminate internal multiples more efficiently, as indicated by the blue arrows.

### 3.2. Synclinal model data example

For the purpose of comprehensively evaluating the performance of this scheme, we show a more complex model in Fig. 10. The velocity model has a size of 4500 m $\times$ 3000 m and consists of a curved interface which forms unconformity with other layers. We compute seismic data using a finite-difference time-domain modeling code and set absorbing boundaries on all sides. A total of 151 shot gathers and 151 traces are distributed on the surface with a uniform lateral interval of 30 m. The source emits a 30 Hz Ricker wavelet and the direct wave is removed.

One representative shot gather graph with internal multiples from forward simulation data is shown in Fig. 11. It is prominent that the deep primaries are interfered by the relatively developed internal multiples. At the beginning, we use the traditional virtual events method to eliminate internal multiples. Fig. 12 displays shot gathers with internal multiple elimination using the traditional virtual events method. In detail, Fig. 12a shows the result after the elimination of internal multiples produced by interface A, Fig. 12b shows the result after the elimination of internal multiples associated with interface A and B, and Fig. 12c shows the result after the elimination of internal multiples related to interface A, B and C. From Fig. 12c, although the vast majority of internal multiples have been eliminated, the residual energy of internal multiples still exists and we can continue to eliminate the multiples at the next interface. Meanwhile, the primaries are impaired severely. Fig. 13 displays the predicted internal multiples related to interface A, interface B and interface C using the traditional virtual events method. We mute the noise associated with time reversal and convolution above the predicted internal multiples.

Internal multiples are eliminated after the first iteration, the first two iterations and the first three iterations and are shown in Fig. 14a, b and c, respectively. Compared with Fig. 12c, the effect of internal multiple elimination in Fig. 14c is more satisfactory. After the three times of iterations, all the internal multiples are basically eliminated and the damage on primaries becomes smaller, especially for the time window between 1.5 and 2.0 s. Fig. 15 shows the predicted internal multiples after the first, first two, and first three iterations, respectively. Owing to the iterative updating of the convolution factor $T$ including time reversal, convolution and other steps, noise will be inevitably generated. To better protect the primaries, we choose a suitable time window to mute the part above

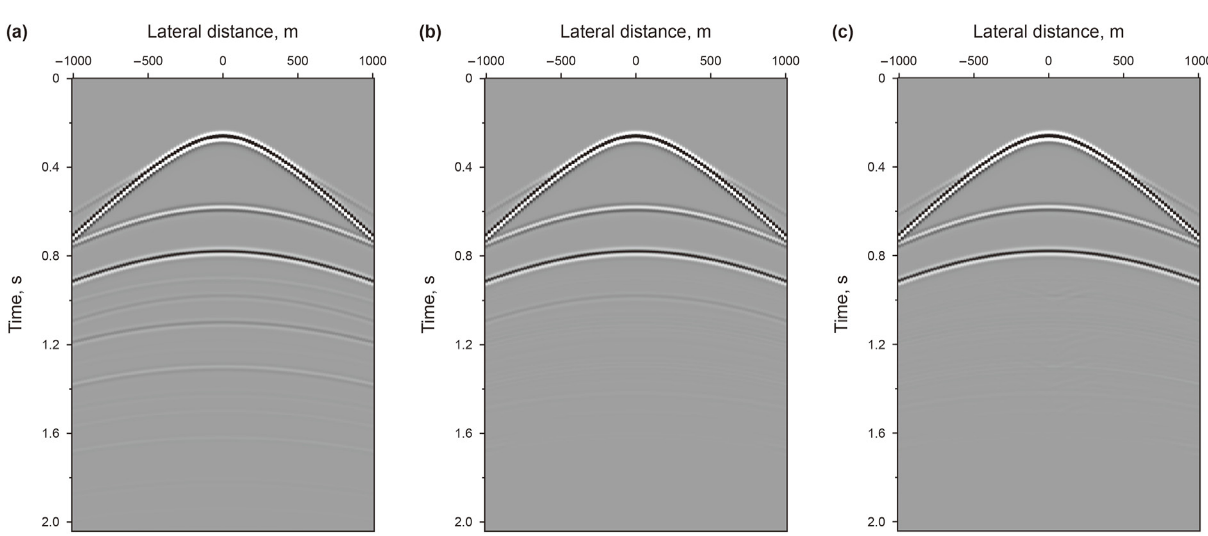


**Fig. 5.** Shot gathers with internal multiple elimination using the traditional virtual events method. (a) Shot gather with internal multiples. (b) Results after the elimination of the internal multiples produced by interface A. (c) Results after the elimination of the internal multiples related to interface A and B.

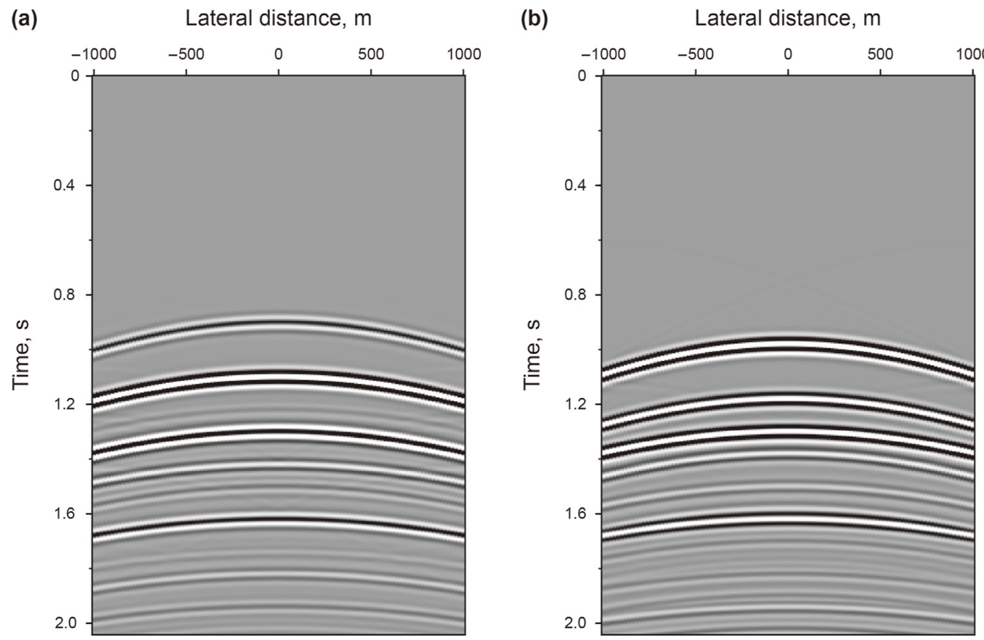


**Fig. 6.** The predicted internal multiples obtained using the traditional virtual events method. (a) The predicted internal multiples produced by interface A. (b) The predicted internal multiples formed from interface B.

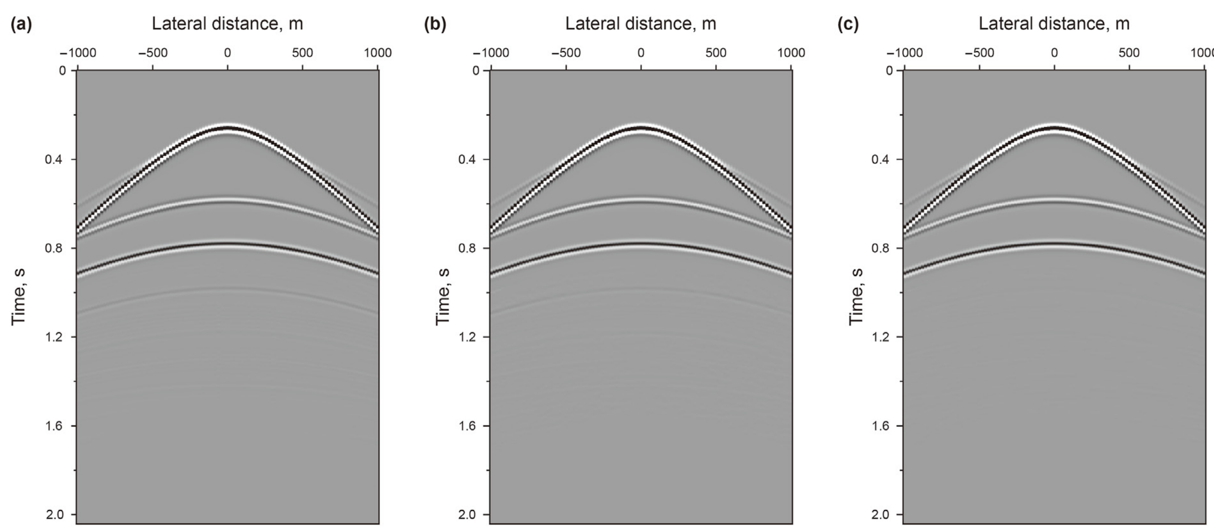


**Fig. 7.** Shot gathers with internal multiple elimination for the first three iterations. (a) Result after the first iteration. (b) Result after the first two iterations. (c) Result after the first three iterations.

the multiples. Fig. 16a shows zero-offset data with internal multiples, Fig. 16b shows the estimated zero-offset primaries obtained after the first iteration, and Fig. 16c shows zero-offset primaries obtained after the first three iterations. Fig. 17 displays the velocity spectrum from the original data set, internal multiple elimination data sets after the first iteration and the first three iterations shown in Figs. 11, 14a and 14c, respectively. It is remarkable that after internal multiples are eliminated, the energy group of primaries on velocity spectrum are more concentrated, and only limited residual energy of multiples exists, indicating that our approach has exerted a better effect than the traditional virtual events method on internal multiple elimination.

## 4. Discussion

In the ‘numerical experiments’ section, first, we use a horizontal stratified model to compare the traditional virtual events method with the improved virtual events method. The results of the traditional virtual events method are shown in Figs. 5 and 6, and the results of the improved virtual events internal multiple elimination method are shown in Figs. 7–9. After that, we use a Synclinal model data example to illustrate the effectiveness of the method. We select predicted internal multiples related to interface A in the internal multiple model of both examples for iteration, due to the fact that the multiples formed from interface A are more abundant (Fig. 1b) and it is also easier for primary extraction.

Compared with the traditional virtual events method, the improved method only needs an initial multiple models which are

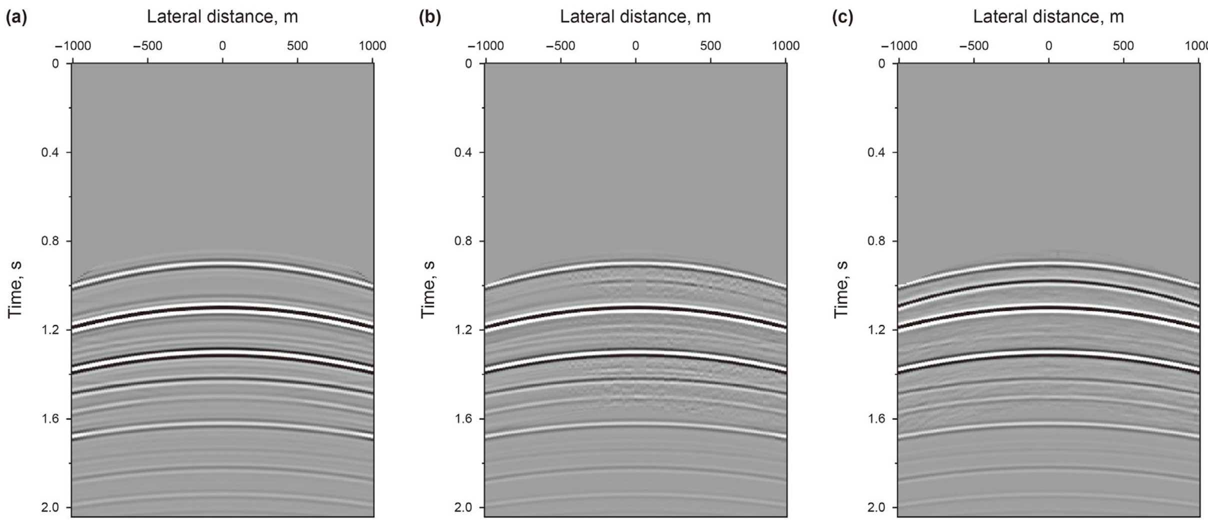


**Fig. 8.** The predicted internal multiples for the first three iterations. (a) The predicted internal multiples after the first iteration. (b) The predicted internal multiples after the first two iterations. (c) The predicted internal multiples after the first three iterations.

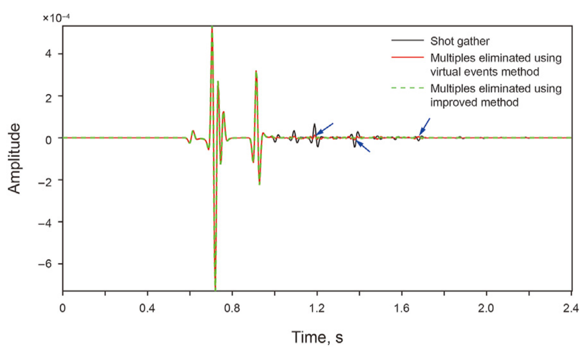


**Fig. 9.** The comparison of zero-offset traces from original and shot gathers with internal multiple elimination data sets.

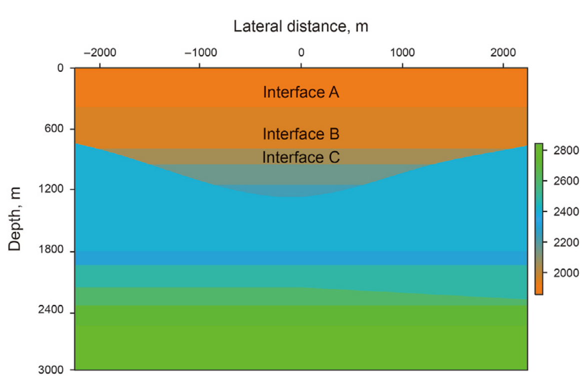


**Fig. 10.** Velocity model for forward simulation.

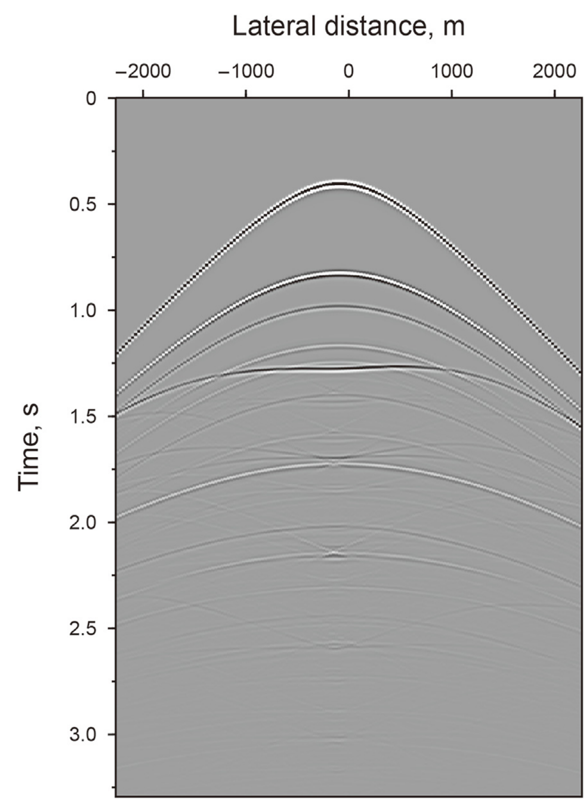


**Fig. 11.** Shot gathers with primaries and internal multiples.

displayed in Fig. 6a or 13a, followed by a step of iteration to obtain all multiples. This not only reduces the dependence on manual operation but also decreases the computational cost and complexity. From the comparison between Figs. 5c and 7c, it is striking that the multiple residuals of Fig. 7c are less than those in Fig. 5c, implying that the adaptive matching filtering method can affect the multiple elimination effect, and the traditional virtual events method may cause error accumulation from surface to subsurface. The same conclusion is obtained in the synclinal model data example. In our study, there is little or no error accumulation and the nonlinear problem existing in adaptive matching filtering which can be solved in the iterative process. In addition, taking the synclinal model for example, the time consumption of our method is 1415 s with three iterations and the value of the traditional virtual events method is 310 s for one interface including 267 s for internal multiple prediction step and 43 s for subtraction step. If the traditional virtual events method is selected to eliminate multiples corresponding to more than five layers, the computational cost will be more expensive. Furthermore, the above time consumption of

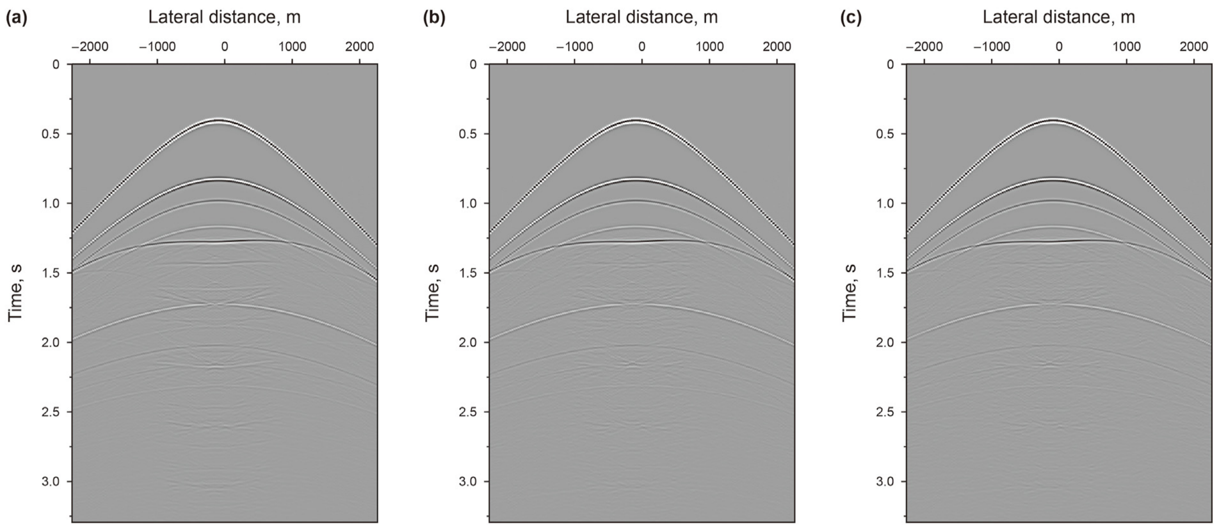


**Fig. 12.** Shot gathers with internal multiple elimination using the traditional virtual events method. (a) Result after the elimination of the internal multiples related to interface A. (b) Result after the elimination of the internal multiples associated with interface A and B. (c) Result after the elimination of the internal multiples related to interface A, B and C.

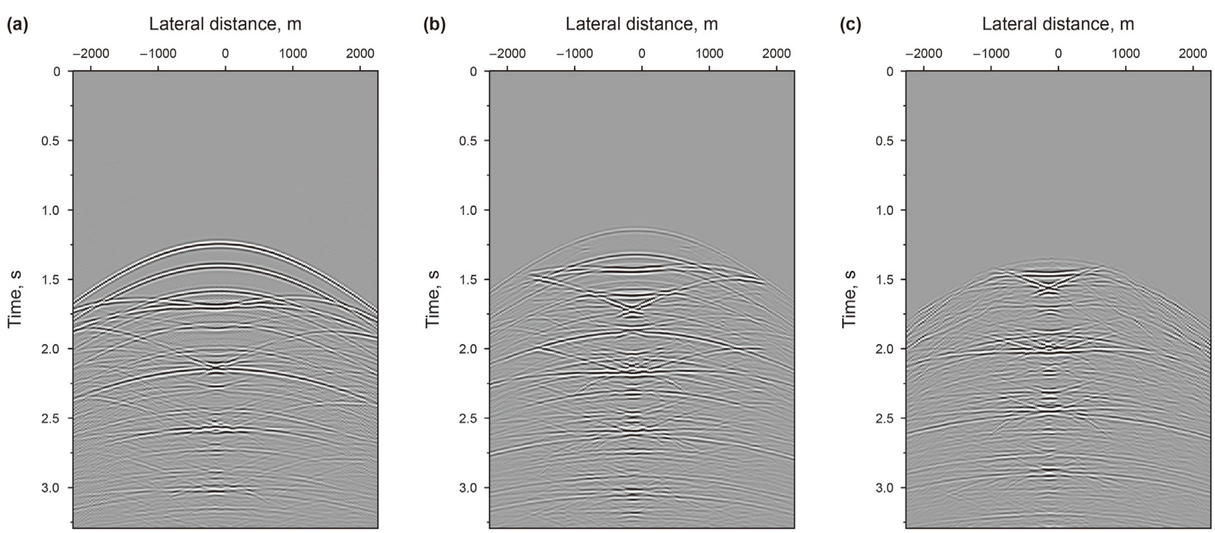


**Fig. 13.** The predicted internal multiples obtained using the traditional virtual events method. (a) The predicted internal multiples generated from interface A. (b) The predicted internal multiples produced by interface B. (c) The predicted internal multiples formed from interface C.

the traditional virtual events method does not include the time-consuming process of manual primary extraction, especially for field data. Therefore, our method has great advantages in the application of field data.

## 5. Conclusion

Internal multiple elimination is one of the most tricky problems in exploration seismology. In our study, the multiple model is established using the virtual events method and the multiple prediction is transformed into an inversion process by the convolution factor $T$, which significantly reduces the calculation amount and the dependence on manual operation. Besides, this method skillfully moves the subsurface scattering point to the surface without explicit information of subsurface structures, realizing the fully data-driven internal multiple elimination. Compared with the traditional internal multiple elimination method, our method reduces the operation complexity, and the accuracy of the internal multiple model is improved both kinematically and dynamically. The approach is feasible in term of single-shot data, zero-offset data, and velocity spectrum through the verification of model data. It demonstrates that our algorithm can separate internal multiples from primaries effectively and practically and exhibit great potential for field data.

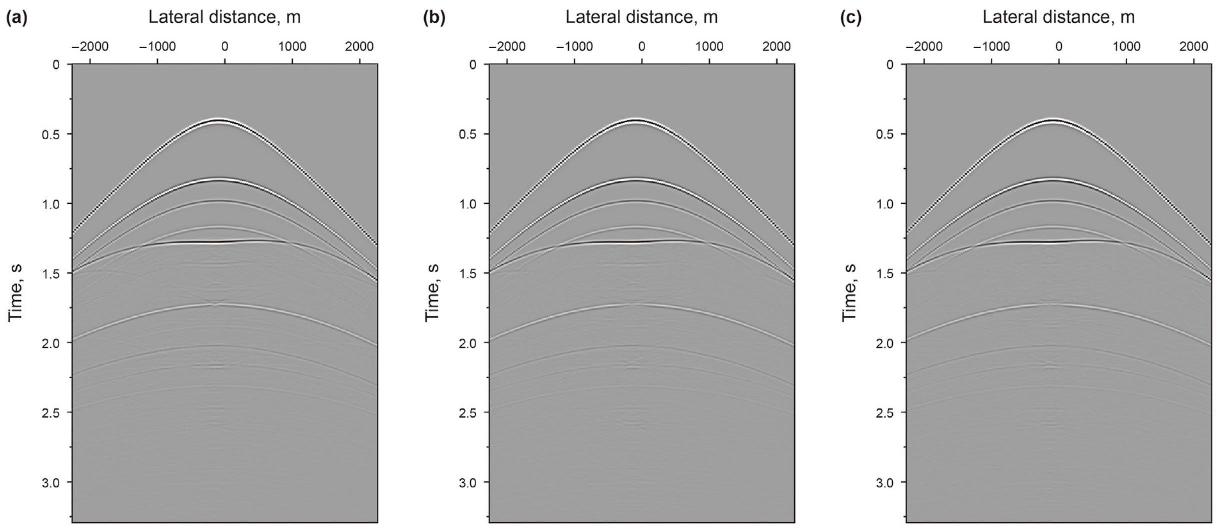


**Fig. 14.** Shot gathers with internal multiple elimination for the first three iterations. (a) Result after the first iteration. (b) Result after the first two iterations. (c) Result after the first three iterations.

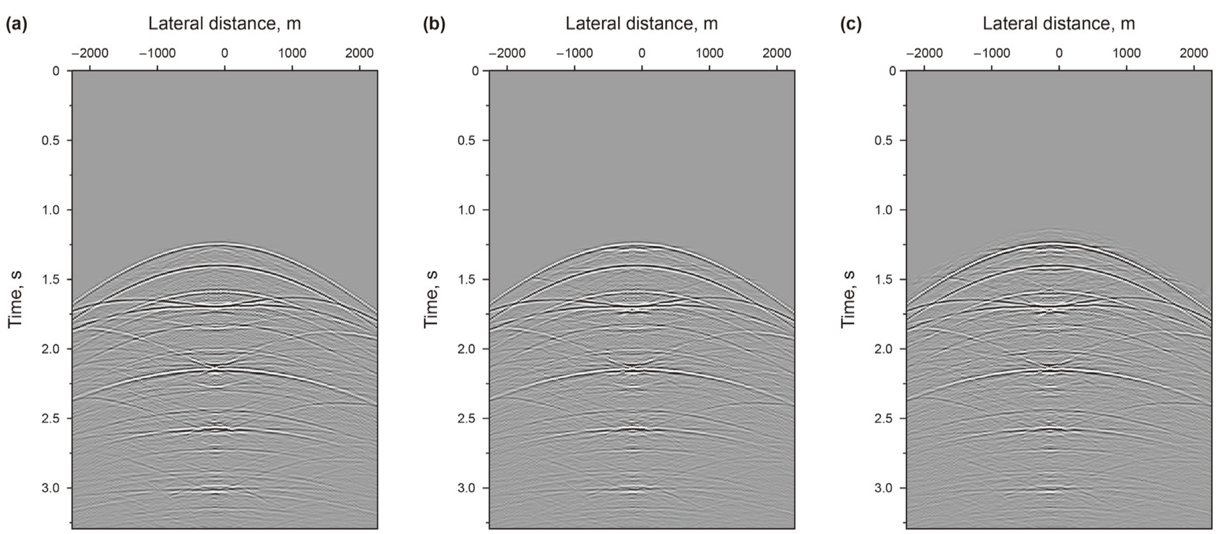


**Fig. 15.** The predicted internal multiples. (a) The predicted internal multiples after the first iteration. (b) The predicted internal multiples after the first two iterations. (c) The predicted internal multiples after the first three iterations.

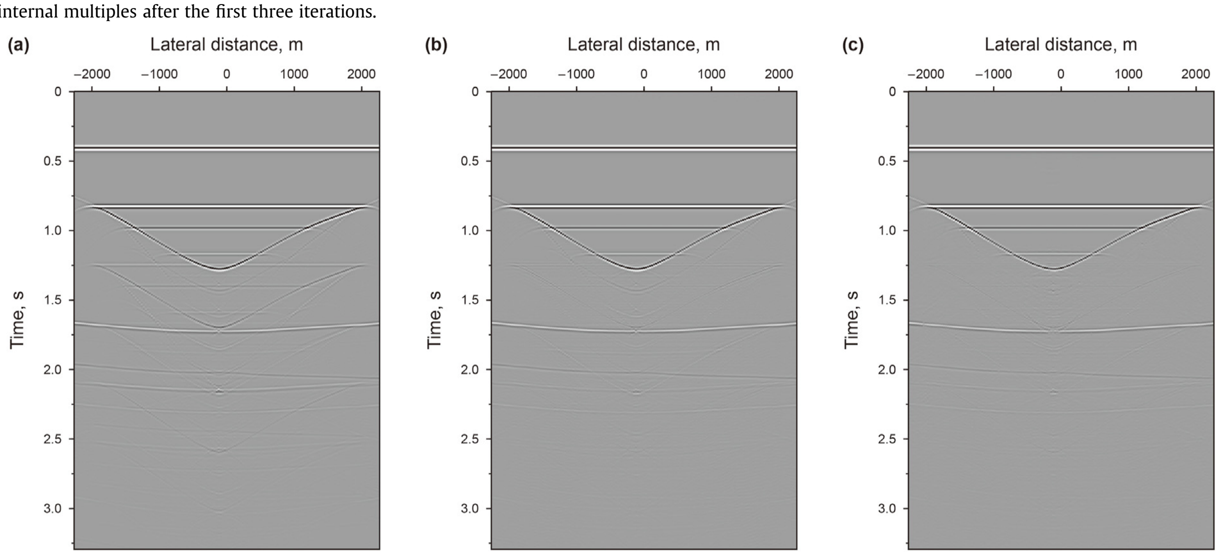


**Fig. 16.** Zero-offset data with internal multiple elimination for the first three iterations. (a) Original zero-offset data with internal multiples. (b) Result after the first iteration. (c) Result after the first three iterations.

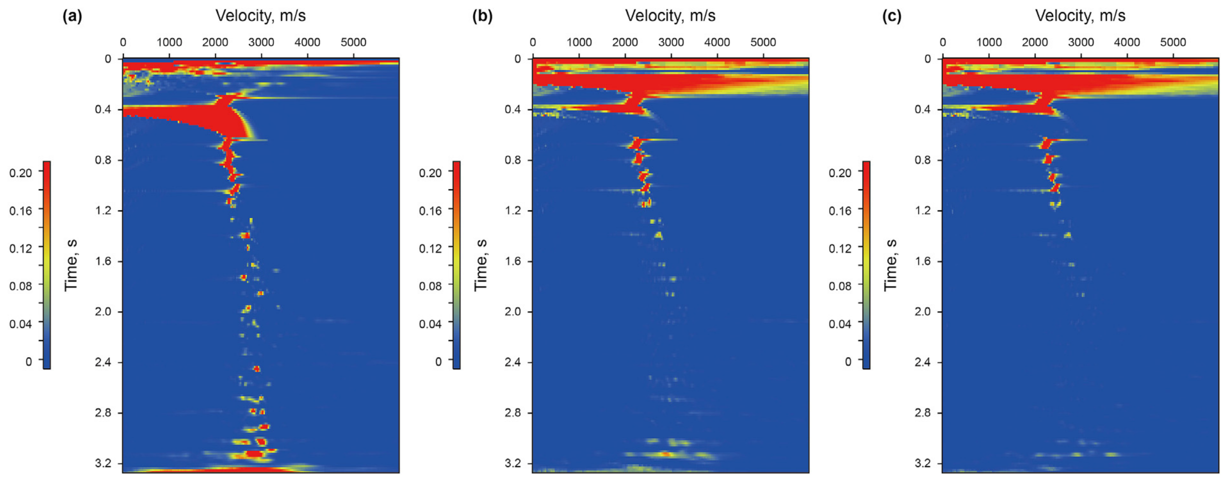


**Fig. 17.** Velocity spectrum with internal multiple elimination for the first three iterations. (a) Velocity spectrum with internal multiples. (b) Velocity spectrum after the first iteration to eliminate internal multiples. (c) Velocity spectrum after the first three iterations to eliminate internal multiples.

## Acknowledgements

We thank the National Natural Science Foundation of China under Grant Nos. 41974116 and 41930431, and Local Universities Reformation and Development Personnel Training Supporting Project from Central Authorities under Grant No. 140119001 for supporting this work.

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