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Original Paper

Refinery optimization in OPEC member states: An integrated inverse DEA approach and economic viability analysis

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ABSTRACT

Refinery optimization is critical for ensuring energy security and economic resilience in Organization of the Petroleum Exporting Countries (OPEC) member states, where persistent inefficiencies and heavy reliance on fuel imports remain major challenges. These issues not only strain foreign exchange reserves but also expose domestic markets to global price volatility. This study applies an integrated inverse data envelopment analysis (IDEA) approach to determine optimal refinery capacity upgrades and throughput adjustments under realistic operational constraints. The methodology is generalizable to OPEC countries and is demonstrated using Nigeria as a case study, given its severe gasoline shortages despite significant investments in refinery expansion. Two model variants were analyzed: the unconstrained model, which suggested impractical capacity upgrades, and the constrained model, which produced feasible adjustments after accounting for gasoline demand shortages. Results indicate that optimized domestic refining could supply approximately 285 thousand barrels of gasoline daily, meeting 81.7% of national demand, while fully satisfying distillate requirements. However, kerosene refining remains economically unsustainable under current market conditions. Complementing the technical analysis, an economic viability assessment shows that gasoline and distillate refining offer highly attractive returns, with payback periods of less than one year, reinforcing the importance of prioritizing these products. Policy implications include adopting data-driven capacity planning across OPEC, implementing selective product prioritization, and channelling investments toward economically viable refinery upgrades to reduce import dependence, stabilize fuel prices, and strengthen energy security. The proposed framework provides a practical decision-making tool for governments and industry stakeholders seeking to enhance refining efficiency and resilience in volatile energy markets.

1. Introduction

Refinery capacity optimization is a critical issue for oil-producing nations, particularly those in the Organization of the Petroleum Exporting Countries (OPEC). Despite being major crude oil exporters, several OPEC member states face persistent shortages of refined petroleum products, leading to heavy reliance on imports and exposure to global price volatility (Ghoddusi and Wirl, 2021; Iheukwumere et al., 2021). This paradox undermines energy security, economic resilience, and sustainable development in these economies. Addressing this challenge is essential in the context of global energy transition and increasing domestic demand for refined products.

Without the refining process, crude oil in its natural form would have minimal practical use, leaving the global energy industry incapable of fulfilling the needs of contemporary society (Armaroli and Balzani, 2007). Advanced refining methods enable the precise separation and processing of hydrocarbons, yielding gasoline, diesel, jet fuel, and essential petrochemical feedstocks, which are key components that drive transportation systems, support industrial expansion, and form the foundation of numerous synthetic products (Favennec, 2022). Beyond its functional importance, refining plays a pivotal role in global economic stability and energy security. Nations with advanced refining capabilities wield greater control over their energy markets (Ruble,

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2019), reducing dependence on external suppliers and mitigating the economic volatility associated with crude oil exports. Moreover, the refining industry is a hub of innovation, continuously evolving to enhance efficiency, reduce environmental impact, and adapt to shifting market trends. With the growing emphasis on sustainability, refiners are implementing cutting-edge technologies to produce cleaner fuels, optimize resource utilization, and align with stringent environmental regulations (Akhtar et al., 2024; Dahiya, 2020). This dynamic sector is not only integral to industrial progress but also deeply intertwined with geopolitical developments, trade policies, and technological advancements. As the world transitions toward diversified energy sources, refining remains a cornerstone of energy production, bridging traditional fossil fuel dependence with emerging sustainable practices.

1.1. Oil refining within OPEC: current trends and challenges

Within OPEC, refining plays a pivotal role in maximizing the economic value of crude oil exports. Several member states have invested in refining infrastructure to strengthen domestic energy security and reduce reliance on imported refined fuels. Leading oil producers such as Saudi Arabia, the UAE, and Kuwait boast expansive refining complexes that cater to both local and international markets. However, disparities exist among OPEC nations, with countries like Venezuela and Nigeria grappling with outdated facilities and operational inefficiencies, forcing them to import refined petroleum despite possessing vast crude reserves.

Driven by projections that fossil fuels will account for 75% of global energy consumption by 2040, reaching 111 million barrels per day (mb/d) (Sato et al., 2022), the refining landscape within OPEC has expanded significantly, particularly across emerging economies. Between 2024 and 2029, global refining capacity is expected to increase by 6.3 mb/d, with the bulk of new installations concentrated in Asia–Pacific (3.2 mb/d), Africa (1.4 mb/d), and the Middle East (1.2 mb/d) (OPEC, 2024).

This shift underscores a broader trend of refining capacity migrating from industrialized nations to regions with surging energy demand. Looking ahead to 2050, refining additions are projected to reach 19.2 mb/d, with nearly 90% of new capacity situated in Asia–Pacific, Africa, and the Middle East (OPEC, 2024). This strategic expansion aligns with OPEC's long-term vision of fortifying refining capabilities among member states, ensuring a resilient supply chain for refined petroleum products while mitigating reliance on external markets.

Despite these anticipated advancements in refining infrastructure and strategic expansion among OPEC nations, the sector continues to grapple with significant challenges that threaten its long-term viability. Refining margins remain under pressure, struggling to recover post-pandemic crisis, averaging \$1.8 per barrel, which is notably below \$4.25 per barrel five-year benchmark (Forbes, 2021). Additionally, structural overcapacity persists, with an excess of 2 mb/d of refining capacity commissioned before the crisis, further straining profitability and market equilibrium (Forbes, 2021). Compounding these issues is the sector's environmental footprint, as refining operations account for 5% of global oil and gas-related emissions, necessitating urgent investments in carbon capture technologies, electrification, and biofuel production to meet evolving sustainability standards. Moreover, the accelerating shift toward electrification in transportation poses a profound challenge to future refining demand. With the proliferation of electric vehicles (EVs) and aggressive energy transition policies, global oil consumption is projected to experience a substantial decline, particularly under scenarios emphasizing rapid decarbonization. If not strategically addressed, these challenges could undermine OPEC's refining competitiveness, necessitating structural reforms, technological innovation, and adaptive market strategies to ensure resilience in an increasingly uncertain energy landscape.

OPEC's refining sector stands at a crossroads, balancing expansion with sustainability while navigating evolving market dynamics. Strategic investments in modernization, efficiency, and environmental compliance will be crucial in ensuring the long-term viability of refining operations within OPEC member states.

1.2. The need for Nigeria's refinery expansion

Nigeria's refining sector has long grappled with inefficiencies, necessitating substantial enhancements in capacity and throughput to meet domestic demand and curtail reliance on imports. Despite possessing a nominal refining capacity of 445,000 barrels per day (b/d), operational challenges have led Nigeria to import over 80% of its refined petroleum products (PwC, 2017). To address these shortcomings, the Nigerian Upstream Petroleum Regulatory Commission (NUPRC) has outlined domestic crude oil refining requirements, emphasizing the need for a stable crude supply to optimize refinery operations. The government has set ambitious targets, including a projected daily crude oil requirement of 770,500 barrels for local refineries, with the Dangote refinery alone expected to process approximately 99.55 million barrels in the first half of 2025 (PwC, 2017). Additionally, reforms have been initiated to revitalize the refining sector, such as the removal of fuel subsidies to stimulate private sector investment. The anticipated completion of refinery upgrades in Port Harcourt and Warri, coupled with the full-scale operation of key refining facilities, is projected to significantly reduce Nigeria's dependence on imported petroleum products (Olujobi et al., 2022).

The pressing need to expand Nigeria's refinery capacity becomes even more evident when examining the imbalance between annual imports and exports of refined petroleum products, as illustrated in Figs. 1 and 2. Fig. 1 highlights the notably low volume of refined product exports, whereas Fig. 2 presents a significantly higher volume of imports. Specifically, the volume of gasoline imports depicted in Fig. 2 exceeds the exports shown in Fig. 1 by over tenfold across the four production years (i.e., 2019–2022). This underscores the essential role of gasoline in Nigerian society, driven by its escalating demand, which is intrinsically tied to the country's growing population. To mitigate the excessive dependence on imported refined gasoline, proactive managerial intervention is crucial. Nigeria's refining capacity must be strengthened through both immediate refinery expansion and

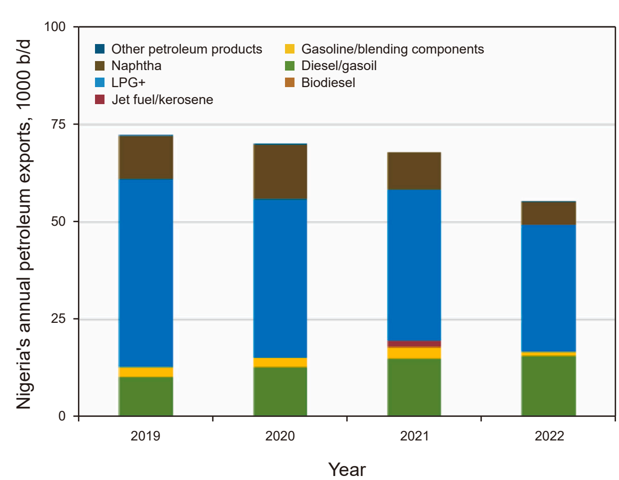

Fig. 1. Nigeria's annual petroleum exports (data source EIA (2023)).

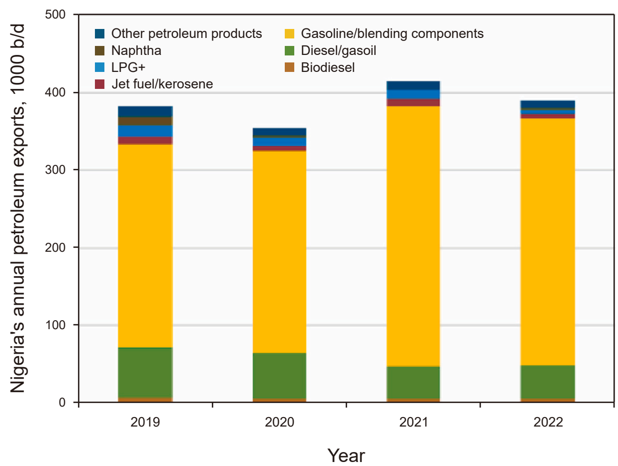

Fig. 2. Nigeria's annual petroleum imports (data source EIA (2023)).

the strategic establishment of modular refineries, providing a lawful alternative to illicit artisanal (Lala et al., 2024) operations.

From a managerial standpoint, mathematical optimization models serve as invaluable tools in determining the necessary capacity and throughput upgrades in Nigeria's refining sector (Gbakon et al., 2022). These models enable policymakers and industry leaders to allocate resources efficiently, minimize costs, and maximize output, all while considering factors like crude oil supply constraints, refinery efficiency, and fluctuating market demand. One effective approach is the transshipment framework, which optimizes crude oil allocation across domestic refining facilities and export markets (Alizadeh and Karimi, 2024; Sharma and Jana, 2009). Through this model, Nigeria can identify the most cost-effective distribution strategy to enhance refinery utilization and curb reliance on imports. Furthermore, optimization techniques can be employed to assess modular refinery operations, ensuring that refining processes are structured to maximize overall efficiency (Michael et al., 2023).

Extending this analysis to inverse data envelopment analysis (DEA), this method offers a means to compute the required upgrades in refining capacity by identifying efficiency frontiers and adjusting input factors to achieve optimal performance. Inverse DEA allows policymakers to evaluate the necessary enhancements in infrastructure, technological advancements, and operational efficiency, aligning Nigeria's refining sector more closely with global benchmarks. Leveraging such analytical methodologies enables strategic planning for refinery expansions and upgrades, ultimately strengthening domestic refining capabilities and bolstering energy security. Thus, the primary motivation for this study is to establish a mathematical framework that systematically computes sustainable upgrades in Nigeria's refining capacity and throughput. These upgrades will empower policymakers and industry stakeholders to advance Nigeria's domestic refining capabilities, simultaneously reducing the nation's dependency on imported petroleum products and fostering long-term energy stability.

1.3. Problem statement

Several OPEC member states continue to face challenges in refining crude oil domestically, with capacities ranging from modest operations to complete absence of infrastructure. For instance, Equatorial Guinea currently lacks a domestic refinery, while nations such as Congo and Gabon operate with minimal capacity. These disparities contribute to widespread reliance on imported refined petroleum products, leaving countries vulnerable to price volatility, supply disruptions, and economic instability.

OPEC countries present a unique context for this study because they collectively account for a significant share of global crude oil production while simultaneously struggling with inadequate domestic refining capacity (Orisaremi et al., 2021, 2023b). This structural imbalance makes them ideal candidates for evaluating optimization strategies that can improve energy security and economic resilience. Nigeria exemplifies this challenge, with chronic shortages of gasoline and other refined products despite being Africa's largest oil producer.

The primary goal of this study is to develop a robust framework that combines inverse DEA modelling with economic viability analysis to support decision-making for refinery investments in OPEC countries. This approach aims to provide actionable insights for policymakers and industry stakeholders seeking to enhance energy security and reduce dependency on imports.

In the existing literature, refinery optimization studies have primarily focused on forward-looking efficiency models or deterministic approaches. However, there is limited research on inverse optimization techniques that determine the required input adjustments to achieve desired output levels under efficiency constraints. Furthermore, the integration of technical optimization with economic viability analysis remains underexplored in the context of OPEC countries. This study seeks to fill these gaps by addressing the following research questions.

1. How can inverse DEA be applied to optimize refinery capacity and throughput for OPEC member states?
2. What are the economic implications of implementing such optimization strategies in real-world scenarios?

By addressing these questions, this research introduces several novel contributions.

1. The study introduces two variants of inverse DEA models: an unconstrained version and another constrained by upper limits on refinery capacity and throughput.
2. Integration of economic viability analysis to complement technical optimization, offering a dual perspective on feasibility.
3. Application of the proposed framework to OPEC member states, with a detailed case study on Nigeria, a country facing gasoline shortages despite its crude oil abundance.
4. Facilitation of short-term refining strategies to increase the share of domestically refined products while reducing reliance on imports.
5. Optimization of refining capacity and throughput to mitigate product shortages, reduce fuel price volatility, and curb inflationary pressures.

The remainder of this paper is structured as follows: Section 2 reviews optimization methods for oil refining and recent applications of DEA and inverse DEA; Section 3 details the research methodology; Section 4 presents findings and analysis; Section 5 discusses managerial implications; Section 6 presents the conclusions; and Section 7 highlights limitations and future directions.

2. Literature review

Optimization methods have played a pivotal role in sustainability research within the energy sector. In this study, we will present a concise review of the relevant literature in two distinct sections. The first section explores the broad applications of optimization models in oil refining, offering insights into their effectiveness and implementation. The second section explores the latest developments in both DEA and inverse DEA, with the latter serving as the chosen methodology for optimizing the oil refining process in this study. By synthesizing findings from both areas, we aim to establish a strong foundation for our study.

2.1. Optimization methods for oil refining

Hou and Wu (2017) implemented an enhanced nondominated sorting genetic algorithm (NSGA) to optimize a specific oil refining schedule, framing it as an assignment problem. Cafaro et al. (2015) developed a mixed integer linear programming (MILP) model to optimize refined product pipeline scheduling. Kancijan et al. (2015) developed a robust optimization model tailored to the production processes of complex oil refineries, incorporating market price fluctuations of refined products. Han et al. (2015) utilized a linear programming technique to analyze and compare the performance of 60 refineries in the U.S. and EU, each processing more than 100,000 barrels of crude oil per day. Kazemi and Szmerekovsky (2015) developed a deterministic MILP model to reduce refinery product costs. Hadidi et al. (2016) introduced a mixed-integer nonlinear programming (MINLP) framework aimed at reducing hazardous gaseous emissions within a refinery in Saudi Arabia. Liu et al. (2018) provided a comprehensive review of optimization methodologies applied to petroleum refineries, offering insights into various strategies. Qiu et al. (2022) introduced a strategic model aimed at harmonizing the supply and demand dynamics of refined petroleum products. Al-Jamimi et al. (2022) employed a hybrid method combining machine learning and genetic algorithms to optimize the petroleum refining desulfurization process. Xu et al. (2022) proposed a multi-objective optimization approach to effectively mitigate shortages in the refined product distribution network, enhancing supply chain efficiency. Jiang et al. (2023) integrated refinery models and life cycle assessment to assess the effects of blending at refineries. Qin and Ye (2024) proposed a multi-objective optimization framework aimed at reducing costs and carbon emissions in a refinery's hydrocracking unit. Qiu et al. (2024) formulated a MINLP model for optimizing the distribution of refined petroleum products.

2.2. Overview of recent applications of DEA and inverse DEA

Licerán-Gutiérrez et al. (2025) employed DEA to assess efficiency within the European banking sector. Zhao et al. (2025) developed a parallel network DEA to enhance training efficiency amid growing postgraduate enrollment. Pishini et al. (2025) utilized DEA to evaluate operational effectiveness in water and sewerage services. Avilés-Sacoto et al. (2025) adapted the DEA framework to evaluate technical efficiency in cases where inputs influence only specific outputs. Wu et al. (2024) applied a network DEA framework to analyze the global airline industry's performance during the COVID-19 pandemic. Wang and Feng (2024) utilized the meta-frontier DEA approach to investigate the underlying factors influencing the decoupling between industrial economic growth and carbon emissions. Borrás et al. (2024) applied DEA benchmarking to refine planning strategies using selected peer comparisons. Sun et al. (2023) leveraged a parallel DEA model to measure the efficiency of higher education systems in China. Eydi and Rastgar (2022) introduced a DEA-based approach to determine the most efficient logistics provider in the context of hospital waste management. Khare et al. (2021) utilized DEA to evaluate sustainability in land use within transit-oriented development frameworks.

2.3. Inverse DEA applications in petroleum supply chain management

To strengthen the rationale for this study, we provide a concise review of recent applications of inverse DEA within petroleum supply chains. This review is essential for understanding the progress achieved to date and for identifying persistent research gaps across the supply chain. Table 1 summarizes all relevant studies, including the current work. As shown in Table 1, while the six previous studies have made notable contributions to operations management in the oil and gas sector, they share two critical limitations: an exclusive focus on upstream activities and the absence of economic viability assessments that could inform managerial decisions for investors and policymakers. The present study addresses these gaps by extending inverse DEA to downstream oil refining processes and incorporating a comprehensive economic viability analysis for each refined petroleum product, thereby bridging technical optimization with managerial economics.

3. Methods

As this study focuses on shortages of desirable outputs, namely refined petroleum products from the oil refining process, it is essential to first adopt a base DEA model that focuses solely on desirable outputs. This means excluding undesirable outputs, such as flare gas, particularly in the upstream sector of the petroleum supply chain. To establish a foundational framework, this study introduces the CCR DEA model as the selected base model. This model will be employed to assess the initial refining efficiency of each OPEC member, utilizing production data sourced from the 2024 OPEC ASB.

3.1. Preliminaries

In this section, we present the traditional CCR model (Charnes et al., 1978) for assessing the refining efficiency of DMU_o, $o \in \{1, 2, \ldots, n\}$

Table 1
Inverse DEA in petroleum supply chains.

Article	Key contribution	Upstream	Downstream	Economic viability assessment (NPV & Rate of return)
Wegener and Amin (2019)	Minimized GHG emissions	✓		
Orisaremi et al. (2021)	Optimized gas-to-wire process	✓		
Orisaremi et al. (2022)	Lean production framework	✓		
Orisaremi et al. (2023a)	Optimized energy mix	✓		
Orisaremi et al. (2023b)	Balanced economic growth with environmental sustainability	✓		
Orisaremi et al. (2025)	Optimal allocation of production quotas	✓		
Current study	1) Optimizing refinery expansion; 2) Economic viability of refinery upgrades		✓	✓

$$\text{M1}: \text{Min } \theta_R^{\text{CRS}}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_R^{\text{CRS}} x_{io}, \; i = 1, 2, \ldots, m \tag{1}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \;\; r = 1, 2, \ldots, s$$

$$\lambda_j \ge 0, \; j = 1, 2, \ldots, n$$

Given that:

DMU_o : decision making unit under evaluation

θ_R^{CRS} : refining efficiency of DMU_o under CRS

x_{ij} : ith input of DMU_j

y_{ro} : rth output of DMU_j

λ_j : weight assigned to DMU_j

If $\theta_R^{\text{CRS}} = 1$, it indicates that the DMU_o functions as an efficient oil refiner, while $\theta_R^{\text{CRS}} < 1$ suggests inefficiency. Using M1 as the base model, its inverse DEA counterpart originally introduced by Wei et al. (2000), can be stated using the same notation as M1 in the following manner:

$$\text{M2}: \text{Min } (\alpha_1, \alpha_2, \ldots, \alpha_m) = (x_{1o+} \Delta x_1, x_{2o} + \Delta x_2, \ldots, x_{mo} + \Delta x_m)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_R^{\text{CRS}} (x_{io} + \Delta x_{io}), \; i = 1, \ldots, m \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \; r = 1, \ldots, s$$

$$\lambda_j \ge 0, \; j = 1, \ldots, n$$

3.2. Basic terminologies and proposed methodology

Before outlining the methodology of this study, it is essential to first clarify key terminologies associated with the oil refining process, as defined by OPEC in its annual statistical bulletin. Since the objective is to identify the optimal refinery expansion required to address shortages in refined products, it is crucial to specify that this study focuses on two primary inputs: refinery capacity R_c and refinery throughput R_T. While these terms are sometimes used interchangeably, it is necessary to delineate their technical distinctions within the context of oil refining.

- Refinery capacity: This refers to the maximum amount of crude oil that a refinery is designed to process over a given time frame, typically measured in barrels per day or metric tons per year. It represents the theoretical maximum output a refinery can achieve under optimal conditions, considering the facility's design, equipment, and configuration.
- Refinery throughput: This is the actual amount of crude oil that a refinery processes during a specific period. It reflects the real-world performance of the refinery and can vary based on factors such as market demand, crude oil supply, maintenance schedules, and external disruptions.

Additionally, the primary products derived from the oil refining process include gasoline, kerosene, distillates, residuals, and other byproducts (OPEC, 2024). However, since residuals and certain byproducts are essentially remnants of the refining process, it is evident that the key refined products in high demand and of significant societal importance are gasoline, kerosene, and distillates. Moreover, it is essential to highlight that one current OPEC member, Equatorial Guinea, lacks a domestic refinery, necessitating its exclusion from our subsequent analysis. Consequently, for these reasons and to enhance the discriminatory power of the base model M1 in assessing the refining efficiency of OPEC members, this study will exclusively consider gasoline, kerosene, and distillates as output variables.

In traditional inverse DEA model M2 and its advancements, past and recent studies commonly assume uniform output increases to drive corresponding input changes, with Δy_{ro} remaining consistent across all outputs. However, this approach is not suitable for our study, as each output faces a different demand shortage. To effectively address these shortages, M2 must be slightly modified to incorporate individual demand variations, as demonstrated in model M3.

$$\text{M3}: \text{Min } (\alpha_1, \alpha_2, \ldots, \alpha_m) = (x_{1o+} \Delta x_1, x_{2o} + \Delta x_2, \ldots, x_{mo} + \Delta x_m)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_R^{\text{CRS}} (x_{io} + \Delta x_{io}), i = 1, \; 2 \tag{3}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \delta_{ro}, \; r = 1, \ldots, s$$

$$\lambda_j \ge 0, \; j = 1, \ldots, n$$

Note that δ_{ro} as the demand shortage of product r in DMU_o.

Regarding the existing framework of model M3, there is a potential risk that it may generate unrealistic or unsustainable projections for upgrades in refinery capacity and throughput, particularly for inefficient OPEC members as identified by the base model M1. This issue can arise in a scenario when a high demand shortage for output r of DMU_o (i.e., δ_{ro}) leads to unsustainable increases in refinery capacity and throughput for the evaluated OPEC member. For instance, given that Nigeria's domestic refineries did not produce a single barrel of gasoline for two consecutive years (2022 and 2023), applying model M3 would yield excessively high upgrades for both refinery capacity and throughput, figures that may be impractical for a developing nation like Nigeria. Therefore, it is imperative to modify or calibrate model M3 to ensure the computation of sustainable upgrades, similar to the adjustments proposed in the study by Orisaremi et al. (2025). To accomplish this, we incorporate two upper-bound constraints, ensuring both technical feasibility and practical applicability in the computed upgrades for refinery capacity and throughput. In this study, these upper bounds are referred to as threshold capacity (T_C) and threshold throughput (T_T), defined as follows:

- Threshold capacity: The refinery capacity among a given of OPEC members, at which any computed upgrade would be deemed unsustainable for the member under evaluation.

- Threshold throughput: The refinery throughput across a group of OPEC members, with all calculated upgrades remaining below this threshold.

In practical application, these constraints are applied to the OPEC member with the largest refinery capacity and throughput. Naturally, such a member is likely to be deemed efficient under model M1 and may experience little to no shortage in any of its refined products. Building on these modifications and ensuring sustainability, we propose the following inverse DEA model to address and resolve shortages in refined products.

$$\text{M4}: \text{Min }(\alpha_1, \alpha_2, \ldots, \alpha_m) = (x_{1o} + \Delta x_1, x_{2o} + \Delta x_2, \ldots, x_{mo} + \Delta x_m)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_R^{\text{CRS}} (x_{io} + \Delta x_{io}),\ i = 1, 2$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \delta_{ro},\ r = 1, \ldots, s \tag{4}$$

$$x_{1o} + \Delta x_{1o} < \text{T}_\text{C}$$

$$x_{2o} + \Delta x_{2o} < \text{T}_\text{T}$$

$$\lambda_j \ge 0,\ j = 1, \ldots, n$$

In its current structure, model M4 considers only two input variables: refining capacity and refining throughput. As previously stated, δ_{ro} represents the demand shortage of refined product r in DMU_o, while T_C and T_T denote the threshold capacity and threshold throughput, respectively. A demand shortage occurs when the demand for a refined product surpasses the supply from domestic refineries. If no shortage exists for a particular refined product, or if domestic supply exceeds demand, then δ_{ro} is set to zero for that product in DMU_o.

3.2.1. Model assumptions and applicability

To enhance comprehension for readers who may be unfamiliar with the inverse DEA methodology, we first state the fundamental assumptions that form the basis of the inverse DEA framework. Subsequently, we highlight the distinct applicability of models M3 and M4 to clarify their respective roles within practical contexts:

1. As an inverse optimization approach, models M3 and M4, along with the inverse DEA model introduced by Wei et al. (2000), assume that the efficiency score of each decision-making unit (DMU) calculated by the base DEA model M1 remains constant or does not decline when a change in an input and/or output variable results in a corresponding optimal adjustment in other input and/or output variables.
2. In this study, forward optimization is defined as the process of evaluating the refining efficiency of each decision-making unit, represented here by an OPEC member. Conversely, the inverse optimization problem focuses on determining the necessary upgrades in refining capacity and throughput required to eliminate shortages in refined petroleum products, such as gasoline, kerosene, and distillate.
3. Similar to the approach in Orisaremi et al. (2025), which introduced and applied two versions of inverse DEA (bounded and unbounded) for practical oil production allocation, this study adopts a comparable methodology by proposing two variants of inverse DEA (M3 and M4) to facilitate practical and sustainable upgrades in refinery capacity and throughput. Specifically, M3 represents the unconstrained (unbounded) inverse DEA, while M4 denotes the constrained (bounded) inverse DEA.
4. The key technical distinction between M3 and M4 lies in their treatment of constraints. M3 estimates theoretical upgrades without incorporating calibrated maximums such as the T_C and T_T. Consequently, M3 may produce results that are not feasible in real-world refining contexts. In contrast, M4 is an improved model that integrates both thresholds as additional constraints, ensuring alignment with practical conditions. From a mathematical standpoint, this guarantees that M4 consistently computes realistic and sustainable upgrades for each producer.
5. Although the calibration or determination of both thresholds T_C and T_T in real-world scenarios is entirely at the discretion of the decision maker, this study, in consultation with industry experts, establishes these thresholds based on a decision-making unit that meets the following criteria: (i) it is an efficient oil refiner, (ii) it has no shortages across its refined products, and (iii) it possesses the largest refinery capacity and throughput.

Drawing on the stated model assumptions and applicability, this study clearly addresses an inverse problem within the broader domain of inverse optimization. A defining characteristic of inverse DEA is its ability to preserve efficiency, positioning it as an advanced managerial tool for practical allocation challenges that demand rigorous and innovative sensitivity analysis in complex multi-input multi-output settings (Orisaremi et al., 2023b). Alternative mathematical approaches such as mixed integer nonlinear programming (MINLP), system dynamics, integer programming, regression and correlation analysis, and life cycle assessment are inadequate for addressing such intricate inverse problems where maintaining evaluated efficiencies within production or service systems is essential.

While our primary focus is on applying model M4 in this study, it is essential to compare its findings with those of M3 to reinforce the validity of our proposed methodology. In the following sections, we will first present the preliminary results of model M1, followed by a comparative evaluation of M3 and M4. It is important to highlight that the final demand fulfillment ratios for each refined product, derived from the optimal solutions of model M4, will serve as the basis for the economic evaluation of all three products. This step is critical in guiding policymakers and prospective investors in the Nigerian petroleum sector toward identifying the most viable products for strategic investment. Accordingly, this forms the second and concluding phase of our study. To enhance clarity and accessibility of our methodology, we present a comprehensive summary of its steps along with a detailed process flow diagram (PFD) in Fig. 3.

4. Application, results, and analysis

This section substantiates our proposed methodology through an empirical evaluation using data from OPEC's Annual Statistical Bulletin (ASB) (2024). The dataset encompasses production years spanning from 2019 to 2023, with our analysis specifically concentrating on the latest available year, 2023. It comprises two input variables: refinery capacity and refinery throughput, as well as three output variables: gasoline, kerosene, and distillates. Notably, all selected inputs and outputs are quantified in units of 1000 b/d.

4.1. Preliminary findings

Employing model M1, we conducted the initial phase of analysis on 11 OPEC member nations, excluding Equatorial Guinea

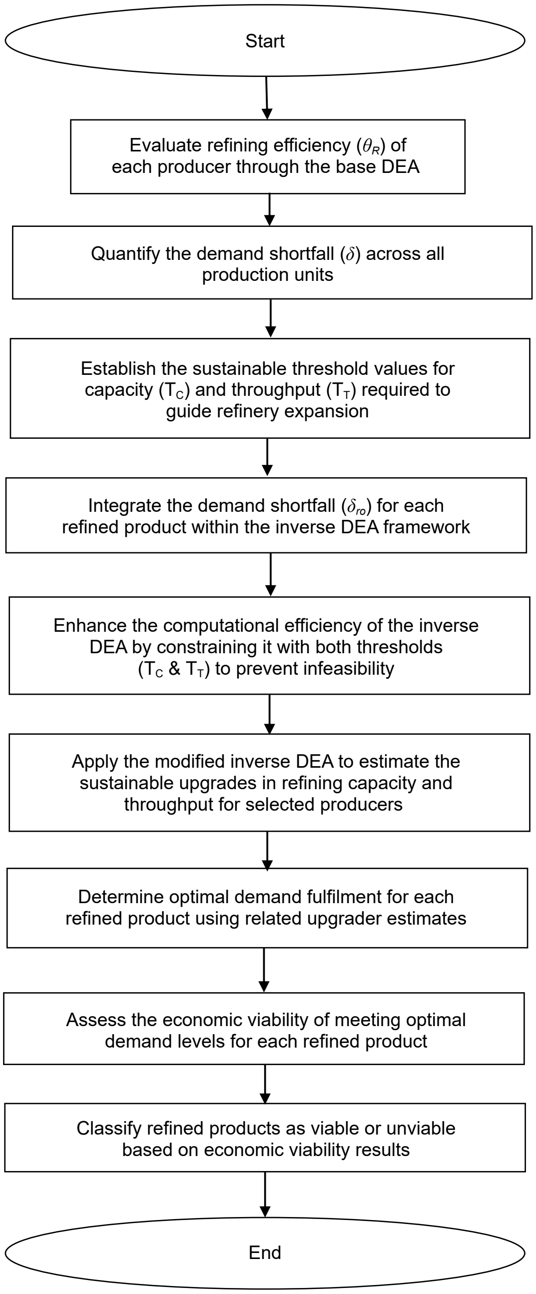

Fig. 3. Summary of research methodology.

from the full cohort of 12 current members due to its lack of a domestic refinery. The corresponding results are presented in Table 1. Based on their refining efficiency scores falling below unity, Algeria, Iran, Iraq, Libya, Nigeria, and Venezuela are classified as inefficient under the application of model M1. Consequently, we delineate our sample set of prospective case studies as follows:

$$A = \{DMU_1, DMU_4, DMU_5, DMU_7, DMU_8, DMU_{11}\}$$

A deeper examination of Table 1 unveils critical insights into the prevailing refinery challenges. Notably, Nigeria is classified as inefficient, registering the lowest efficiency score among the assessed OPEC members while also exhibiting the most severe shortages across all three refined products. Notably, its gasoline shortfall surpasses those of Iran, Iraq, and the UAE by more than

Table 2
Preliminary results under M1

DMU	Refining efficiency θ_R^{CRS}	Demand shortage		
		Gasoline δ_1	Kerosene δ_2	Distillates δ_3
1-Algeria	0.9972	0	0	1
2-Congo	1.0000	0	0	0
3-Gabon	1.0000	5	1	25
4-Iran	0.9711	159	0	0
5-Iraq	0.4982	112	0	15
6-Kuwait	1.0000	10	0	0
7-Libya	0.5076	92	0	48
8-Nigeria	0.4319	349	11	78
9-Saudi Arabia	1.0000	0	0	0
10-UAE	1.0000	154	0	0
11-Venezuela	0.7339	0	1	16

twofold. This highlights the critical need for managerial action and supports refinery expansion in Nigeria via our proposed inverse DEA. Conversely, Table 2 indicates that both Congo and Saudi Arabia, despite being classified as efficient, experienced no shortages in any of the three refined products. While this reflects a commendable performance from both nations, it is crucial to highlight Saudi Arabia's significantly superior efficiency compared to Congo, attributable to its substantial surpluses in domestically refined products. This efficiency advantage aligns with Saudi Arabia's status as the OPEC member with the largest refinery capacity and throughput. Consequently, in our subsequent analysis, we establish the threshold capacity and throughput for this study based on Saudi Arabia's refinery benchmarks. Figs. 4 and 5 provide a clear depiction of these initial findings.

4.2. Main findings

To determine the necessary refinery upgrades for all members in set A, we employ our proposed models, M3 and M4, with the summarized results presented in Table 3. Under M3, all members, except Nigeria, require refinery capacity and throughput enhancements of significantly less than one thousand. A closer examination reveals that Algeria and Venezuela require only minimal improvements, with Algeria needing enhancements in refinery throughput and Venezuela in refinery capacity. Moreover, Table 3 indicates that no upgrades exist for Algeria's refinery throughput or Venezuela's refinery capacity, suggesting that these nations might benefit more from managerial strategies or refined operational policies to mitigate their product shortages during the production year.

Nigeria, however, presents a substantial managerial challenge, as evidenced by the exceptionally high refinery capacity and throughput upgrades required to address its supply shortfalls. This

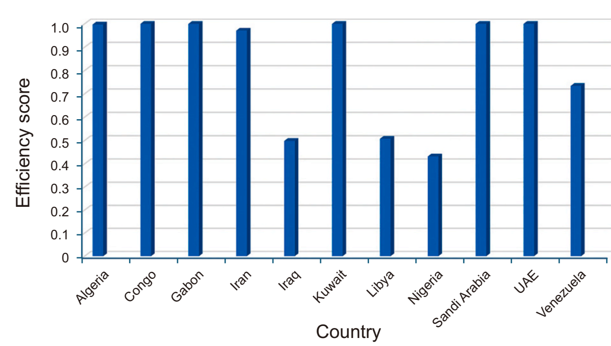

Fig. 4. Refining efficiency across OPEC members.

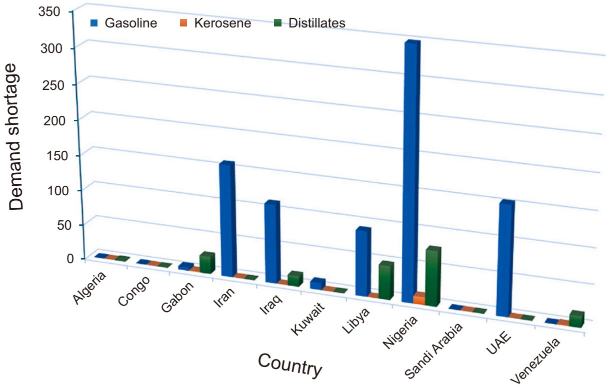

Fig. 5. Product demand shortage across OPEC members.

Table 3
Optimal refinery upgrades under M3 and M4

DMU_o	Optimal refinery upgrades, 1000 b/d			
	M3		M4	
	Δx_{1o}	Δx_{2o}	Δx_{1o}	Δx_{2o}
1-Algeria	2.9946	0.0000	2.9946	0.0000
4-Iran	483.7373	583.6248	483.7373	583.6248
5-Iraq	380.1983	726.8450	380.1983	726.8450
7-Libya	176.8747	646.6485	176.8747	646.6485
8-Nigeria	2271.8410	3043.618	-	-
11-Venezuela	0.0000	0.0035	0.0000	0.0035

predicament stems from the considerable demand shortages across all three products for Nigeria, as detailed in Table 2. Furthermore, it is crucial to recognize that Nigeria's refinery upgrade requirements far exceed sustainable levels. This is underscored by the fact that its computed upgrades surpass those of Saudi Arabia, as reported in the OPEC bulletin. Specifically, Nigeria's projected refinery throughput enhancement of 3043.618 thousand barrels per day in Table 3 exceeds Saudi Arabia's actual throughput, recorded at 2514 thousand barrels per day by OPEC. This comparison is not intended to undermine Nigeria's refining capabilities, but rather to illustrate the distinction between theoretical projections and practical feasibility. Nigeria's increase in refining capacity by 650 thousand barrels per day, achieved through the completion and commissioning of the Dangote refinery in 2023, took decades to materialize. Thus, achieving the estimated refinery capacity upgrade of 2271.841 thousand barrels per day, as calculated in Table 3, would require extensive long-term planning and strategic management rather than immediate implementation. Consequently, it is evident that the refinery upgrades computed for Nigeria in Table 3 are not viable as short-term solutions but must be considered long-term projects.

To address the issue of unsustainable upgrades for Nigeria, we turn to model M4, a constrained variant of M3 that imposes a safe upper limit on refinery enhancements. Interestingly, when applied, model M4 produces identical upgrade computations for all members except Nigeria. In fact, the model proves infeasible for Nigeria, as demonstrated by the absence of calculated upgrades for the nation in Table 3. This presents a dilemma: M3 has already projected unrealistic refinery upgrades for Nigeria, while M4 fails to yield any feasible solution. The causal factor for this infeasibility can be discussed from two perspectives. Firstly, it is imperative to note that Nigeria has historically struggled with oil refining over the last two decades, as evidenced by the fact that large percentages of refined products are processed in foreign refineries

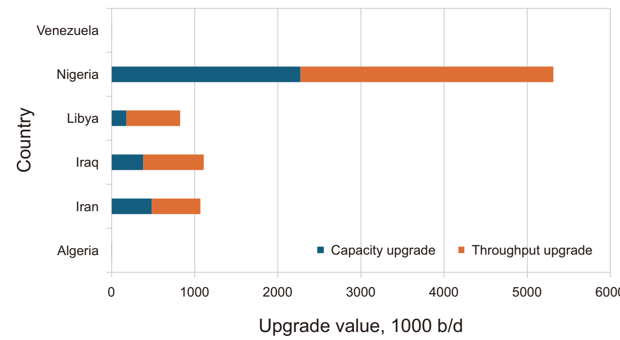

Fig. 6. Capacity and throughput upgrades under M3.

(Iheukwumere et al., 2020). In fact, before the commissioning of the Dangote refinery, Nigeria had one of the lowest refinery utilization rates (i.e., 8.67%) among OPEC members (Orisaremi et al., 2021). This poor record of capacity utilization has been linked to inadequate maintenance of existing refineries and insufficient supply of crude oil (Ogbuigwe, 2018). Hence, the infeasibility of model M4 is clearly reflected in these real-world challenges encountered in the downstream sector of the Nigerian petroleum supply chain.

From a mathematical perspective, this infeasibility is evident as it stems from the additional constraints incorporated within the framework of model M4. In fact, this inherent infeasibility of inverse DEA due to structural modifications is a common limitation highlighted by researchers in recent years (Lu et al., 2024; Zhang and Cui, 2016). Nevertheless, this study addresses this infeasibility through a rigorous sensitivity analysis of product mixes and demand scenarios.

Given that Nigeria's demand shortage across all three products is the highest among members, as indicated in Table 2, this may be a key factor contributing to M4's infeasibility for the nation. As with the previous section, we illustrate these computed upgrades under both models in Figs. 6 and 7.

4.2.1. Sensitivity analyses

To explore the feasibility region of model M4 for Nigeria's sustainable refinery upgrades, we will conduct a series of sensitivity analyses. The first step involves examining the optimal product mix of gasoline, kerosene, and distillates to determine whether any trade-offs or compromises can be made to establish a feasible solution for Nigeria. If a viable product mix is identified under M4, we will then refine the analysis to determine the proportion of the demand shortage for each product that can be satisfied within this mix.

As outlined in Table 4, four distinct product scenarios were considered under M4 for Nigeria. Interestingly, when M4 was

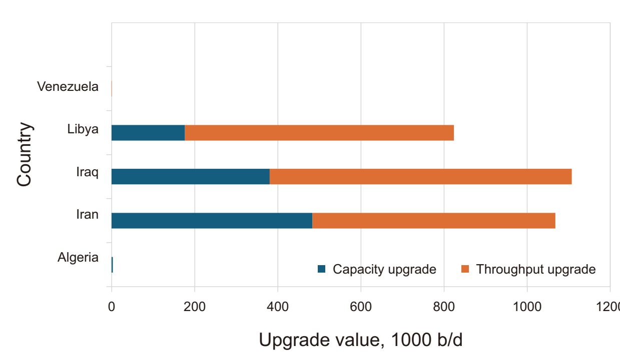

Fig. 7. Capacity and throughput upgrades under M4.

Table 4
Sensitivity analysis for a feasible product mix under M4

DMU	Scenario 1		Scenario 2		Scenario 3		Scenario 4	
	Gasoline		Gasoline & Kerosene		Gasoline & Distillates		Kerosene & Distillates	
	Δx_{1o}	Δx_{2o}	Δx_{1o}	Δx_{2o}	Δx_{1o}	Δx_{2o}	Δx_{1o}	Δx_{2o}
Nigeria	-	-	-	-	-	-	0.0000	389.3772

configured to address only the demand for gasoline, excluding kerosene and distillates, it remained infeasible for Nigeria. Although this may seem counterintuitive, the high gasoline demand shortage likely plays a key role in Nigeria's infeasibility under M4. To further investigate, two alternative product mixes were tested: one where M4 computed the necessary upgrades to satisfy the combined demand for gasoline and kerosene, and another where M4 addressed only gasoline and distillates. However, both scenarios remained infeasible for Nigeria, mirroring the outcome observed when gasoline was considered independently. At this juncture, it is becoming increasingly evident that gasoline demand is the root cause of infeasibility for Nigeria. For final validation, we examined a fourth scenario in which M4 was tasked with fulfilling demand for kerosene and distillates alone. This configuration successfully yielded feasible refinery upgrades for Nigeria. Consequently, we can reasonably conclude that Nigeria's exceptionally high demand shortage for gasoline is the sole factor rendering M4 infeasible from the outset.

It is reasonable to assume that a portion of Nigeria's significant gasoline demand shortage could be met through the application of model M4. To explore this possibility, we follow the approach outlined by Orisaremi et al. (2025) and conduct an additional sensitivity analysis, adjusting the gasoline demand shortage either upward or downward. Specifically, we examine five distinct scenarios: 70, 140, 210, 280, and 349, with 349 representing the actual demand shortage for Nigeria. The primary objective of this secondary analysis is to determine whether reducing the demand shortage to lower levels would make M4 feasible for Nigeria. The findings from this sensitivity analysis are presented in Table 5. As evidenced in the table, model M4 remained feasible and fully operational across the first four scenarios, up to a demand shortage of 280 thousand barrels. Predictably, however, the fifth and final scenario reflecting Nigeria's actual gasoline demand shortage proved infeasible for M4. This clearly establishes that Nigeria's significant gasoline demand shortfall, combined with its low refining efficiency, constrains the feasibility region of model M4. Conversely, reducing demand shortages and possibly enhancing refining efficiency could expand the feasibility region of model M4 for Nigeria (see Figs. 8 and 9). This indicates that a portion of Nigeria's unmet gasoline demand will still need to be covered by imports. However, on a positive note, M4 successfully fulfills the demand for both kerosene and distillates in full.

To pinpoint the demand shortage threshold where model M4 stays feasible, an analyst can adjust the sensitivity analysis in

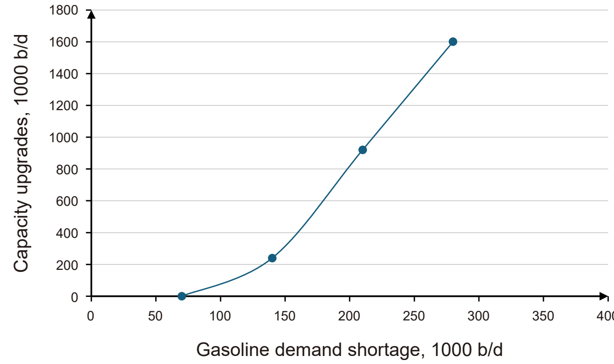

Fig. 8. Effect of gasoline demand shortage on capacity upgrades for Nigeria.

Table 5, focusing on the range between the fourth and fifth scenarios. To achieve this, we examine smaller incremental changes in demand shortage within the 280-to-349-thousand-barrel range, specifically evaluating three additional scenarios: 282, 284, and 286 thousand barrels. The findings of this supplementary analysis, presented in Table 6, reveal that M4 remains feasible up to a gasoline demand shortage of 284 thousand barrels. However, from the third scenario onward, where the demand shortage is set at 286 thousand barrels, M4 becomes infeasible. By conducting a more precise analysis with finer intervals, the final feasible threshold for M4 is identified at 285 thousand barrels of gasoline demand.

For managerial decision-making, if an approximate estimate is required, then 285 thousand barrels is the highest feasible demand shortage that can be accommodated within the total unmet demand of 349 thousand barrels. Mathematically, this indicates that our proposed model M4 can address 81.7% of the gasoline demand, along with fully meeting the kerosene and distillates demand, through appropriate upgrades in both refinery capacity and throughput. At optimal efficiency, the computed upgrade values corresponding to these achievement levels are $\Delta x_{1o} = 1649.475$ and $\Delta x_{2o} = 2480.525$. Based on our comprehensive analysis of Nigeria's refining capabilities, we propose a refining mix consisting of 81.7% domestically refined gasoline and 18.3% imported gasoline to meet the nation's demand. However, no mixes are necessary for kerosene and distillates, as the optimal refinery upgrades, we have calculated, are sufficient to fully satisfy the demand for these products.

4.3. Results of the economic analysis

Following the completion of the technical component of this study, it is essential to assess the profitability and economic viability of each refined product. This requires a comparative economic analysis based on the extent to which demand was fulfilled, as determined by the sensitivity analysis presented in the preceding section. As shown in Table 2, Nigeria experienced

Table 5
Sensitivity analysis for gasoline demand shortage under M4

DMU_o	Scenario 1 (Demand shortage $\delta_1 = 70$)		Scenario 2 (Demand shortage $\delta_1 = 140$)		Scenario 3 (Demand shortage $\delta_1 = 210$)		Scenario 4 (Demand shortage $\delta_1 = 280$)		Scenario 5 (Demand shortage $\delta_1 = 349$)	
	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade,1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade,1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}
Nigeria	0.0000	607.3251	239.4263	1204.767	920.1394	1820.650	1600.853	2436.533	-	-

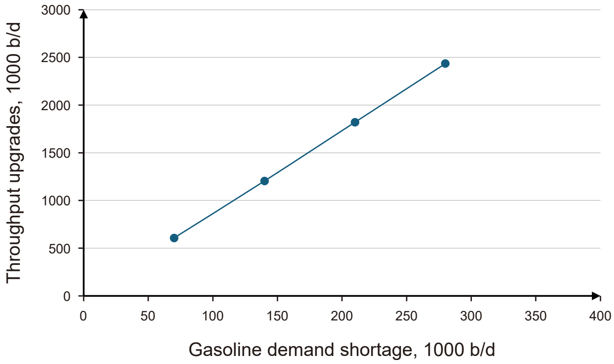

Fig. 9. Effect of gasoline demand shortage on throughput upgrades for Nigeria.

demand shortfalls of 349, 11, and 78 thousand barrels for gasoline, kerosene, and distillates, respectively.

However, the sensitivity analysis revealed that model M4 met only 81.7% of gasoline demand, equivalent to 285 thousand barrels, while the demands for kerosene and distillates were fully satisfied at 100%. Accordingly, the economic evaluation in this section will be based on the following optimal demand fulfilment levels:

- Gasoline: 285 thousand barrels
- Kerosene: 11 thousand barrels
- Distillates: 78 thousand barrels

Due to insufficient data on refinery plant construction in Nigeria, a widely accepted cost estimation method for chemical plants, based on chemical engineering principles, developed by Towler and Sinnott (2021), was used to determine the capital and operational costs, which were then adjusted to the current year. This value was further adapted to the Nigerian context using the Nigerian location factor. A location factor (LF) modifies the base plant cost to account for regional differences in construction and operating conditions, including costs of labor, freight, taxes, duties, procurement, design, and project administration. Currency exchange rates heavily influence LF and hence fluctuate over time (Towler and Sinnott, 2021). The Nigeria-specific location factor, as cited by Oladipupo (2023) and Dibia et al. (2025), was used to adjust the costs to fit the Nigerian basis. The CAPEX and OPEX values for the refinery plant, as reported by Ali et al. (2021), were updated from 2016 to 2023 using the chemical engineering plant index (CEPCI). The CEPCI was 541.7 in 2016 and increased to 797.9 by 2023. Lastly, the sixth rule was also applied to adjust the capacity of the base refinery to the assumed refinery. The base capacity of the refinery is 320 tons/h, which is equivalent to 56,820 b/d. The economic parameters evaluated were the net annual benefit, payback period, and the net present value. Furthermore, in collaboration with industry experts, we assumed an 8% discount rate and a 20-year plan was adopted for the analysis.

4.3.1. Payback period

Payback Time refers to the duration needed to recover the initial investment, and it can be determined using the following equation.

$$\text{Payback period} = \frac{\text{Capital investment}}{\text{Net annual benfit}} \tag{5}$$

4.3.2. The net annual benefit

The net annual benefit is calculated by taking away the operating cost from the annual revenue. It was calculated using Eq. (6)

$$\text{Net annual benefit} = \text{Annual revenue} - \text{OPEX} \tag{6}$$

4.3.3. The net present value

The net present value (NPV) of a project is the sum of the present values of future cash flows. A discount rate of 8% and a plant life of 20 years was used in the calculation of the NPV. The details of the parameters used in the economic analysis are shown in Table 7, while the obtained results are presented in Table 8.

The NPV for gasoline and distillate was positive, indicating that the discounted benefits exceed the associated costs and confirm the investment's financial viability. In contrast, the net annual benefit of kerosene is negative, signalling that the project experiences a yearly net loss. Under such conditions, the project fails to generate sufficient positive cash flow to recover the initial investment, rendering the payback period indeterminable. The financial outlook suggests that kerosene refining would consistently incur losses. For gasoline and distillate, the payback periods are approximately 0.471 years and 0.84 years, respectively. These short durations reflect the high annual returns relative to capital expenditure (CAPEX), which is a promising indicator for potential investors seeking rapid capital recovery. The absence of a payback period for kerosene, due to its negative net annual value, further reinforces its lack of economic viability. In conclusion, the financial metrics clearly favour strategic investment in gasoline and distillate refining, both of which demonstrate strong profitability and swift return on investment. Kerosene, while technically feasible to produce domestically, does not meet the economic threshold for viable investment. These findings provide a data-driven foundation for prioritizing resource allocation in Nigeria's refining sector, enabling policymakers and investors to focus on products with the highest potential for economic impact and energy security.

4.4. Discussion of further findings

To further validate our proposed methodology, we conduct an additional experiment using a new set of production data. In the previous section, we analyzed data from the 2023 production year, which corresponds to the most recent year in the 2024 OPEC ASB. In this section, however, we focus on data from the preceding year, 2022. A key question we seek to address is whether the commissioning of the Dangote refinery in Nigeria in 2023 had an impact on Nigeria's refinery efficiency, as estimated in Table 2. Similar to

Table 6
Extended sensitivity analysis for gasoline demand shortage under M4

DMU_o	Scenario 1 (Demand shortage $\delta_1 = 282$)		Scenario 2 (Demand shortage $\delta_1 = 284$)		Scenario 3 (Demand shortage $\delta_1 = 286$)	
	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}	Capacity upgrade, 1000 b/d Δx_{1o}	Throughput upgrade, 1000 b/d Δx_{2o}
Nigeria	1620.301	2454.130	1639.750	2471.727	-	-

Table 7
Parameters used for economic analysis.

Steps	Description	Value
1	CAPEX for (2016 basis)	$880,900,000
2	Base plant capacity	56,820 b/d
3	Six-tenth rule factor	0.6
4	CEPCI ratio	797.9/541.7 = 1.47
5	Nigeria location factor	0.98
6	Adjusted CAPEX for Nigeria basis (for the year 2023)	Step (1*3*4*5)
7	OPEX for the (2016 basis)	$53 M
8	Adjusted OPEX for Nigeria basis (for the year 2023)	Step (7*3*4*5)
9	Plant life	20 years
10	Refinery base capacity	320 tons/h

Table 8
Results of the economic analysis.

Parameters	Gasoline	Kerosene	Distillate
Payback period	0.471 years	-	0.84 years
Net present value	$65,986,125,705.2	−$715,100,732.52	$15,381,790,984.25
Annual net benefit	$7,058,822,784	−$24,572,473.14	$1,815,153,097.02

section 4.1, we assess the refining efficiency of all members for this production year, with the results presented in Table 9. An initial review of Table 9 highlights an important observation: Nigeria exhibits a refining efficiency of unity, indicating its efficiency for this production year. This finding underscores a decline in refining efficiency compared to the year 2023 production, when the Dangote refinery was commissioned. The primary reason for this reduction is that the new refinery is either not fully operational or has yet to reach its expected capacity, as evidenced by persistent gasoline shortages in Nigeria throughout 2023. Given that Nigeria is classified as efficient for this production year, we will identify the efficient DMUs in Table 9 as our prospective case studies, in accordance with the approach outlined in Section 4.2. This results in a two-sided investigation, where the previous section focused on inefficient DMUs, while this section exclusively examines efficient ones. Based on the findings presented in Table 9, we define our set of efficient DMUs as follows:

$$B = \{\mathrm{DMU}_6, \mathrm{DMU}_8, \mathrm{DMU}_{10}\}$$

Congo and Saudi Arabia are the efficient DMUs excluded from set B, as they do not experience shortages in any refined products.

We then proceed to assess potential upgrades for all members within set B using models M3 and M4. The resulting upgrades are summarized in Table 10. An important observation from this table highlights a key difference from Table 3, where model M4 was previously deemed infeasible for Nigeria. In this case, however, M4 is feasible, with both models yielding identical upgrades for all three producers. The feasibility of M4 for Nigeria can be attributed to the nation's relatively efficient status in this experiment. Despite the increased gasoline shortage demand in Nigeria for this experiment, M4 remains viable for the nation. It successfully meets the demand for all three products while staying within both threshold capacity and throughput constraints. Overall, Nigeria's refining efficiency plays a pivotal role in determining the feasibility of M4 in this extended analysis. Nonetheless, the experimental results from both production years offer valuable insights for policymakers, reinforcing the notion that inverse DEA is highly sensitive to fluctuations in refining efficiency and product demand shortages.

Table 9
Extended results under M1

DMU	Refining efficiency θ_R^{CRS}	Demand shortage		
		Gasoline δ_1	Kerosene δ_2	Distillates δ_3
1-Algeria	0.8484	0	0	8
2-Congo	1.0000	0	0	0
3-Gabon	0.7858	4	1	25
4-Iran	0.9653	150	0	0
5-Iraq	0.4771	122	4	30
6-Kuwait	1.0000	8	0	0
7-Libya	0.5633	88	0	41
8-Nigeria	1.0000	420	14	69
9-Saudi Arabia	1.0000	0	0	0
10-UAE	1.0000	141	0	0
11-Venezuela	0.5433	1	0	16

5. Managerial implications

The oil refining industry remains essential for both the short and long term, especially for developing OPEC nations. Despite ongoing advancements in the global energy transition, refined petroleum products continue to play a significant role beyond meeting energy demands. Key products such as jet fuel and petrochemical feedstocks are indispensable to aviation and synthetic manufacturing, respectively. Given the vital role of refined petroleum products, their ongoing scarcity in an oil-rich nation like Nigeria remains perplexing. Chronic shortages, particularly in gasoline, have long fueled inflation and rising fuel prices, deteriorating citizens' living standards. In response, the Nigerian government has played a pivotal role in commissioning the Dangote refinery and is actively pursuing modular refinery expansion for long-term growth. However, an immediate solution is necessary to curb Nigeria's heavy dependence on imported refined products. This study proposes an inverse DEA methodology as a short-term

Table 10
Extended results under M3 and M4

DMU_o	Optimal refinery upgrades, 1000 b/d			
	M3		M4	
	Δx_{1o}	Δx_{2o}	Δx_{1o}	Δx_{2o}
6-Kuwait	21.0480	18.6796	21.0480	18.6796
8-Nigeria	1292	1507	1292	1507
10-UAE	383.2852	329.9575	383.2852	329.9575

strategy to optimize oil refining, aiming to strengthen domestic refining capacity while minimizing reliance on imports. Nevertheless, managers must consider certain implications when applying this approach.

This study introduces two variations of inverse DEA to optimize refinery expansions among OPEC nations. The key technical distinction lies in the integration of thresholds for refinery capacity and throughput. While the first version remains feasible for all members, it often produces impractical refinery upgrade projections when faced with significant demand shortages of refined products. Conversely, the second version, though more effective in generating realistic upgrades, proves infeasible under high demand shortage conditions. Consequently, decision-makers may employ the first version for theoretical long-term analysis while opting for the bounded model for pragmatic short-term assessments. Additionally, both models can be strategically combined to enhance policy formulation. Sensitivity analysis reveals that lower demand shortages improve the feasibility of the second inverse DEA model. Thus, addressing excessive demand shortages necessitates a balanced approach, integrating a higher proportion of domestically refined products with a lower reliance on imports. Lastly, refining efficiency is a key determinant of our proposed model's feasibility. Our findings indicate that when a unit operates efficiently, the model remains viable even under significant demand shortages. In such cases, sensitivity analyses may not be necessary, as both models applied in this study yield equivalent upgrades in refining capacity and throughput.

6. Conclusions

Taking a practical approach, this study introduces an innovative method for optimizing refinery expansion among OPEC nations through two distinct versions of the inverse DEA model. The primary aim is to mitigate shortages in refined petroleum products, including gasoline, kerosene, and distillates, using Nigeria as the focal case study. This framework enables industry stakeholders to determine sustainable upgrades in refinery capacity and throughput, an increasingly crucial task given Nigeria's prolonged petroleum product scarcity over the past two decades. By adopting this approach, Nigeria's petroleum sector can strengthen its domestic refining capabilities, reducing dependence on imported refined products.

The methodology provides valuable insights for industry policymakers. Initial findings indicate that the unconstrained inverse DEA model successfully computed optimal refinery upgrades for Algeria, Iran, Iraq, Libya, Nigeria, and Venezuela. However, for Nigeria, this model generated impractical projections, necessitating the adoption of a constrained inverse DEA model with upper limits on refinery expansion. While the constrained model proved effective for other OPEC nations, it was initially infeasible for Nigeria due to the severe gasoline demand shortage. To address this challenge, extensive sensitivity analyses were conducted. The first analysis sought a feasible product mix to make the constrained model viable for Nigeria, while subsequent assessments defined the feasible region for refinery expansion. Findings revealed that Nigeria's infeasibility stemmed from an excessive gasoline shortage, despite the model successfully meeting kerosene and distillate demand. This suggested that a portion of gasoline demand would need to be supplemented through imports. Further investigation determined that reducing Nigeria's gasoline shortage to 285 thousand barrels would enable feasibility, leading to the formulation of a refining strategy: 81.7% domestically refined gasoline combined with 18.3% imported gasoline, ensuring full demand coverage for kerosene and distillates. The economic viability assessment reveals that, although it is technically feasible to fully meet the demand for both kerosene and distillates, the local refining of kerosene remains economically unviable. This underscores the need for a more pragmatic strategy among domestic and international investors, as well as policymakers within the petroleum industry.

Beyond solving immediate feasibility concerns, these findings underscore broader implications for refinery planning and energy policy. The inverse DEA models, both unconstrained and constrained, demonstrate their efficacy in refinery expansion strategies, marking a significant advancement in managerial problem-solving. Furthermore, the constrained model excels in scenarios with lower gasoline demand shortages, offering a cost-effective alternative by preventing excessive refinery expansion expenditures. Given these strengths, we advocate for its application across industries to optimize plant expansions, equipment upgrades, facility layouts, and production floor planning. Additionally, this framework could inform broader policy discussions on energy resilience, regional refining strategies, and sustainable resource allocation.

7. Limitations and recommendations for future studies

Although this study provides valuable insights into refinery optimization using an integrated inverse DEA approach and economic viability analysis, some limitations should be acknowledged. First, gasoline demand may significantly increase due to factors such as population growth, subsidy removals, or unforeseen economic disruptions. These dynamics were not fully captured in the current modelling framework and could influence decision-making processes based on the findings of the proposed models. Second, the scope of refined products was limited to gasoline, kerosene, and distillate. While these products are critical for domestic energy security, other refined outputs such as LPG, jet fuel, and petrochemical feedstocks should be incorporated in future studies. Thirdly, the models adopted static assumptions regarding policy and market conditions, overlooking the potential impact of regulatory changes, subsidy reforms, and environmental mandates on refinery economics.

To address these limitations, future research should incorporate stochastic or scenario-based modelling to account for demand uncertainty, particularly for gasoline under varying economic and policy conditions. Specifically, a stochastic inverse DEA framework should be developed to capture demand variability and uncertainty in optimization outcomes. Additionally, expanding the product coverage to include LPG, jet fuel, and petrochemical derivatives would provide a more holistic perspective on refinery optimization. Integrating exchange rate volatility and inflation-adjusted financial metrics into economic viability assessments will enhance the accuracy of investment projections in macroeconomically unstable environments. Furthermore, a dynamic inverse DEA model should be developed to evaluate the impact of regulatory shifts and policy changes on refinery operations over time, enabling more adaptive and resilient optimization strategies. Finally, future studies should also align refinery optimization strategies with global energy transition trends, assessing compatibility with renewable energy adoption and carbon reduction targets.

CRediT authorship contribution statement

Kelvin K. Orisaremi: Writing – original draft, Software, Methodology, Investigation, Data curation, Conceptualization. **Felix T.S. Chan:** Writing – review & editing, Supervision, Methodology, Formal analysis. **Felister Dibia:** Methodology, Investigation, Data curation. **Xiao-Wen Fu:** Writing – review & editing, Supervision, Methodology. **Yue Chen:** Investigation, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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