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Adaptive structure-guided impedance inversion using implicit sparse constraint

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ABSTRACT

Structure-guided inversion has been increasingly adopted to enhance structural features and improve the interpretability of seismic impedance imaging. However, the limited bandwidth of seismic wavelets leads to low-resolution poststack records, which inevitably compromises the accuracy of structural estimates in discontinuous regions. Consequently, the success of this approach critically depends on the appropriate use of structural information as geometric constraints. To address this limitation, we propose an adaptive structure-guided approach that maintains large-scale structural continuity while preserving fine-scale variability. The average structural similarity (ASSIM) index and structural constraint sensitivity (SCS) index are introduced to adjust the structural penalty intensity across adjacent traces. The former metric determines the upper bound of penalty intensity for uniform structural constraints, while the latter one characterizes regions prone to distortion where a local reduction in lateral smoothing is required. In addition, the sparse constraint is separated from the objective function and implicitly applied via an iteratively refined reference model, which is generated by an adaptive edge-preserving filter (AEPF) followed by L_2 -TV denoising. This combined strategy promotes blocky impedance reconstruction and effectively preserves thin-layer details. Numerical and field examples demonstrate the superiority of the proposed approach. These improvements establish a more rational and geologically consistent framework for incorporating prior structural information, thereby enhancing the practicality and imaging performance of the structure-guided inversion in complex scenarios.

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1. Introduction

Convolution model-based impedance inversion is a crucial technique for resolving subsurface petrophysical structures and characterizing lithological characteristics from poststack data (Robinson, 1967; Madiba and McMechan, 2003; Wagner et al., 2012; Wang et al., 2020). In this theory, the inversion is inherently a one-dimensional (1-D) inverse problem and is traditionally implemented trace by trace. However, the absence of geometry

constraints crossing adjacent traces usually leads to laterally discontinuous impedances, particularly when data contain high noise levels, thereby compromising subsequent elastic and reservoir interpretations (Wang et al., 2022). To overcome this challenge, the inversion has been gradually extended from the 1-D to two-dimensional (2-D) formulations over the past decade by incorporating 2-D spatial constraints, such as Tikhonov and total variation (TV) regularization schemes, into simultaneous multi-trace impedance reconstruction (Gholami, 2015; Hamid and Pidlisecky, 2015; Li et al., 2018; Zhao et al., 2023). While these 2-D structural constraints generally improve the imaging continuity, they often struggle to describe fine-scale targets, such as faults and inclined strata. As a result, to better uphold the consistency between seismic waveforms and impedance structures, a novel

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class of methods, known as structure-constrained, structure-guided, or structure-oriented inversion, has gained significant popularity for its superior performance in reconstructing complex structures. These methods leverage structural attributes derived from seismic data itself to guide the spatial reconstruction of inverted models (Hamid and Pidlisecky, 2016; Wu, 2017; Hamid et al., 2018; Huang et al., 2021a; Zhang et al., 2021, 2022; Yang et al., 2023; Li et al., 2024). Specifically, the structure-guided inversion reformulates the model gradient calculation to align with directions parallel and perpendicular to local structures. By doing so, it ensures that the 2-D geometric constraint is both physically meaningful and consistent with prior structures.

Most existing structure-guided inversion algorithms impose the uniform structural constraint on all impedances. Assigning larger penalty intensities usually tends to yield more continuous structures, while this enhanced lateral continuity does not necessarily translate to improvements in lateral resolution. This contradiction primarily arises from the inherent inaccuracies in prior structural information, denoted as local slopes or dips. These inaccuracies mainly stem from data noise and structural distortions contained in the data itself. For data noise, denoising algorithms and robust structural estimation algorithms can be applied to mitigate its impact. However, if the structure in poststack data is distorted even in the absence of noise, it becomes more challenging to eliminate such interference. This structural distortion is attributed to the finite bandwidth of seismic wavelets. Although poststack data can intuitively indicate the general pattern of impedance structures, the finite wavelet bandwidth inevitably reduces the spatial and temporal resolution at reflection interfaces with complex structures, thereby resulting in biased structural estimates. This type of inaccuracy can be well suppressed if the dominant frequency of seismic wavelets is sufficiently high. However, in practice, the dominant frequency of seismic wavelet is always limited. Therefore, this limitation remains unavoidable in current structure-guided inversion methods.

For instance, Fig. 1 shows a portion of the Marmousi model, corresponding noisy data with a signal-to-noise ratio (SNR) of 2 dB

simulated using a 30 Hz Ricker wavelet, and local slopes estimated by the plane-wave destruction (PWD) method (Fomel, 2002). As circled by the dotted frames, the seismic data exhibit a lower resolution in these discontinuous interfaces compared to the true model, but the PWD yields the local slope angle estimates of approximately 50° over a broader range surrounding these regions. Owing to this inherent limitation, none of the PWD, structure tensor (Wu and Janson, 2017), or correlation analysis (Yin et al., 2020) can accurately estimate the local slopes. Under the guidance of such biased structural attributes, the structure-guided inversion results obtained by the method of Li et al. (2024) with a large lateral penalty are shown in Fig. 1(d). These incorrect estimates lead to the lateral impedance smoothing across fault zones and thus distort the recovered impedance structures, resulting in a reduction in lateral resolution and accuracy. Similar artifacts can also be found in the research of structure-guided seismic interpolation (Chen et al., 2016; Li et al., 2022). Applying a small lateral penalty intensity can alleviate this issue, whereas it may be inadequate for suppressing model noise and enhancing structures. Some recent studies have proven that adjusting the relative penalty intensity along two extracted orientations is effective in preserving discontinuous structures and accelerating model convergence (Gholami and Gazzola, 2025; Li and Guo, 2025). However, their effectiveness still heavily relies on the accuracy of estimated structural attributes and it is hard to address the artifacts shown in Fig. 1(d) when using inaccurate prior structural information.

In recent years, deep learning (DL)-based inversion approaches have been widely applied to seismic inversion, and exhibit great potential for characterizing the spatial heterogeneity in subsurface formations. For instance, Li et al. (2025) developed a DL-based multi-trace pre-stack inversion framework. This method integrates geologic and well-log data as probabilistic labels to optimize the estimate from well locations to non-well locations. While promising, these methods often rely heavily on high-quality and representative training datasets. Inadequate data quality or sparse spatial coverage may degrade the inversion performance. These

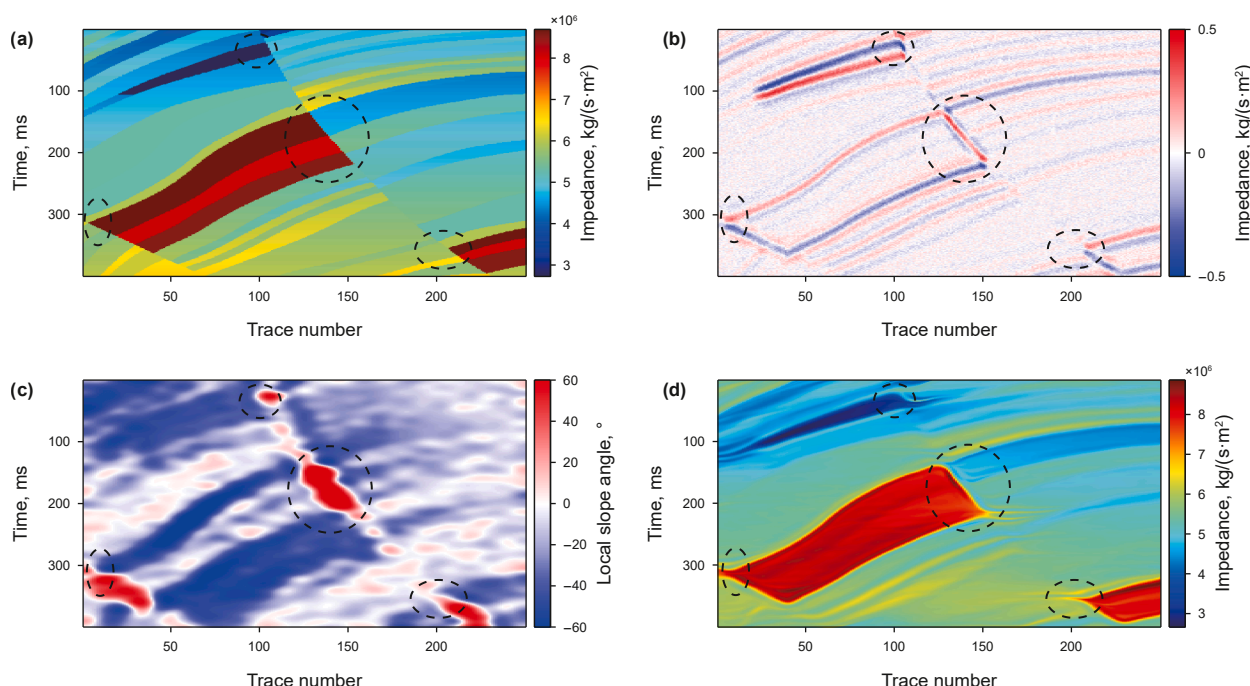


Fig. 1. (a) Synthetic model 1, (b) noisy data (SNR = 2), (c) estimated local slope angles, and (d) structurally distorted impedances using overly large lateral penalty intensities.

approaches also face challenges like difficult reproducibility, insufficient generalization to unseen complex geology conditions, and dependence on prior well-log data for labeled samples. In contrast, our work follows a purely physics-driven paradigm, founded upon the seismic convolution model and geologically consistent structural constraints. It aims to establish an interpretable structure-guided inversion framework that does not require prior well-log data, thereby ensuring good generalizability and physical interpretability. The DL-based methods are not within the scope of this study, as we focus on optimizing physics-based constraint mechanisms, which remains an open challenge, particularly given that the unreliability of prior structural estimates.

The constraint parallel to the local slope enhances the lateral continuity across traces, while the perpendicular one enforces the smoothness of the inverted impedances. In previous studies, both constraints are incorporated into the objective function. Given that the inversion is also constrained by a reference model, balancing the optimization among at least four terms introduces considerable complexity. In addition, this 2-D structural constraint is more appropriate for the full waveform inversion (FWI), because its physical modeling is at least a 2-D problem, while the convolution model is only a 1-D model. Therefore, it is unnecessary to directly impose the structure-guided vertical constraint on 2-D impedance models. Alternatively, the enforcement of sparsity can be decoupled from the structure-guided inversion itself. To achieve this, the adaptive edge-preserving filter (AEPF) serves as an ideal mechanism for integration into the structure-guided inversion framework. The AEPF uses a multi-window mean filtering process to enhance blocky features without explicit sparse constraints. The filtered results are then fed back as the initial model for the next round of iteration. It has been demonstrated to outperform the L_1 -type constraints in preserving thin layers (Dai et al., 2019; Xu et al., 2023, 2025). This implicit sparse constraint strategy is conceptually analogous to the modified total variation (MTV) regularization proposed by Lin and Huang (2014). In the MTV regularization approach, the sparse TV constraint is individually applied to the inversion results constrained by only a reference model, and this reference model is then updated by the denoised results, obtained by solving an L_2 -TV denoising problem decoupled from the data-fitting process, for the next round of inversion. However, the trace-by-trace implementation of the AEPF risks compromising the lateral continuity of the structure-guided inversion results. Such discontinuity may not be well suppressed, and even diffuse to final inversion results under large structural penalties and introduce artifacts. Besides, the soft-thresholding shrinkage in the L_2 -TV denoising process tends to diminish impedance gradients if using inappropriate denoising parameters, blurring sharp boundaries and fine-scale details. Thus, it is reasonable to combine the strengths of both methods. Under the sparsity assumption, this combined approach allows the use of small denoising parameter values to eliminate the AEPF-induced discontinuity, while preserving sharp boundaries. In turn, this yields an updated reference model with sparse reflection coefficients, enabling the realization of implicit sparse constraints.

In this study, we develop an average similarity-powered adaptive structure-guided inversion framework based on an implicit sparse constraint strategy for high-resolution impedance imaging. The proposed framework comprises four steps. First, a series of different penalty intensities are applied for the structure-guided inversion with uniform lateral constraints. Second, we introduce an average structure similarity (ASSIM) index, the average of the structure similarity (SSIM) index between each result and all others, and the penalty intensity associated with the result with the maximum ASSIM value is selected as the absolute

lateral penalty intensity to promise lateral continuity. Third, the structural constraint sensitivity (SCS) index, defined as the variance of each inverted impedance across all inversion results, is used to differentiate the relative lateral penalty, with the aim of suppressing structural distortion. Fourth, the structure-guided inversion is performed to reconstruct multi-trace impedances simultaneously using the previously determined parameters. At this step, the reconstructed impedance model is constrained by the lateral constraint and a reference model. This model undergoes post-processing via the AEPF followed by L_2 -TV denoising in each iteration, and the denoised results are subsequently fed back as an updated reference model for the next round of iteration. The proposed approach is validated using the Marmousi model and field data.

2. Method

2.1. Impedance forward modeling

Based on the convolution theory, the forward simulation of a single trace can be expressed as the convolution of a seismic wavelet with reflection coefficient series, as given by

$$\mathbf{d} = \mathbf{G}_S \mathbf{m} \quad (1)$$

with

$$\mathbf{G}_S = \frac{1}{2} \mathbf{W} \mathbf{D} \quad (2)$$

where $\mathbf{d}$ is the seismic data, $\mathbf{m}$ is the natural logarithm of the impedance vector, $\mathbf{G}_S$ is the single-trace forward operator comprising the wavelet matrix $\mathbf{W}$ and the first-order difference matrix $\mathbf{D}$. The multi-trace forward operator can be further represented as the concatenation of all single-trace forward operators, as given by

$$\mathbf{d} = \mathbf{G}_M \mathbf{m} \quad (3)$$

Eq. (3) can be expressed in detail as

$$\begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_N \end{bmatrix} = \begin{bmatrix} \mathbf{G}_S & & \\ & \mathbf{G}_S & \\ & & \ddots \\ & & & \mathbf{G}_S \end{bmatrix} \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \vdots \\ \mathbf{m}_N \end{bmatrix} \quad (4)$$

where $\mathbf{d}_i$ and $\mathbf{m}_i$ are the simulated data and impedance vector of the i th seismic trace, respectively. To simplify the notation, $\mathbf{G}_M$ is hereafter referred to as $\mathbf{G}$.

2.2. Structure-guided multi-trace inversion

If only the lateral constraint is considered, the objective function of the conventional structure-guided multi-trace inversion is defined as

$$\varphi(\mathbf{m}) = \|\mathbf{Gm} - \mathbf{d}\|_2^2 + \lambda_1 \|\mathbf{m} - \mathbf{u}\|_2^2 + \lambda_2 \|\mathbf{Rm}\|_2^2 \quad (5)$$

where λ_1 and λ_2 are regularization factors, $\mathbf{u}$ is the reference model. $\mathbf{R}$ is the rotated gradient operator that is parallel to the local slope, and it can be constructed by

$$\mathbf{R} = \mathbf{D}_{\cos} \mathbf{D}_x + \mathbf{D}_{\sin} \mathbf{D}_z \quad (6)$$

where $\mathbf{D}_x$ and $\mathbf{D}_z$ are the first-order horizontal and vertical gradient operators in the Cartesian coordinate system, respectively. $\mathbf{D}_{\cos}$ and $\mathbf{D}_{\sin}$ are the diagonal matrices of local slope angles for coordinate rotation as given by

$$\mathbf{D}_{\cos} = \begin{bmatrix} \ddots & & \\ & \cos \theta_i & \\ & & \ddots \end{bmatrix} \quad (7)$$

and

$$\mathbf{D}_{\sin} = \begin{bmatrix} \ddots & & \\ & \sin \theta_i & \\ & & \ddots \end{bmatrix} \quad (8)$$

where θ_i is the local slope angle of the i th parameter.

The closed-form solution of Eq. (5) is given by

$$\mathbf{m} = (\mathbf{G}^T \mathbf{G} + \lambda_1 \mathbf{I} + \lambda_2 \mathbf{R}^T \mathbf{R})^{-1} (\mathbf{G}^T \mathbf{d} + \lambda_1 \mathbf{u}) \quad (9)$$

where $\mathbf{I}$ denote the identity matrix.

2.3. Adaptive structural constraint based on ASSIM and SCS indices

To determine an appropriate upper bound of λ_2 and differentiate the uniformly imposed lateral constraint, two auxiliary metrics independent of prior information, the ASSIM and SCS indices, are designed for adaptive lateral constraints. As illustrated in Eq. (10), the lateral constraint intensity is formulated as a combination of the absolute penalty intensity λ_2 and the relative penalty intensities $\mathbf{W}$.

$$\varphi(\mathbf{m}) = \|\mathbf{Gm} - \mathbf{d}\|_2^2 + \lambda_1 \|\mathbf{m} - \mathbf{u}\|_2^2 + \lambda_2 \|\mathbf{WRm}\|_2^2 \quad (10)$$

The ASSIM index helps determine the acceptable upper bound of λ_2 , and the SCS index differentiates the penalty intensity among impedances. In this scheme, we first obtain a series of structure-guided inversion results through Eq. (9) using different λ_2 values, with λ_1 fixed as a constant. Subsequently, we calculate the ASSIM value of each inverted impedance image following

$$\text{ASSIM}(\mathbf{m}_i) = \frac{1}{K} \sum_{j=1}^K \text{SSIM}(\mathbf{m}_i, \mathbf{m}_j) \quad (11)$$

with

$$\text{SSIM}(\mathbf{m}_i, \mathbf{m}_j) = \frac{(2\mu_i \mu_j + c_1)(2\sigma_{ij} + c_2)}{(\mu_i^2 + \mu_j^2 + c_1)(\sigma_i^2 + \sigma_j^2 + c_2)} \quad (12)$$

where K is the number of used λ_2 values, $\mathbf{m}_i$ is the impedance image using the i th λ_2 , μ_i and μ_j are the average of $\mathbf{m}_i$ and $\mathbf{m}_j$, σ_i and σ_j are their standard deviations, σ_{ij} is the associated covariance, and c_1 and c_2 are two small constants for stability. To better capture structural features across different scales, the multi-scale SSIM function (Wang et al., 2003) is employed to evaluate the similarity between inversion results.

In this step, λ_1 can be previously set using traditional methods, such as the L-curve method, or assigned a slightly large value with λ_2 fixed to zero. Considering that the reference model will be gradually updated during iterations, the inversion results are not highly reliant on this parameter. When the initial model is of good quality, this parameter can be set slightly larger to ensure stability. However, when λ_1 is excessively large, much larger λ_2 values are required for structural distortion to occur. Then, a series of values of λ_2 will be tested. When an overly small value of λ_2 is employed, the lateral constraint may be too weak to sufficiently suppress noise. With the increase of λ_2 , the recovered impedance structure will be gradually enhanced and become closer to true structures, while excessively large λ_2 will

eventually lead to structural distortion. Therefore, within a reasonable range of λ_2 , the ASSIM index will initially rise slowly or almost remain stable, but redrop rapidly once λ_2 becomes too large. The inversion results corresponding to a specific λ_2 yielding the maximum ASSIM value indicates the highest structural similarity with all other results. This selected λ_2 can be regarded as a favorable balance between the under-constrained and over-constrained conditions, and is referred to as the absolute penalty intensity in this study. It should be noted that the number of excessively large λ_2 values should not dominate the tested range, as this may lead to an overestimation of λ_2 . An upper limit for the misfit of the data term, such as 10% or 15%, can be set as a threshold to constrain the maximum of tested λ_2 values.

Although λ_2 can now be selected based on variation of the ASSIM index, it still imposes a uniform constraint across all impedances, without differentiating the penalty intensity between continuous and discontinuous regions. To further mitigate the impact of inaccurate structural estimates, the SCS index is introduced to weaken the lateral penalty in regions prone to structural distortion. The SCS index is defined as

$$\text{SCS}_i = \frac{1}{K} \sum_{j=1}^K (\bar{m}_i - m_j^i)^2 \quad (13)$$

where i denotes the index of impedance, $\bar{m}_i$ is the average of the i th impedance over all the inversion results obtained using different λ_2 values, and m_j^i is the i th inverted impedance using the j th λ_2 . SCS_i is the variance of the i th inverted impedance derived from all the inversion results. It characterizes the sensitivity of each impedance to λ_2 . A low SCS index indicates that the inverted impedance is less sensitive to the change in λ_2 . This typically occurs in geologically continuous regions. In contrast, a high SCS index suggests greater sensitivity to the lateral constraint, which is typically observed in regions with sharp discontinuities or complex structures. To address this, a new weighting matrix $\mathbf{W}$ is introduced to differentiate the uniform penalty intensity as defined by

$$\mathbf{W} = \begin{bmatrix} \ddots & & \\ & W_i & \\ & & \ddots \end{bmatrix} \quad (14)$$

where W_i , referred to as the relative penalty intensity in this study, is calculated using an exponential decay function of the logarithmically normalized SCS (NSCS) index as given by

$$W_i = e^{-\left(\frac{\text{NSCS}_i}{\alpha}\right)^\beta} \quad (15)$$

with

$$\text{NSCS}_i = \frac{\ln(\text{SCS}_i) - \ln(\text{SCS}_{\min})}{\ln(\text{SCS}_{\max}) - \ln(\text{SCS}_{\min})} \quad (16)$$

where α controls the start of the decay, while β determines the steepness of the decay curve. As shown in Fig. 2, larger α values shift the decay onset toward higher NSCS values, which in turn means fewer impedances are subject to weakened lateral penalties. Meanwhile, larger β values lead to a sharper transition from one to zero occurring near a horizontal position of α . In this way, the final lateral constraint intensity is adaptively differentiated, and the modified lateral constraint term can be redefined as $\|\mathbf{WRm}\|_2^2$.

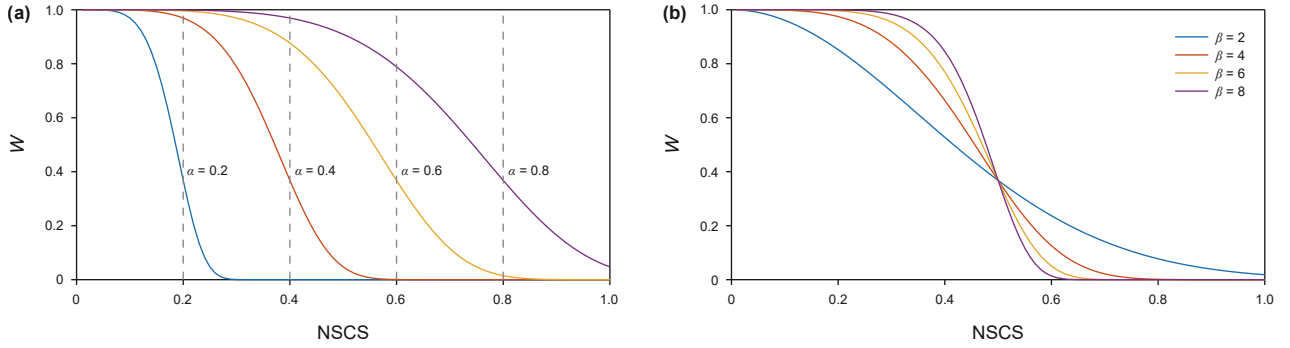


Fig. 2. Decay curves of Eq. (14) with (a) varying α with β fixed at 5, and (b) varying β with α fixed at 0.5, respectively.

2.4. AEPF-assisted implicit sparse constraint based on MTV regularization scheme

Since the inversion does not incorporate any vertical constraints, the least-squares solution tends to exhibit excessive vertical smoothing and may fail to delineate sharp boundaries. To improve the vertical imaging resolution, the structure-guided inversion is integrated into the MTV regularization framework (Lin and Huang, 2014). Through continuously updating the reference model via L_2 -TV denoising, the sparse constraint is implicitly enforced. Furthermore, the AEPF is introduced in the denoising process to further reinforce the sparsity of inversion results and fine-scale features (Dai et al., 2019; Xu et al., 2023, 2025). The adaptive filtering can be expressed as

$$\mathbf{m}_A = \mathbf{F}_A(\mathbf{m}) \quad (17)$$

where $\mathbf{F}_A$ is the mean filter with multiple window sizes for 1-D inverted impedances, and the AEPF is trace-by-trace applied to the structure-guided inversion results $\mathbf{m}$. For each seismic trace, a series of mean filters with varying window sizes are employed. At each location, these filters slide along the trace and generate multiple local windows for each window length. The final filtered result of the current impedance is determined as the average of the window with the lowest variance. This approach not only enhances blocky features without the need for explicit sparse constraints, but also outperforms the L_1 -norm constraint in resolving thin-layer targets. Theoretically, the filtered results can then be fed back as the reference model $\mathbf{u}$ to promote the sparsity. However, since the AEPF is essentially a 1-D filter, it outputs satisfactory results only when inverted impedances oscillate uniformly around true values. Otherwise, the filtered results might become biased to one side. In addition, the trace-by-trace implementation inevitably compromises the lateral continuity. Using discontinuous results as the reference model may lead to instability, as the lateral constraint could amplify such discontinuities. Thus, the AEPF should not be used as the final step in the denoising process, so we apply the L_2 -TV denoising to mitigate the slight discontinuity in $\mathbf{m}_A$ before using it to update $\mathbf{u}$. Another advantage of this ordering lies in the fact that structural features have already been enhanced by the AEPF. That is, we only need to address the minor discontinuities it introduces. This allows for the use of relatively small denoising parameters and better preserves the sharp boundaries. In general, the MTV regularization shares conceptual similarities with the AEPF-based inversion, as both promote sparsity by optimizing the reference model rather than explicitly constraining the primary inverse problem. This simple combination of the 1-D filtering and 2-D denoising is easy to implement and enhances application flexibility. The L_2 -TV denoising can be expressed as

$$\mathbf{u} = \underset{\mathbf{u}}{\operatorname{argmin}} \left\{ \|\mathbf{u} - \mathbf{m}_A\|_2^2 + \lambda_3 \|\mathbf{u}\|_{\text{TV}} \right\} \quad (18)$$

where $\mathbf{u}$ is the denoised model of $\mathbf{m}_A$, and it can be solved by alternately solving two subproblems using the split-Bregman method as shown in Eqs. (19)–(21).

$$\mathbf{u}^{k+1} = \underset{\mathbf{u}}{\operatorname{argmin}} \left\{ \|\mathbf{u} - \mathbf{m}_A\|_2^2 + \lambda_4 \left\| \mathbf{w}_x^k - \nabla_x \mathbf{u} - \mathbf{b}_x^k \right\|_2^2 + \lambda_4 \left\| \mathbf{w}_z^k - \nabla_z \mathbf{u} - \mathbf{b}_z^k \right\|_2^2 \right\} \quad (19)$$

$$\mathbf{w}_x^{k+1}, \mathbf{w}_z^{k+1} = \underset{\mathbf{w}_x, \mathbf{w}_z}{\operatorname{argmin}} \left\{ \lambda_3 \|\mathbf{u}\|_{\text{TV}} + \lambda_4 \left\| \mathbf{w}_x^k - \nabla_x \mathbf{u} - \mathbf{b}_x^k \right\|_2^2 + \lambda_4 \left\| \mathbf{w}_z^k - \nabla_z \mathbf{u} - \mathbf{b}_z^k \right\|_2^2 \right\} \quad (20)$$

$$\mathbf{b}_x^{k+1}, \mathbf{b}_z^{k+1} = \mathbf{b}_x^k + \left(\nabla_x \mathbf{u}^{k+1} - \mathbf{w}_x^{k+1} \right), \mathbf{b}_z^k + \left(\nabla_z \mathbf{u}^{k+1} - \mathbf{w}_z^{k+1} \right) \quad (21)$$

where k is the iteration number in the denoising process rather than in the alternating optimization process of $\mathbf{m}$ and $\mathbf{u}$, ∇_x and ∇_z are the same as $\mathbf{D}_x$ and $\mathbf{D}_z$, $\mathbf{w}_x^k$ and $\mathbf{w}_z^k$ are auxiliary variables, and $\mathbf{b}_x^k$ and $\mathbf{b}_z^k$ are the Bregman distance. Eq. (19) is a simple L_2 -norm problem, and Eq. (20) can be solved using the shrinkage formula as given by

$$\mathbf{w}_x^{k+1} = \max \left(\mathbf{q}^k - \frac{\lambda_3}{2\lambda_4}, 0 \right) \frac{\nabla_x \mathbf{u}^k + \mathbf{b}_x^k}{\mathbf{q}^k} \quad (22)$$

$$\mathbf{w}_z^{k+1} = \max \left(\mathbf{q}^k - \frac{\lambda_3}{2\lambda_4}, 0 \right) \frac{\nabla_z \mathbf{u}^k + \mathbf{b}_z^k}{\mathbf{q}^k} \quad (23)$$

with

$$\mathbf{q}^k = \sqrt{\left| \nabla_x \mathbf{u}^k + \mathbf{b}_x^k \right|^2 + \left| \nabla_z \mathbf{u}^k + \mathbf{b}_z^k \right|^2} \quad (24)$$

where λ_4 controls the penalty intensity in Eq. (19), and $\frac{\lambda_3}{2\lambda_4}$ determines the soft-threshold for total impedance gradients, where values larger than $\frac{\lambda_3}{2\lambda_4}$ are attenuated, while those below this threshold will be shrunk toward zero. Since impedance gradients are actually twice reflection coefficients, in practice, $\frac{\lambda_3}{2\lambda_4}$ can be identified through pilot tests on prior well-log traces, followed by the set of λ_4 , which affects the convergence speed of the denoising. In the absence of prior well-log data, under the assumption that

reflection coefficients are sparse, the distribution of reflection coefficients of the AEPF-filtered results can be statistically characterized, and $\frac{\lambda_2}{2\lambda_4}$ can thus be set based on these characteristics, which will be introduced in the next section. In addition, to better preserve large gradient features, we can also set a second threshold for Eqs. (22) and (23). Impedance gradients exceeding this threshold will remain constant during the denoising process. Moreover, to ensure the lateral resolution, a relatively small penalty weight can be assigned for the term involving $\nabla_x \mathbf{u}$ or using anisotropic TV denoising algorithm (Shi and Chang, 2013). The alternating iteration in the proposed inversion framework follows

$$\mathbf{m}^{k+1} = \underset{\mathbf{m}}{\operatorname{argmin}} \left\{ \|\mathbf{G}\mathbf{m} - \mathbf{d}\|_2^2 + \lambda_1 \|\mathbf{m} - \mathbf{u}^k\|_2^2 + \lambda_2 \|\mathbf{W}\mathbf{R}\mathbf{m}\|_2^2 \right\} \quad (25)$$

and

$$\mathbf{u}^{k+1} = \underset{\mathbf{u}}{\operatorname{argmin}} \left\{ \|\mathbf{u} - \mathbf{F}_A(\mathbf{m}^{k+1})\|_2^2 + \lambda_3 \|\mathbf{u}\|_{\text{TV}} \right\} \quad (26)$$

where k is the iteration number, and the complete workflow is illustrated in Fig. 3.

3. Numerical examples

The model shown in Fig. 1(a) is consistently used to validate the effectiveness of the proposed approach. In this section, we first introduce how to use the ASSIM curve to determine the acceptable upper bound of the absolute penalty intensity λ_2 , and then illustrate the effect of the SCS index in preserving discontinuous features, and finally evaluate the performance between the combined implicit sparse constraint strategy and other methods.

To intuitively present the impact of the uniformly imposed lateral constraint with different intensities, a series of the structure-guided inversion results are obtained from Eq. (9), using varying λ_2 values and a fixed λ_1 of one. Fig. 4(a) and (b) show the inversion results of the noisy data with an SNR of 4 dB and 2 dB,

respectively. The structural features used for constraint are also calculated separately from these two datasets. As noted in Fig. 4, excessively large values of λ_2 strengthen the smoothing along the lateral direction, which drives the diffusion of these impedance anomalies across discontinuous interfaces, thereby compromising the imaging accuracy. The root-mean-square error (RMSE) of the data fitting term, ASSIM index, true SSIM index, mean absolute error (MAE), and peak SNR (PSNR) metrics are provided in Fig. 5. The true SSIM index is calculated between each result shown in Fig. 4 and the true model. The variation in RMSE indicates that the lateral constraint interferes with the data fitting. This interference is acceptable, because the discontinuous features are inevitably distorted by inaccurate lateral continuous constraints. Therefore, the RMSE increases when the lateral penalty intensity becomes stronger. As λ_2 becomes overly large, both the ASSIM and SSIM indices show a significant decrease because of the artifacts.

In the proposed method, λ_2 with the largest ASSIM index is determined for the subsequent inversion, while this procedure relates closely to the testing ranges of λ_2 values. As shown in Fig. 5(a) and (b), the purple, green and blue curves represent the ASSIM index derived from the first 10, 13, and 16 inversion results, respectively. The noted points denote their maximum values. It is evident that these maximum values increase with the increasing upper limit of λ_2 , because the calculation includes more distorted results. Besides, the selected λ_2 values tend to increase as the SNR decreases, which can be attributed to the fact that low SNR data inversion requires stronger structural constraints. Although the λ_2 values denoted by the maximum of the two purple curves are close to those shown on the true SSIM curves, they are not suitable to determine the acceptable upper bound of λ_2 , because they do not show a significant decrease with the increase of λ_2 . This downward trend can be observed in the green curves, and it is more pronounced in the blue curves. For the 4 dB data, the RMSE values of the 13th and 16th inversion results exceed the minimum RMSE by 11% and 22%, respectively. When the threshold is set around to 10%, the λ_2 is determined as 2.56 from the first 14 inversion results. For the 2 dB data, the corresponding RMSE increments are 8% and 16%, respectively, with the λ_2 identified as 2.56 as well. The determined λ_2 values from the green and blue curves are larger than those of the true SSIM curves, while the SSIM values at these λ_2 positions are quite close to their own maximum. Additionally, the trend of the true SSIM curves is observed to align closely with that of the ASSIM curves, with both of which exhibiting a rapid decline when λ_2 exceeds the mentioned values. Moreover, as shown in Fig. 5(c) and (d), the λ_2 values according with the maximum of the PSNR curves are identical to those of the true SSIM curves, while the relevant λ_2 values on the MAE curves are larger. This suggests that when the true model is available as a reference, the optimal choice of λ_2 may show slight discrepancies across different evaluation metrics. Nevertheless, the values at the λ_2 positions estimated by the ASSIM curves remain close to their optimal values. Given that the true model is unavailable in practice, this consistency facilitates the determination of appropriate λ_2 , where the RMSE threshold could serve as an effective constraint for setting the upper bound of the tested range of λ_2 .

To further validate the effectiveness of the ASSIM metric, two additional models from other regions of the Marmousi model are provided for validation. As shown in Fig. 6, these two models exhibit distinct structural complexities. The model 2 contains more discontinuous interfaces, while the model 3 shows greater lateral continuity, with only a low-impedance target embedded within it. Their local slope angles are estimated from synthetic data with an SNR of 2 dB. As the structure of the model 3 is relatively simple, the RMSE threshold for these two models are set to 10% and 15%, respectively. Fig. 7 shows the mentioned metrics for

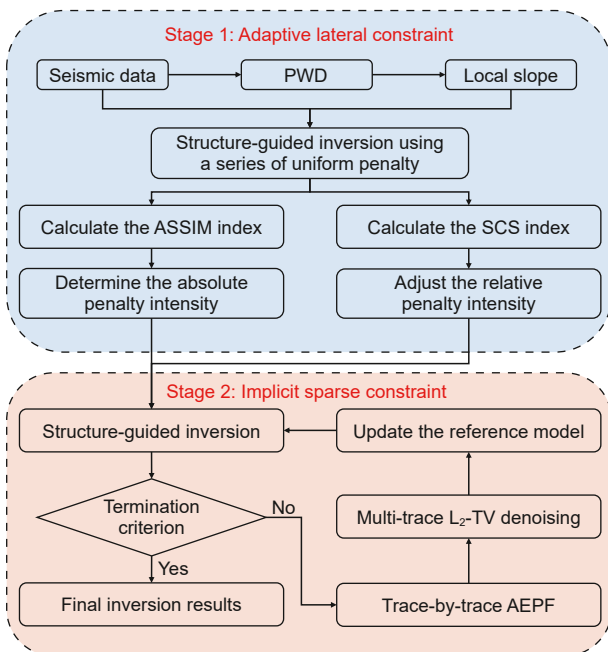


Fig. 3. Workflow of the proposed inversion approach.

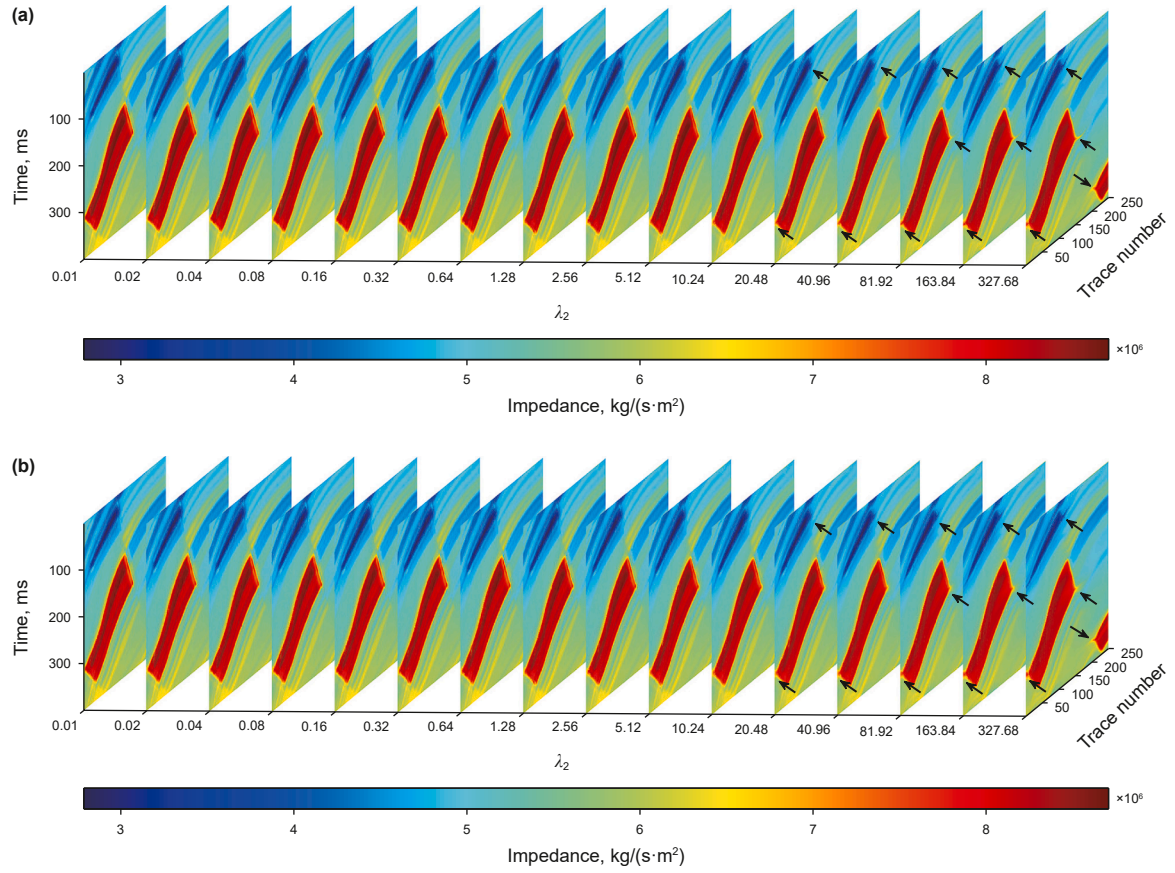


Fig. 4. Inversion results of noisy data with an SNR of (a) 4 dB and (b) 2 dB using Eq. (9) with varying λ_2 values and $\lambda_1 = 1$.

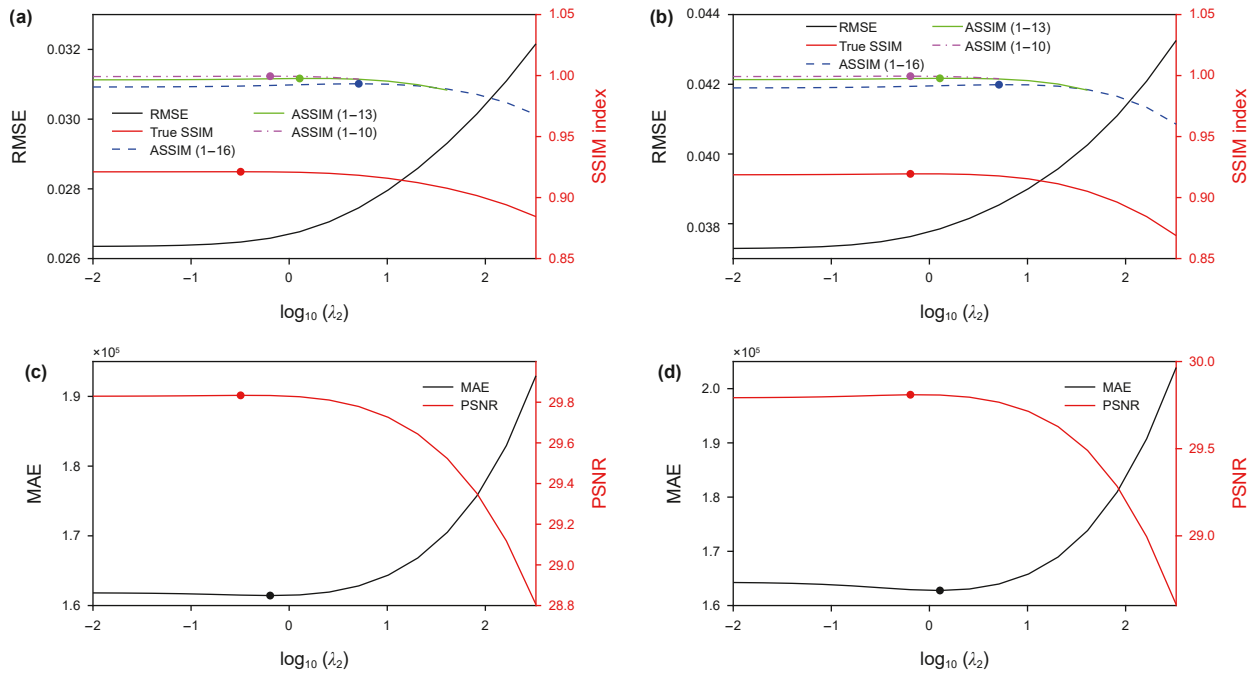


Fig. 5. Variations of (a) RMSE, ASSIM and SSIM, and (b) MAE and PSNR in the 4 dB data inversion. Variations of (c) RMSE, ASSIM and SSIM, and (d) MAE and PSNR in the 2 dB data inversion.

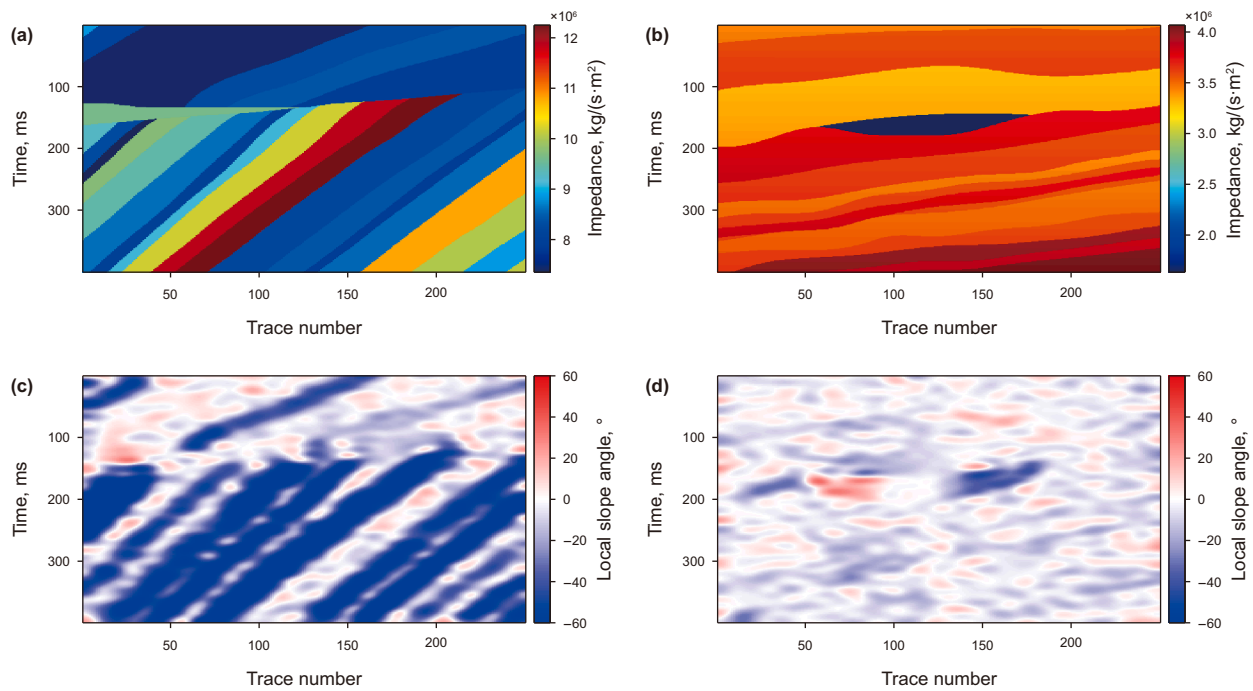


Fig. 6. (a) Synthetic model 2 and (c) estimated local slope angles from noisy data with an SNR of 2 dB. (b) Synthetic model 3 and (d) estimated local slope angles from noisy data with an SNR of 2 dB.

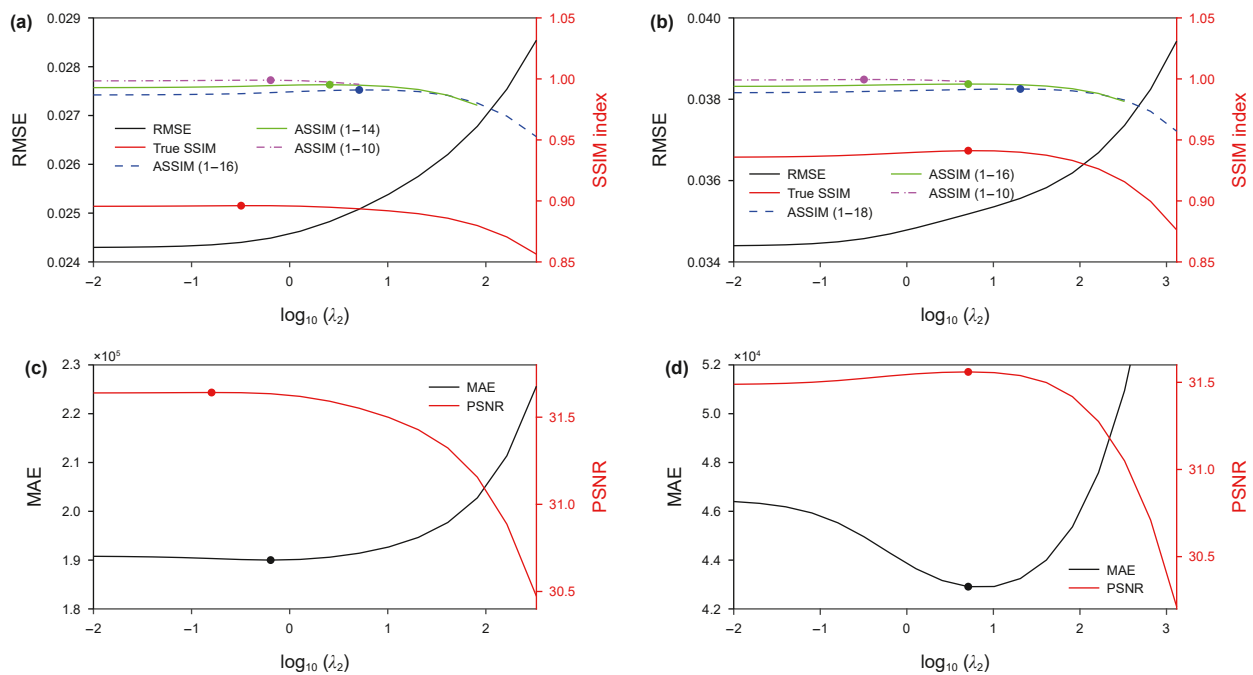


Fig. 7. Variations of (a) RMSE, ASSIM and SSIM, and (b) MAE and PSNR in the 2 dB data inversion of the model 2. Variations of (c) RMSE, ASSIM and SSIM, and (d) MAE and PSNR in the 2 dB data inversion of the model 3.

these two models. It can be observed that these results still adhere to the rules illustrated in Fig. 5. Especially for the model 3, the ASSIM-determined λ_2 is in full agreement with the optimal choice indicated by the true SSIM and MAE curves. By comprehensively integrating the true SSIM, PSNR, and MAE across the three synthetic models, the ASSIM index proves to be a reliable metric for determining appropriate λ_2 values. Although the determined λ_2

may not be the optimal parameter under the uniform lateral constraint scheme, it ensures no significant structural distortion occurs. Considering that the inversion will then adopt non-uniform lateral constraints and continuously refined reference models, the ASSIM index offers a feasible way to determine the upper bound of the lateral penalty intensity that can be applied without perceptible structural distortions.

Appropriate λ_2 values can be identified based on the ASSIM curve, whereas the inherent limitation of the uniformly imposed lateral constraint is not entirely mitigated. This still carries the risk of introducing structurally distorted artifacts at the fine scale, and such constraints fundamentally impede the attainment of high-resolution imaging. Therefore, it is desirable to differentiate the lateral penalty intensity between continuous features and regions prone to distortion. Fig. 8 shows the NSCS index of the model 1 calculated from the inversion results shown in Fig. 4 with different λ_2 ranges. As shown in Fig. 8(a) and (b), the NSCS indices indicate only a few discontinuities as circled, because the inversion results are not obviously distorted within this λ_2 range. As more structurally distorted results are incorporated, the areas prone to distortion are more obvious as the arrows noted in Fig. 8(c) and (d). The indices shown in Fig. 8(e) and (f) are obtained exceeding the upper limit of the 10% RMSE threshold, while they show the high similarity. However, in this stage, not only the regions according to the biased estimated local slopes are displayed, but also the continuous formation interfaces are highlighted. Meanwhile, same phenomenon can be found in Fig. 9. Fig. 9(c) and (d) show the NSCS indices of the models 2 and 3 under the threshold constraint of 10% RMSE. These results are already capable of characterizing distorted regions as noted by the circles, whereas results under the 15%

RMSE threshold constraint indicate the regions would be affected if λ_2 continues to increase as noted in Fig. 9(e) and (f). These examples illustrate that the passively defined NSCS index primarily serves as a diagnostic indicator for existing structural distortions, rather than a proactive identifier of discontinuous features. Furthermore, it reveals that these distortions caused by inaccurate structural estimates originate from lateral discontinuous regions.

To investigate the impact of different λ_2 ranges on the effectiveness of the non-uniform constraints, we use the NSCS indices shown in Fig. 8(a), (c), and (f) for the 4 dB data inversion of the model 1. Here, α and β are set to 0.6 and 8, respectively. The inversion results of the three distinct λ_2 ranges are shown in Fig. 10. The related RMSE, SSIM, MAE, and PSNR metrics are shown in Fig. 11. Both SSIM and PSNR metrics across the three inversions stay constant as λ_2 increases. It is suggested that the inversion results do not suffer from structural distortions even when using an overly large λ_2 , which demonstrates the effectiveness of differentiating the lateral constraint intensities guided by the SCS index in suppressing the structural distortions. Meanwhile, it does not indicate that the lateral constraint is invalid. On the contrary, the trend of the three MAE curves, and particularly the λ_2 values at which their minima occur, underscores the necessity of the lateral constraint for reducing the reconstruction

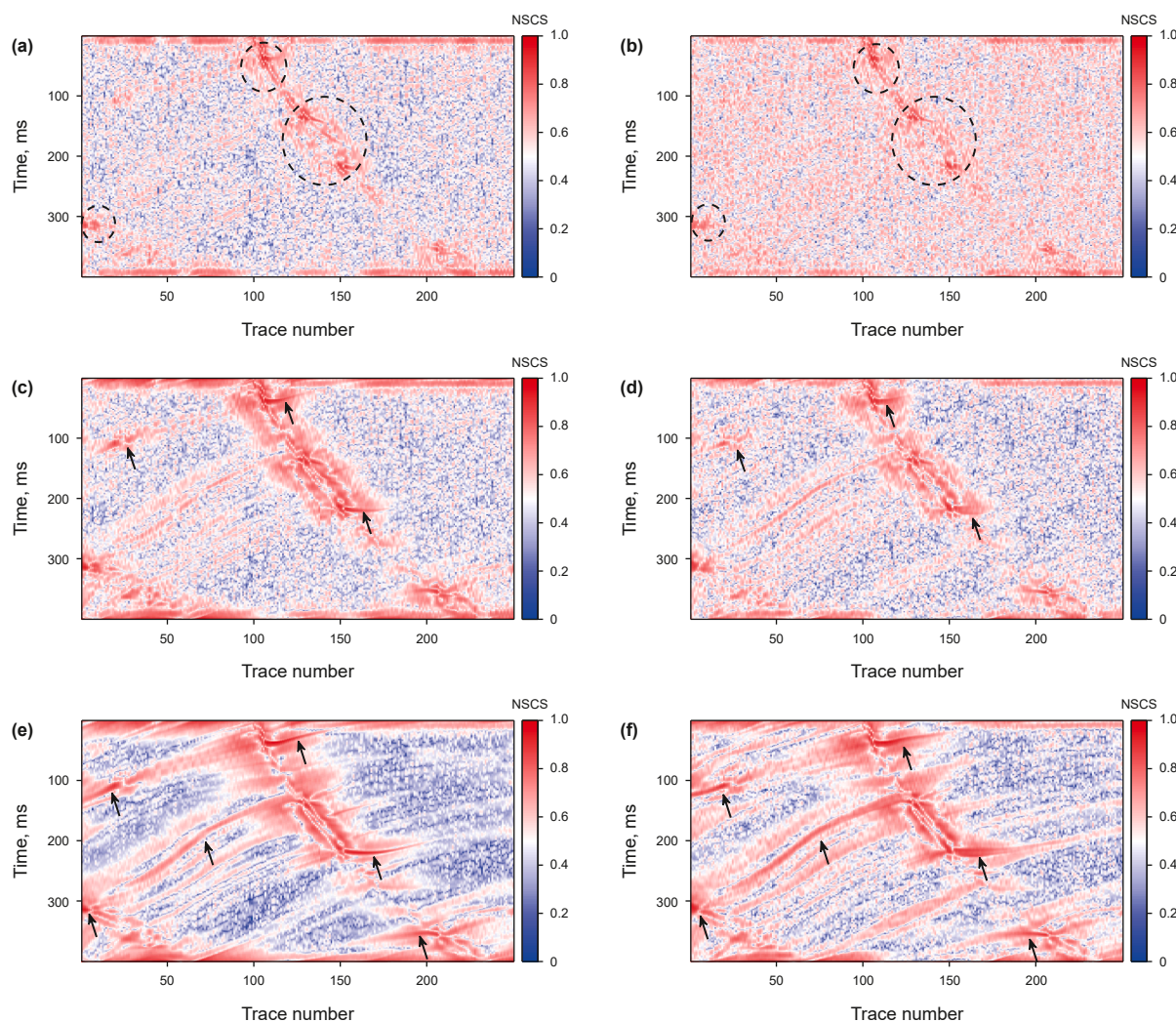


Fig. 8. (a), (c), and (e) are the NSCS indices calculated from the first 10, 13, and 16 inversion results of the 4 dB data, respectively. (b), (d), and (f) are the NSCS indices calculated from the first 10, 13, and 16 inversion results of the 2 dB data, respectively.

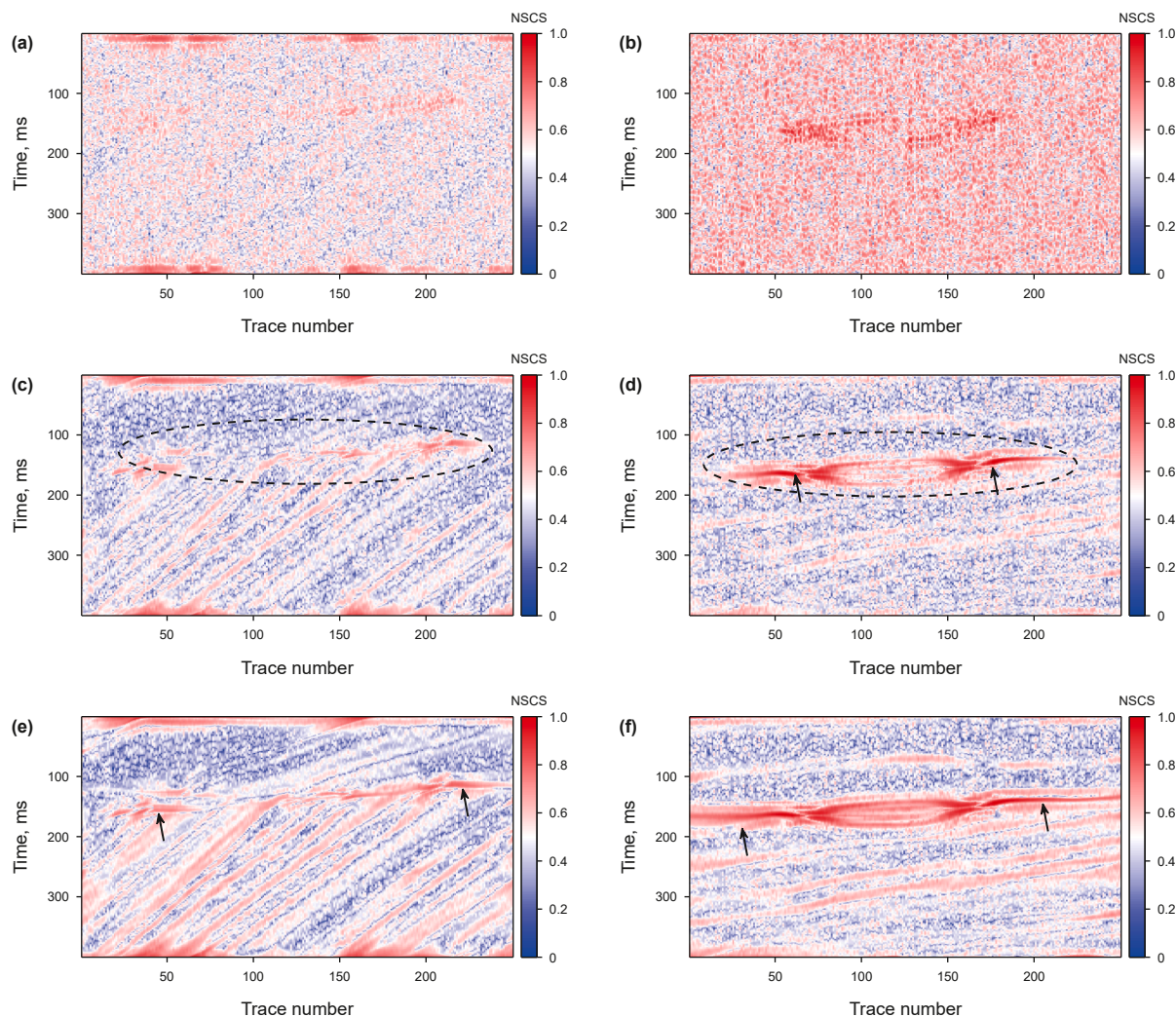


Fig. 9. (a), (c), and (e) are the NSCS indices calculated from the first 10, 14, and 16 inversion results of the 2 dB data generated from the model 2, respectively. (b), (d), and (f) are the NSCS indices calculated from the first 10, 16, and 18 inversion results of the 2 dB data generated from the model 3, respectively.

error. The consistency in these metrics stems from two key factors. First, all the presented NSCS indices indicate the locations prone to distortion to a certain extent, and the larger the range of λ_2 values, the larger the regions identified. Second, the given α and β sufficiently suppress the lateral penalty within these areas even when the NSCS indices shown in Fig. 8(a) are used. This suggests that the structural distortion can be alleviated as long as the continuous lateral constraints at discontinuous locations are weakened. However, it should be noted that an excessively small range of λ_2 may be unable to effectively characterize structurally distorted regions. For instance, most NSCS indices shown in Fig. 9(b) are large and close to one, indicating that the λ_2 values in this range have not yet produced apparent structural distortions. Assigning the relative penalty intensity based on these results would likely weaken the lateral penalty intensity in continuous areas, rendering the constraint ineffective. Considering the differentiation of the lateral penalty intensity is realized through the cooperation of the NSCS index, α and β , even if the NSCS index is derived from overly large λ_2 values, its impact can be mitigated by adjusting the value of α and β . Therefore, it is advised to continuously use the range constrained by the RMSE threshold for the NSCS index calculation, while maintaining the consistency with the step of determining λ_2 .

To illustrate the impact of α and β on inversion results, it is necessary to analyze the respective roles and appropriate value ranges of these two parameters. Fig. 12(a) shows the NSCS index distribution derived from the first 13 inversion results of the 2 dB data of the model 1. After the logarithmic normalization, this distribution is quite close to a normal distribution. We first investigate the role of α . Based on the three-sigma rule, the mean value μ of this distribution, along with the values corresponding to the mean plus one, two, and three standard deviations (i.e., $\mu + \sigma$, $\mu + 2\sigma$, $\mu + 3\sigma$), are selected as candidate α values. These values are reported with two decimal places. Additionally, an α value of 0.2 is supplemented for comparison with scenarios where the lateral constraint is insufficient. Meanwhile, β is set to 100 to achieve rapid decay. The corresponding inversion results are shown in Fig. 12(b)–(f), respectively, and the model evaluation metrics are presented in Table 1. It can be observed in Fig. 12(a) that few portions of impedances are imposed with the lateral constraint when α is set to 0.2. Consequently, the inversion results exhibit certain noise as noted. The model noise is gradually suppressed as α increases, while the metrics achieve the best performance when α is set to 0.74 as shown in Table 1. The metrics of the inversion results using an α of 0.85 is slightly lower than those of 0.74. This is because the lateral penalty intensity for regions prone to structural

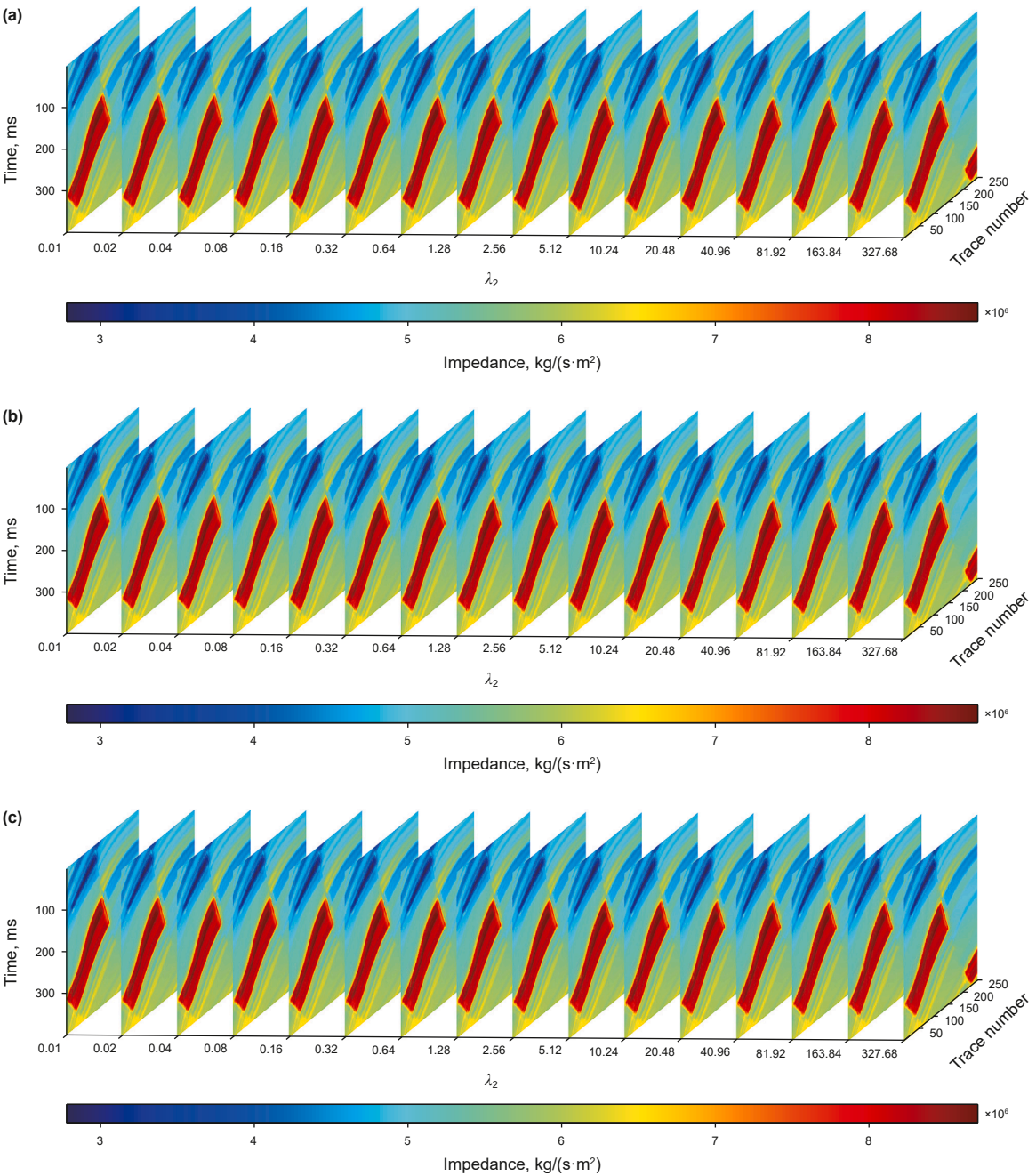


Fig. 10. Inversion results of the 4 dB data of the model 1 using the relative penalty intensity calculated from the first (a) 10, (b) 14, and (c) 16 results.

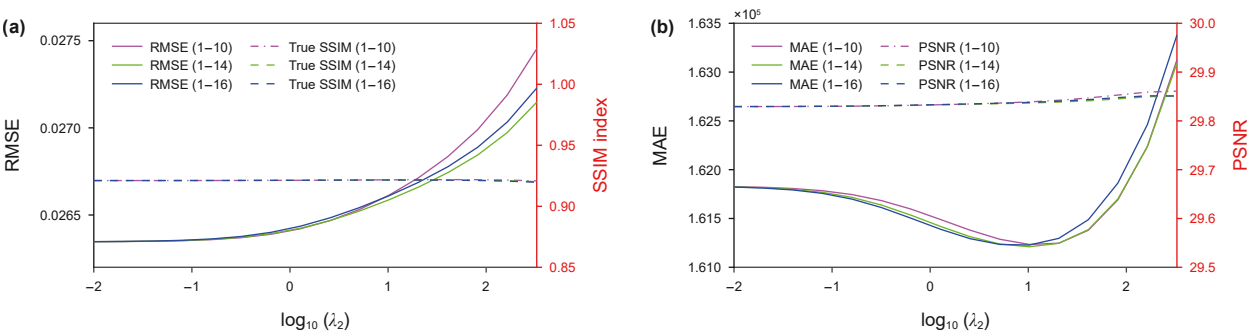


Fig. 11. Variations of (a) RMSE and SSIM, and (b) MAE and PSNR in the 4 dB data inversion of the model 1.

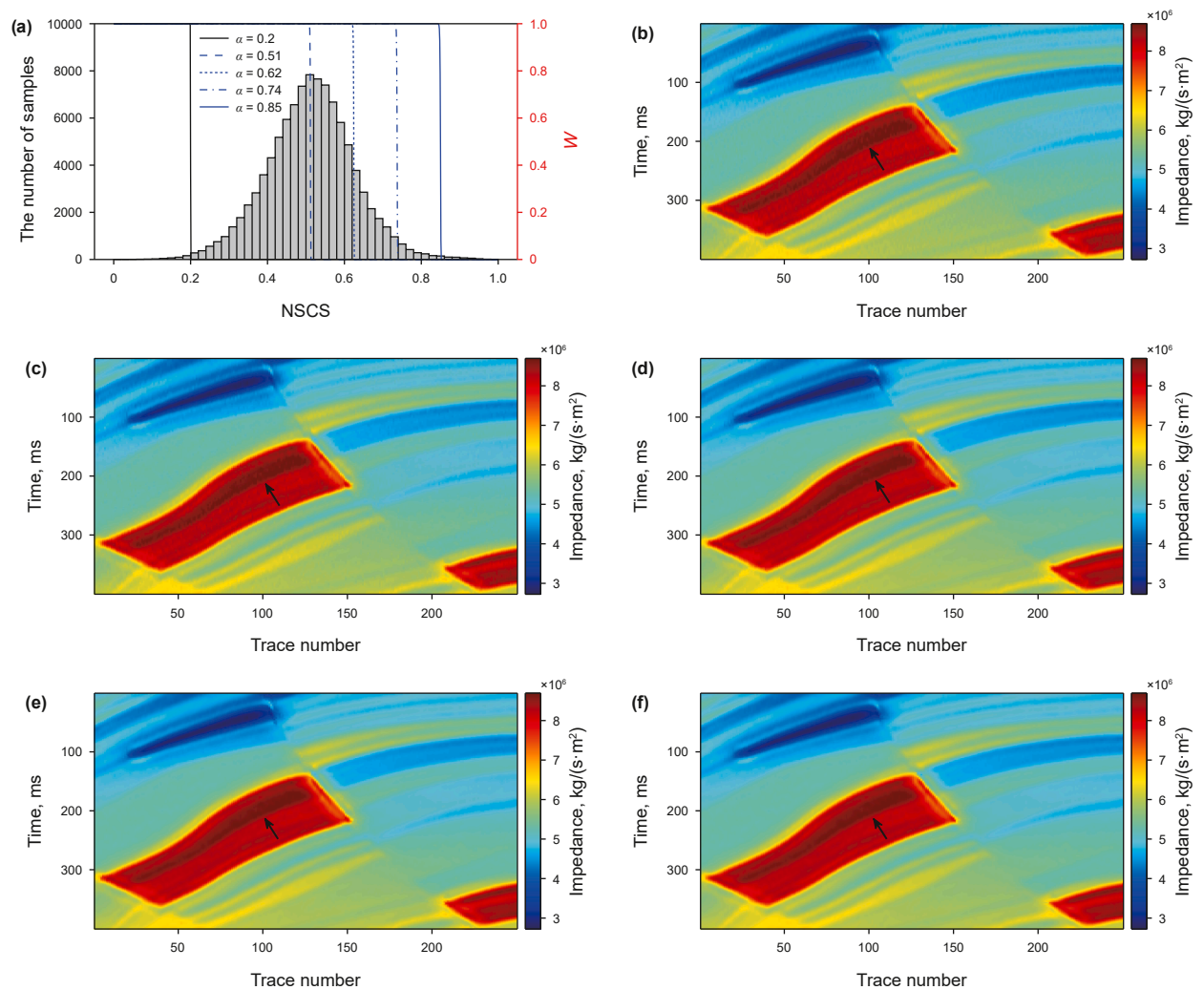


Fig. 12. (a) NSCS index distribution derived from the first 13 inversion results of the 2 dB data of the model 1 and decay curves of different α values, and their corresponding inversion results using α values of (b) 0.2, (c) 0.51, (d) 0.62, (e) 0.74, and (f) 0.85 with a fixed β of 100, respectively.

distortion is not sufficiently suppressed. Nevertheless, it still performs better than that with relatively small α values.

Based on the optimal α of 0.74, a series β values of 1, 5, 8, 20, and 100 are used to investigate the influence of β . As shown in Fig. 13(a), a smaller β value results in a slower decay rate of the NSCS. For instance, the lateral constraint for all impedances is weakened when β is set to 1. This leads to the insufficient lateral constraint for most impedances and inadequate suppression of regions with high NSCS index, thereby yielding the worst performance, as shown in Table 2. With the increase of β , the decay onset shifts closer to 0.74. This ensures that the majority of impedances are constrained by a penalty intensity of 2.56, while enabling more

effective suppression of the lateral constraint for impedances with associated NSCS index larger than 0.74. Compared with the impact of α , the inversion is less sensitive to β . Nevertheless, it is recommended to avoid using excessively small values for either parameter.

Having established the lateral constraint configuration, this research next aims to assess the effectiveness of the implicit sparse constraint. In this step, the adaptive lateral constraint is applied to all test cases, and we focus exclusively on the performance of different implicit sparse constraint strategies. To intuitively describe the characteristic of each strategy, ablation experiments are implemented, in which the AEPF, MTV, and proposed methods are individually applied to generate the reference model $\mathbf{u}$. Specifically, in the AEPF-based inversion, $\mathbf{u}$ is obtained through applying the trace-by-trace AEPF. In the MTV-based inversion, it is derived via the L_2 -TV denoising. In the proposed approach, the reference model is generated as described in Eq. (25). The filtering window size plays a crucial role in the performance of the AEPF. In its earlier application, the filtering window size is set to the range of 3 to 21 (Dai et al., 2019). It is important to note that the maximum window size should not be set too small, because this may diminish the ability to maintain blocky impedance features. Although some recent studies show that a maximum window size

Table 1
Evaluation metrics of the inversion results shown in Fig. 12.

α	MAE	PSNR	SSIM
0.20	1.6431×10^5	29.7917	0.9187
0.51	1.6366×10^5	29.7986	0.9190
0.62	1.6267×10^5	29.8152	0.9198
0.74	1.6264×10^5	29.8235	0.9200
0.85	1.6280×10^5	29.8114	0.9197

The bold contents denote the best performance under each metric.

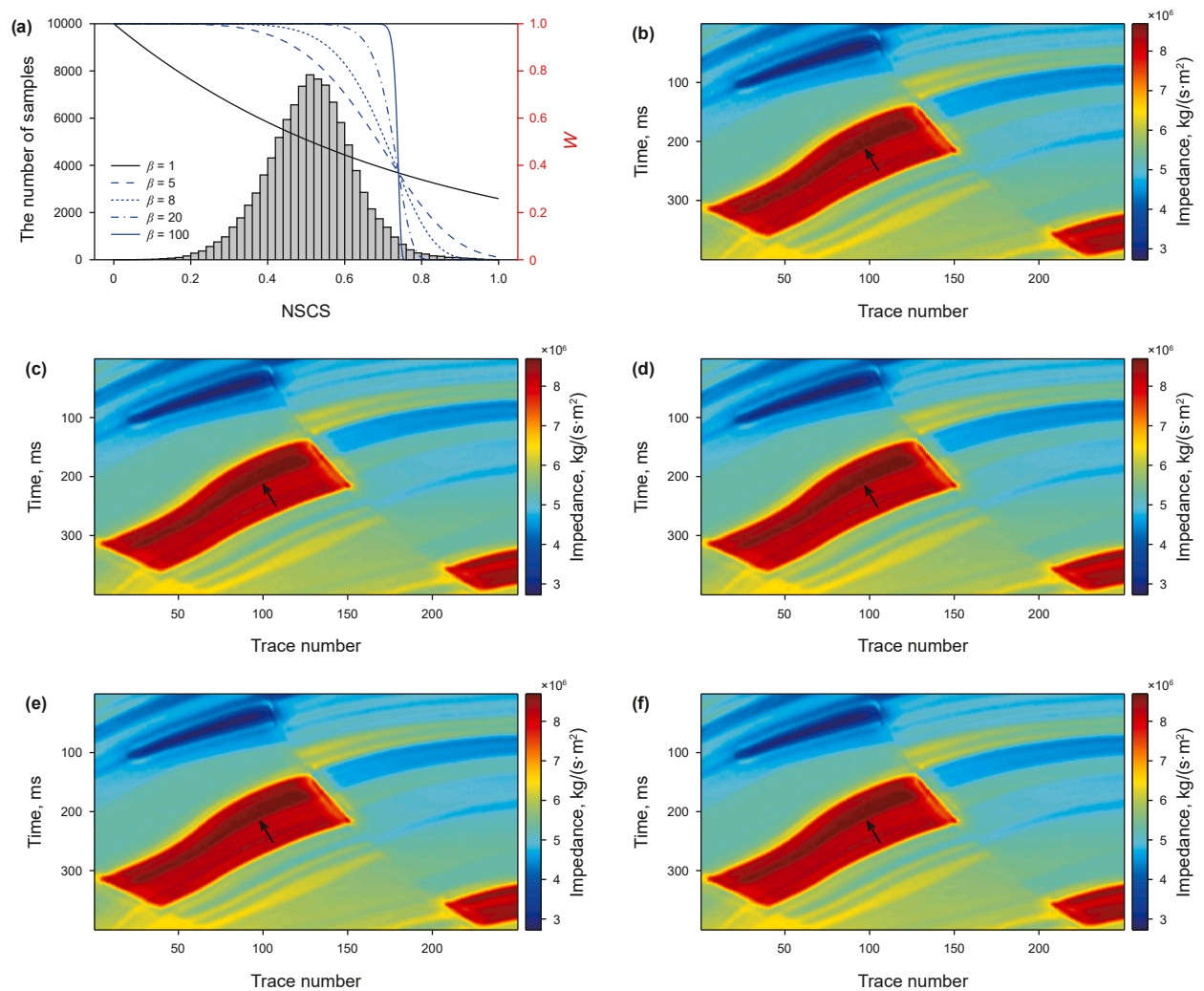


Fig. 13. (a) NSCS index distribution derived from the first 13 inversion results of the 2 dB data of the model 1 and decay curves of different β values, and their corresponding inversion results using β values of (b) 1, (c) 5, (d) 8, (e) 20, and (f) 100 with a fixed α of 0.74, respectively.

around 10 is also effective (Xu et al., 2023, 2025), the window size in this research is set to the range of 3 to 21 to mitigate potential risks. Given that the optimal maximum window size is highly dependent on the feature of specific research targets, a practical strategy is to perform preliminary tests with different upper window limits and identify the value beyond which the results no longer change significantly. This value can then be selected as the maximum window size to achieve the balance between computational cost and filtering performance. However, as this topic falls beyond the main scope of the current study, it will not be discussed in further detail here.

Table 2
Evaluation metrics of the inversion results shown in Fig. 13.

β	MAE	PSNR	SSIM
1	1.6293×10^5	29.8110	0.9195
5	1.6260×10^5	29.8190	0.9199
8	1.6257×10^5	29.8215	0.9199
20	1.6260×10^5	29.8236	0.9200
100	1.6264×10^5	29.8235	0.9200

The bold contents denote the best performance under each metric.

Regarding the denoising parameter $\frac{\lambda_3}{2\lambda_4}$, it determines the minimum magnitude of reflection coefficients to be preserved. In the conventional MTV regularization scheme, the L_2 -TV denoising serves for the solution of $\mathbf{m}$ without sparse constraints, which is typically smooth at the start of the iteration. That is, the MTV regularization promotes the sparsity of reflection coefficients through the denoising process, whereas it is highly dependent on the choice of $\frac{\lambda_3}{2\lambda_4}$. In contrast, the L_2 -TV denoising in the proposed approach is applied to the results enhanced by the AEPF, and its effect is to suppress slight model noise rather than sharpen boundaries. Fig. 14(a) shows the results without any sparse constraint, and Fig. 14(b) shows the AEPF-based inversion result. Although it exhibits excellent performance as quantified by Table 3, the circled regions present certain discontinuities, which are initially caused by the trace-by-trace implementation of the AEPF and then propagate from the reference model to the inversion results under the guidance of the lateral constraint during the iteration and finally compromises the structural fidelity. Fig. 14(c) shows the distribution of reflection coefficients of filtered inversion results after one, five, and ten iterations, respectively. As the iteration process advances, the reflection coefficients become concentrated around zero, with respective standard deviations of 0.0101, 0.0185, and 0.0189. Considering that these distributions are

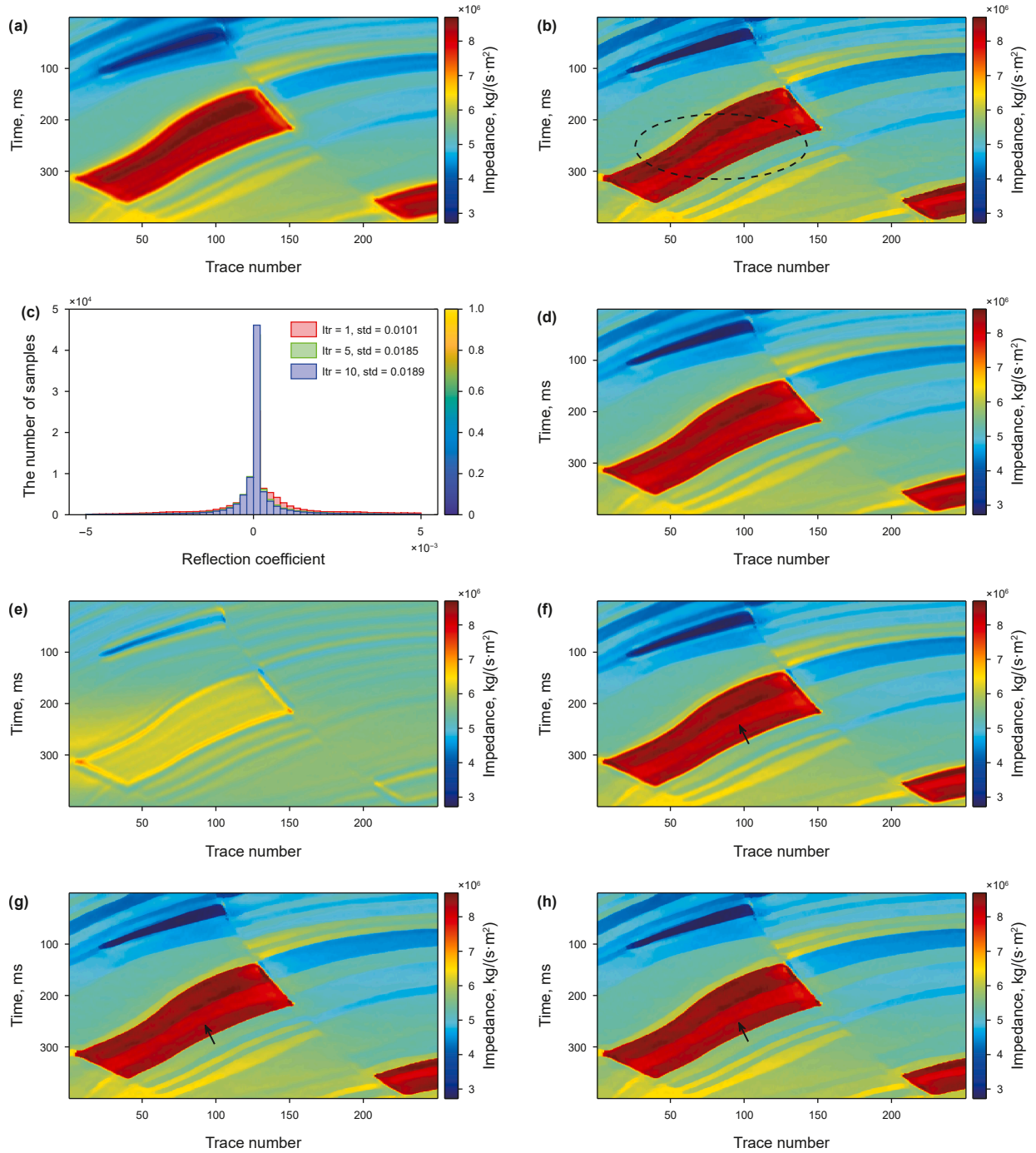


Fig. 14. (a) Inversion results without sparse constraints, (b) AEPF-based inversion results and (c) associated reflection coefficients distribution of filtered inversion results at different iterations. (d)–(f) are MTV-based inversion results with $\frac{\lambda_3}{2\lambda_4} = 0.03$ and $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-4}$, $\frac{\lambda_3}{2\lambda_4} = 0.03$ and $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-5}$, and $\frac{\lambda_3}{2\lambda_4} = 0.003$ and $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-5}$, respectively. (g)–(h) are inversion results enforced by the proposed approach with $\frac{\lambda_3}{2\lambda_4} = 0.03$ and $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-4}$, and $\frac{\lambda_3}{2\lambda_4} = 0.003$ and $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-5}$, respectively.

also close to a normal distribution, the magnitudes of the vast majority of reflection coefficients are below the standard deviation. Therefore, the standard deviation offers a basis for setting denoising parameters. To maintain a single control variable, we select 0.015 as the denoising target, and employ 0.03 and 0.003 as the denoising parameters for comparative analysis. In practical applications, this value can be determined based on the standard deviation of well-log data or derived from AEPF-based inversion

results. Here, $\frac{\|\mathbf{u}^k - \mathbf{u}^{k-1}\|_2}{\|\mathbf{u}^k\|_2} \leq 10^{-5}$ is used as the termination criterion in the denoising process.

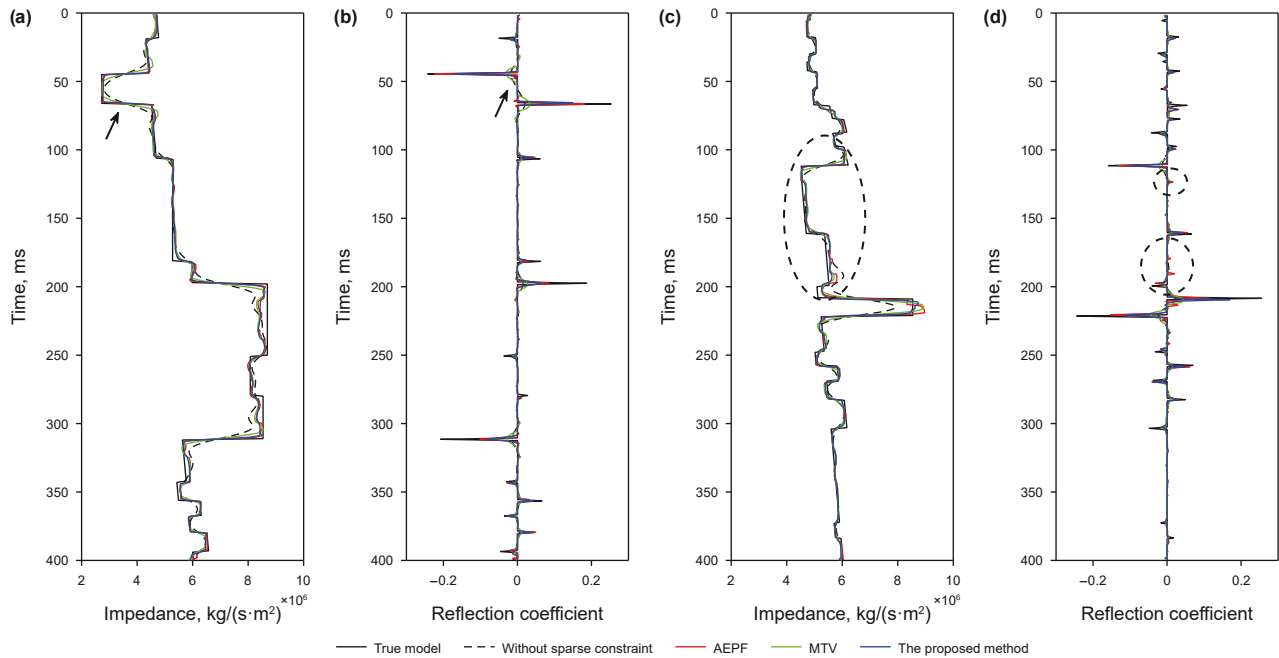
Denoising performance is not only affected by the denoising parameters but also by the number of iterations. As compared in Fig. 14(d) and (e), both examples employ 0.03 as the denoising parameter, but Fig. 14(d) uses a convergence criterion of 10^{-4} , while Fig. 14(e) employs a criterion of 10^{-5} . Since the soft

Table 3

Evaluation metrics of the inversion results shown in Fig. 14. DTC denotes the denoising termination condition.

Method	DTC	MAE	PSNR	SSIM	L ₀ norm
AEPF		9.4112×10^4	34.0119	0.9641	2.8756×10^4
MTV (0.03)	$\ \mathbf{u}^k - \mathbf{u}^{k-1} \ _2 \leq 10^{-4}$	1.3318×10^5	31.8570	0.9462	3.9332×10^4
MTV (0.03)	$\ \mathbf{u}^k \ _2 \leq 10^{-5}$	7.2440×10^5	18.3963	0.6299	4.3254×10^4
MTV (0.003)	$\ \mathbf{u}^k - \mathbf{u}^{k-1} \ _2 \leq 10^{-5}$	1.2359×10^5	31.9774	0.9563	3.7570×10^4
MTV (0.003)	$\ \mathbf{u}^k \ _2 \leq 10^{-5}$				
The proposed (0.03)	$\ \mathbf{u}^k - \mathbf{u}^{k-1} \ _2 \leq 10^{-4}$	1.0003×10^5	33.9803	0.9652	2.7719×10^4
The proposed (0.03)	$\ \mathbf{u}^k \ _2 \leq 10^{-4}$				
The proposed (0.003)	$\ \mathbf{u}^k - \mathbf{u}^{k-1} \ _2 \leq 10^{-5}$	9.3534×10^4	34.1284	0.9690	2.1929×10^4
The proposed (0.003)	$\ \mathbf{u}^k \ _2 \leq 10^{-5}$				
The proposed (0.003)	$k = 10$	9.3652×10^4	34.0227	0.9671	2.5480×10^4

The bold contents denote the best performance under each metric.


Fig. 15. (a) Inversion results and (b) corresponding reflection coefficients of the 75th trace, and (c) and (d) are the associated results of the 150th trace.

threshold shrinkage reduces impedance gradients with each iteration, excessive denoising iterations may lead to the blurring of vertical impedance features. Meanwhile, 10^{-5} can be set as the denoising convergence threshold when $\frac{\lambda_3}{2\lambda_4}$ is set to 0.003, which exhibits superior performance, as supported by Table 3. To preserve fine-scale details, it is recommended to employ a relatively small $\frac{\lambda_3}{2\lambda_4}$, such as one-fifth of the standard deviation of the reflection coefficients derived from the AEPF-processed inversion results. Fig. 14(g) and (h) show the inversion results obtained by the proposed method. Here, reflection coefficients exceeding 0.015 are protected from the soft-thresholding shrinkage process. These results exhibit better lateral continuity compared to Fig. 14(b), particularly as indicated by the arrows, the slightly lower impedance strips within the high-impedance targets are clearly reconstructed. Meanwhile, they exhibit sharper boundaries compared to Fig. 14(d) and (f). Furthermore, another advantage of the proposed method is that when it is difficult to determine an appropriate

termination condition, the number of denoising iterations can serve as an alternative condition. For instance, setting a $\frac{\lambda_3}{2\lambda_4}$ of 0.003 and performing ten or slightly more denoising iterations also yields satisfactory results, as proved by Table 3.

To provide an intuitive comparison of the vertical imaging resolution, the inverted impedances and reflection coefficients of the 75th and 150 traces are shown in Fig. 15. For greater clarity, only the optimal results of each method are presented. The AEPF-based inversion shows superior resolution, especially in delineating sharp boundaries, as noted by the arrows. However, the oscillatory impedances in the circled flat areas indicate the limitation of the AEPF in adequately maintaining blocky characteristics. The MTV-based inversion generates relatively smooth results, whereas the recovered amplitudes of the reflection coefficients are suppressed due to the denoising algorithm. The proposed approach achieves better reconstructions that simultaneously preserve the sharp boundaries and fine-scale details. In addition,

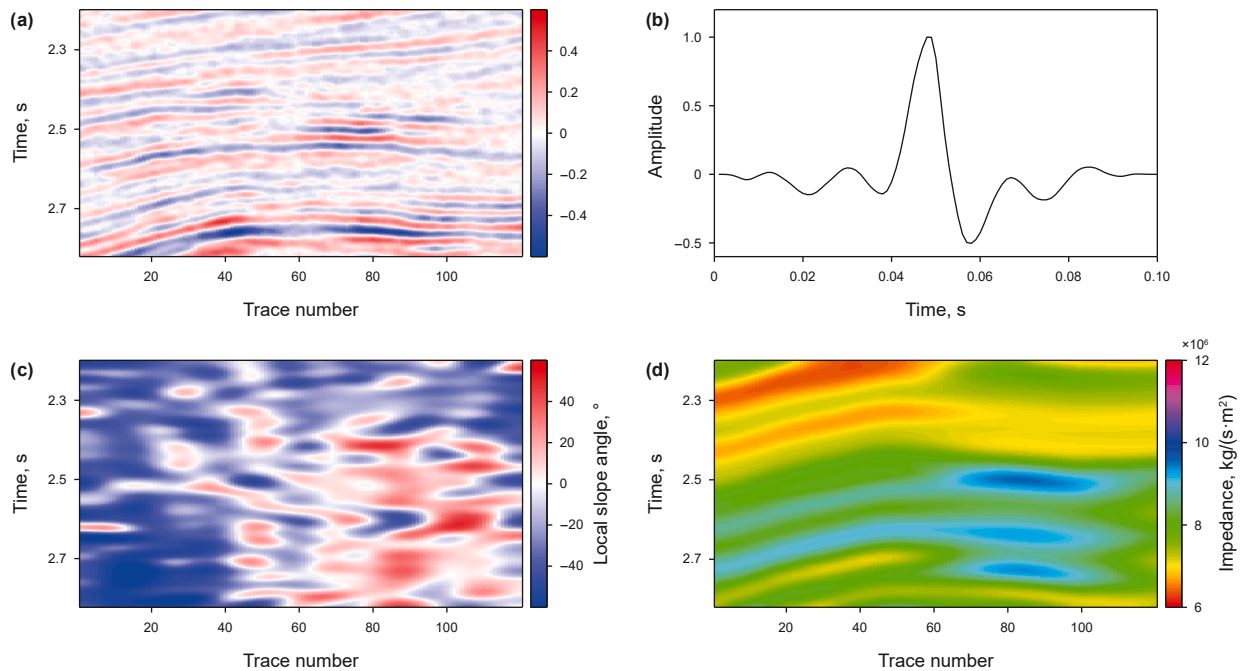


Fig. 16. (a) Normalized poststack data, (b) statistical source wavelet, (c) estimated local slope angles, and (d) initial model.

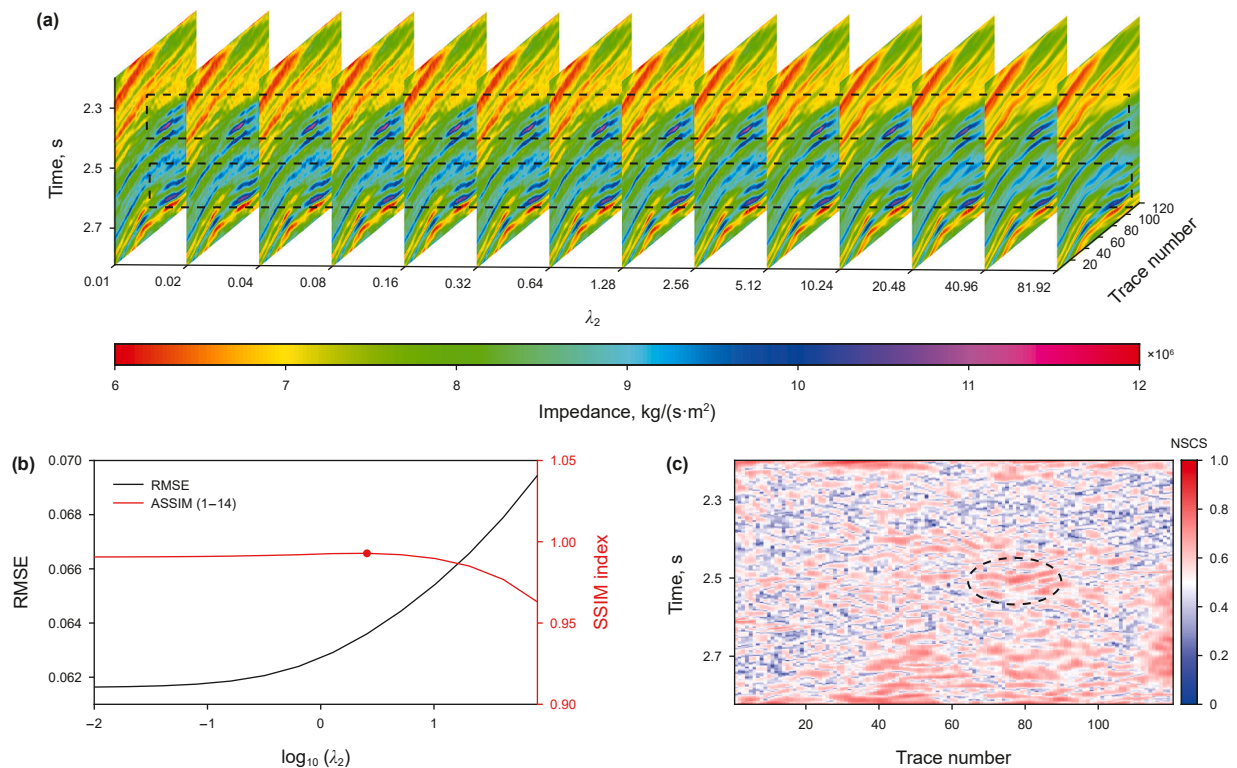


Fig. 17. (a) Inversion results of the field data using Eq. (9) with varying λ_2 values, (b) RMSE and ASSIM index variations, and (c) NSCS index distribution.

we use a pseudo L_0 norm to assess the sparsity of recovered reflection coefficients, which denotes the number of reflection coefficients greater than 0.0015. The evaluation results shown in Table 3 verify that the proposed method can recover sparser reflection coefficients and possesses a stronger ability to preserve blocky features. Furthermore, these cases also demonstrate that

the sparse constraint need not be explicitly coupled with the data term. Instead, it can be achieved through continuously enhancing the blocky features of the reference model, functioning independently of the data-fitting process. Hence, the inversion can produce blocky models under the constraints of the reference model and lateral constraint alone.

4. Field example

The proposed inversion approach is applied to field data from the research of Wang et al. (2021). The obtained dataset comprises gathers of 43 different angles ranging from 3° to 45° , which have been processed by attenuation compensation, noise suppression, normal moveout correction. Fig. 16(a) presents the normalized poststack data, while Fig. 16(b) presents the average of five statistical wavelets extracted from these gathers, respectively. This average wavelet, with a dominant frequency of 25 Hz, serves as the source wavelet for the field data inversion. Fig. 16(c) and (d) show the local slope angles and initial model, respectively. The initial model is built from well-log data after a crucial time-depth conversion to match the depth-domain logs with the time-domain seismic data. Then, the low-frequency components of multiple wells are interpolated in the time-domain to generate the initial model.

Fig. 17(a) shows 14 inversion results using different λ_2 values. As shown in the circled regions, the high-impedance anomalies become smooth with the increase of λ_2 . Fig. 17(b) presents the corresponding RMSE and ASSIM indices with a determined λ_2 of 2.56. Here, we extend the RMSE threshold to 15% to highlight the distorted areas, as noted in Fig. 17(c). Meanwhile, it is observed

that the NSCS values in these areas are relatively close to the mean value of the NSCS plus twice its standard deviation. Therefore, to effectively mitigate the unnecessary structural smoothing, α is set to the mean value plus its standard deviation 0.62, while β is set to 8 to prevent the complete truncation. Based on the logging results from two wells at the 41st and 84th traces, the denoising parameter is set to 0.0092 and iterate ten times in the denoising process, and the rest parameters are set to be the same as in the synthetic examples. Given that our goal is to verify the effectiveness of the proposed method in preserving the inherent discontinuity, we also use an overly large λ_2 of 100 for comparison.

Fig. 18 presents the well-through results reconstructed by different inversions. Fig. 18(a) shows the inversion results using the conventional MTV regularization method without the structural constraint. Fig. 18(b) and (c) are the inversion results obtained by the structure-guided MTV (SGMTV) method (Li et al., 2024). With the introduction of the structural constraint, the continuity of the inversion results is improved. However, as λ_2 increases to 100, the lateral continuity becomes overly enhanced, and this in turn causes the circled high-impedance anomalies to be smoothed by the background. Although the structural constraint improves the lateral continuity, it overly intensifies this effect with this penalty intensity, and results in the decreased lateral

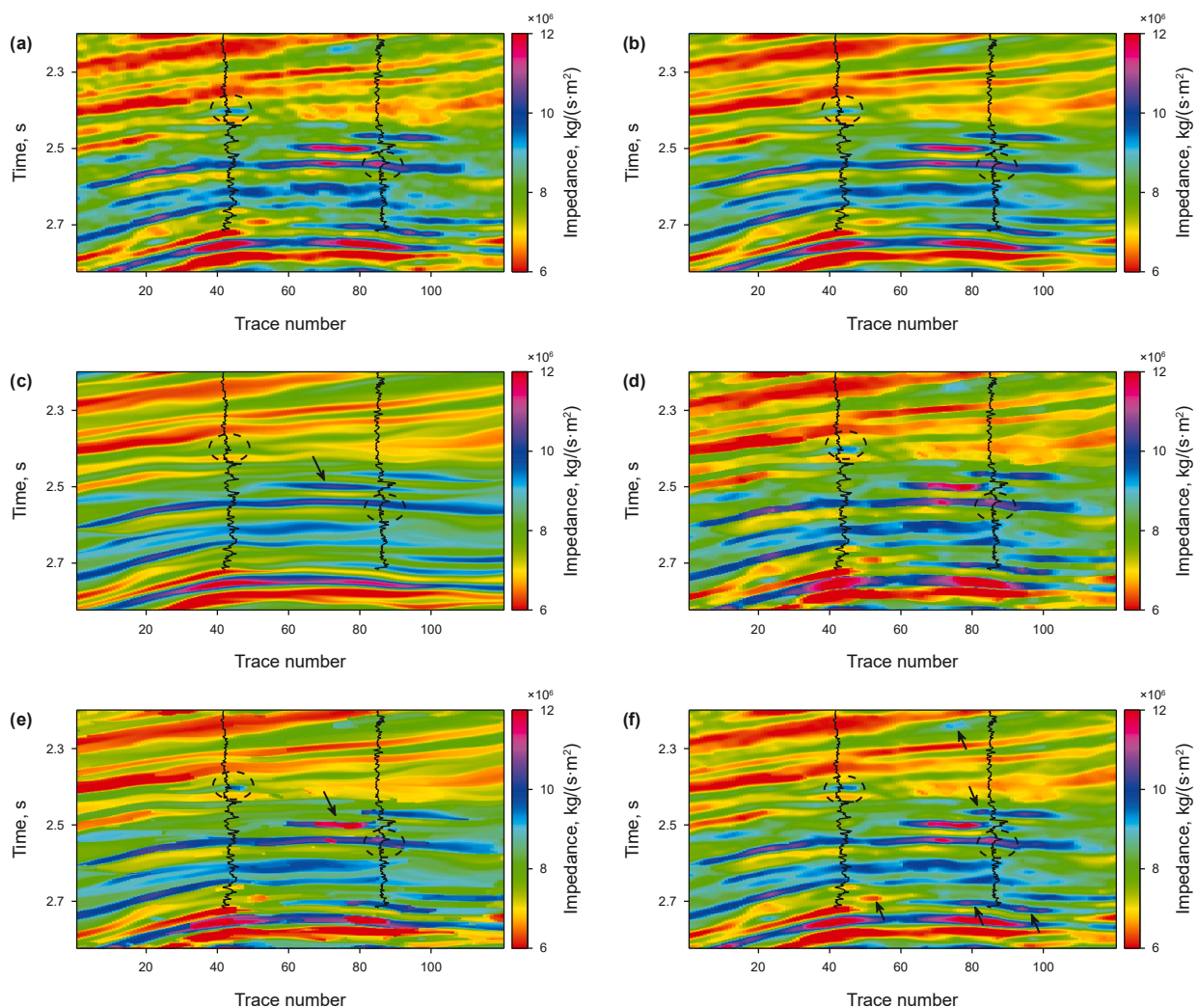


Fig. 18. Inversion results obtained by (a) the conventional MTV method without the structural constraint with a λ_2 of 2.56, the SGMTV method with λ_2 values of (b) 2.56 and (c) 100, the proposed method with λ_2 values of (d) 2.56 and (e) 100, and (f) the proposed method using solely the L_2 -TV denoising with a λ_2 of 2.56.

resolution. In comparison, Fig. 18(d) and (e) show the inversion results obtained by the proposed approach. When λ_2 is set to 2.56, the results are quite similar to Fig. 18(b), with the only notable difference being a sharper definition of the vertical interfaces. As λ_2 increases to 100, the lateral continuity is enhanced across most regions, while the circled and noted discontinuous targets are well preserved. This is further supported by the 1-D impedance comparison shown in Fig. 19, and the associated correlation coefficients provided in Table 4. Since the 41st trace lies entirely within the continuous region, the inversion results shown in Fig. 18(a) without the lateral constraint show that the impedances of its adjacent traces on both the left and right sides are quite close. As a result, the inverted impedances from the mentioned four inversions vary relatively smooth and show high consistency. Thus, our focus is more on the preservation of discontinuous targets at the 84th trace. As noted by the arrows in Fig. 19(b), under the excessively strong uniform lateral constraint, the impedances are suppressed by their lateral neighboring impedances, which limits the recovery of such features and yields the lowest correlation coefficients. In contrast, the results of the other three inversions are relatively similar.

The logging results indicate that the impedance does not present a perfect blocky structure in the vertical direction, so the vertical resolution of the proposed method is not significantly different from that of the previous SGMTV approach. However, if only the AEPF are individually applied, the inversion results would like to exhibit obvious staircase-like results even when true impedances are smooth based on previous studies (Dai et al., 2019;

Xu et al., 2023, 2025), and may not always achieve an ideal match with the logging data. In contrast, our approach uses the model processed by the AEPF and L_2 -TV denoising as a reference model, thereby allowing blocky features to be recovered when the true impedance structure is blocky, while avoiding the presence of the AEPF-caused artifacts when the true impedance structure is smooth. Meanwhile, in scenarios where the impedance does not exhibit blocky features, we also recommend using only the L_2 -TV denoising to yield the reference model **u**. As shown in Fig. 18(f), the resulting inversion results are smoother than Fig. 18(d), while also preserving more of the fine-scale discontinuous details indicated by the arrows compared to Fig. 18(b). The related correlation coefficients of the 41st and 84th traces are 0.7043 and 0.7651, respectively, higher than the results shown in Fig. 18(b)–(e).

It also should be noted that the main innovation of this research lies in the adaptive structural constraint scheme, which focuses on mitigating structural distortion in lateral and preserving discontinuous features. The adaptive structure-guided constraint scheme and the implicit sparsity constraint are independent strategies, and do not require mutual validation. Instead, they can be flexibly applied according to practical needs. The field examples demonstrate that exceedingly strong lateral constraint intensities significantly impact the accuracy of reconstructed discontinuous impedance targets. However, when the lateral constraint intensity is appropriately applied, the inversion results obtained by different methods are comparable in vertical, which is reasonable.

5. Discussion

Different from existing structure-guided inversion methods that uniformly impose structural constraints, this study proposes a novel adaptive scheme to adjust the lateral penalty intensity to mitigate potential structural distortions. The lateral penalty intensity is governed by the absolute penalty intensity λ_2 , and the relative penalty intensity W_i . Both are determined based on a series of structure-guided inversion results obtained by Eq. (9) with varying λ_2 values. The initial bounds set for λ_2 critically influence both the parameters, especially its upper bound. Relatively small values lead to a weak lateral constraint, whereas overly large values may distort the recovered impedance structure. Consequently, the λ_2 determined by the ASSIM with excessively large values may be overestimated. To address this, a threshold, such as 10% above the minimum RMSE, can be applied to constrain the upper bound of λ_2 values. For relatively simple models that have few lateral discontinuities, this threshold can be set slightly higher, such as 15%. Fortunately, the relative penalty intensity W_i partially compensates for the issue. Although overly large λ_2 values can highlight the artifacts, they can also increase the SCS index within the regions prone to distortion, which in turn suppresses the excessive lateral constraint. Besides, the initial reference model **u** is also crucial in estimating the SCS index. Since the SCS index is calculated from a series of the closed-form solutions, using poor initial models may fail to accurately capture structural distortions. Thus, it is recommended to use good initial models, which can be derived from structure-guided interpolation results from well-log data or outputs from multiscale inversion. Moreover, in the numerical case, the high-impedance anomalies diffuse into the low-impedance areas under overly large lateral penalty intensities, whereas in the field example, the high-impedance anomalies are restricted from full recovery by their surrounding low-impedance background. This discrepancy indicates that the SCS index cannot be regarded as an equivalent indicator of discontinuous regions. As a passively defined metric, its true function is to identify regions prone to structural distortion. While these regions tend to exhibit discontinuous characteristics, this is not always the case.

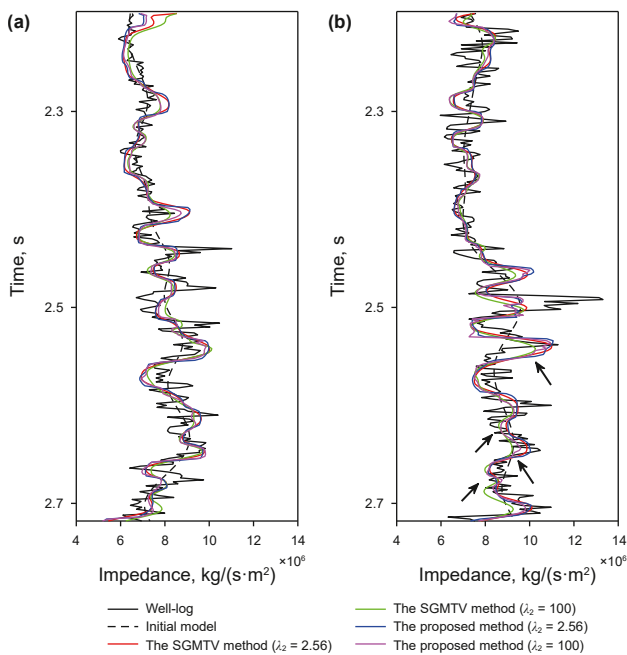


Fig. 19. Comparison of the inverted 1-D impedances at the (a) 41st and (b) 84th traces.

Table 4
Correlation coefficients of the inverted impedance at the 41st and 84th traces.

Method	SGMTV		The proposed approach	
λ_2	2.56	100	2.56	100
The 41st trace	0.7485	0.7383	0.7509	0.7593
The 84th trace	0.7035	0.6595	0.6836	0.6948

Existing structure-guided inversion approaches typically impose explicit vertical constraints along the direction perpendicular to local structures. However, the presence of too many constraint terms in the objective function complicates the balance among different terms. In contrast, the proposed method enforces the constraint in the vertical direction implicitly. This design enhances the vertical imaging resolution via the AEPF, followed by the L_2 -TV denoising within the MTV regularization framework to further improve the imaging quality. Here, the lower and upper bounds of filtering window size of the AEPF play a crucial role. The lower bound is typically set to three to preserve sharp boundaries, while the upper bound should be set sufficiently large to ensure the blocky characteristic. Regarding the L_2 -TV denoising, the ratio $\frac{\lambda_3}{2\lambda_4}$ governs the degree of smoothing applied to impedance gradients. In practice, it should be carefully selected based on prior well-log information to preserve fine-scale details. Meanwhile, to retain sharp boundaries, particularly large gradients should be avoided. In the absence of prior well-log data, it can be set based on the statistical distribution of reflection coefficients of the AEPF-filtered results. The current implementation of this step relies on this combined strategy, which has demonstrated high efficiency. Given that the AEPF is designed for 1-D signals and the conventional L_2 -TV denoising is sensitive to certain parameters, it would be beneficial to develop an adaptive 2-D edge-preserving denoising algorithm to enhance the applicability of the proposed inversion framework. Theoretically, the AEPF can be extended for 2-D images, while its relevant performance, such as computational cost and effectiveness in enhancing resolution, and noise suppression remains unstudied and requires in-depth verification. This falls outside the scope of the present study. In future study, we will investigate the effectiveness of the 2D AEPF as well as denoising techniques with boundary enhancement to optimize the current scheme.

This study proposes a purely physics-driven inversion framework for scenarios where prior structural information is not necessarily fully reliable. However, this study does not fundamentally address the inherent defect of structural estimation for poststack seismic data, and high-precision structural estimation remains the cornerstone of high-resolution imaging. In future work, two further aspects are worthy of investigation. The first concerns the resolution of the data used to generate structural features. Future work can first apply deconvolution to poststack data to mitigate the effect of limited wavelet bandwidth, and then calculate local slope angles from the reflection coefficient profile with relatively higher resolution. The second relates to approaches for calculating local slope angles. Given that PWD struggles with complex structures due to its local planar wave assumption, the multiscale ST approach (He et al., 2023) and deep learning-based methods (Huang et al., 2021b; Zu et al., 2022) can be adopted as alternatives.

6. Conclusion

Since the inherently limited bandwidth of the wavelet inevitably compromises the accuracy of structural estimation in discontinuous areas, conventional structure-guided inversion methods that apply uniform structural constraints may introduce artifacts and lead to biased interpretations when overly strong penalties are applied. To address this challenge, a novel structure-guided inversion framework is developed to adaptively formulate the structural constraint. In this framework, the structural constraint is applied only in the lateral direction, with the intensity jointly governed by the absolute penalty intensity and relative penalty intensity. Given the strong correlation between the ASSIM

and the true SSIM, this index can evaluate whether the inversion is subject to excessive lateral constraints. Thus, it serves to reliably determine the appropriate upper bound of the absolute penalty intensity. In parallel, the relative penalty intensity is modulated by the SCS index, which intrinsically characterizes regions prone to structural distortion. By distinguishing these areas from continuous regions, it preserves fine-scale features from being corrupted by inaccurate structural information. Furthermore, the sparse constraint is decoupled from the objective function and implicitly enforced through the MTV regularization scheme, where the AEPF is integrated prior to the L_2 -TV denoising to enhance boundary delineation. By continuously updating the reference model, the blocky characteristic of the reconstructed impedance can be highlighted. Moreover, this strategy avoids imposing blocky artifacts where the true impedance structure is smooth. Both numerical and field examples underscore the crucial role of appropriate structural constraints and demonstrate the proposed approach as a promising and practical tool for high-resolution seismic imaging.

CRedit authorship contribution statement

Hao Li: Writing – original draft, Software, Methodology, Conceptualization. **Yi-An Cui:** Writing – review & editing, Validation, Funding acquisition. **Pu Wang:** Writing – review & editing, Funding acquisition, Formal analysis. **Jing-Ye Li:** Formal analysis. **Jian-Xin Liu:** Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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