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An unsupervised CNN–Transformer method for transient electromagnetic logging inversion under physical–prior constraints



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ABSTRACT

Transient electromagnetic (TEM) logging can reflect the electrical properties of formation around the borehole in the time domain, serving as an important method for detecting oil and gas reservoirs, coal seams, groundwater and geothermal resources. However, conventional optimization inversions suffer from high computational costs and poor convergence stability, while supervised deep learning methods rely heavily on high-quality training datasets, limiting their applicability in complex geological environments. To address these challenges, this study proposes an unsupervised time-domain inversion framework based on physical–prior constraints. The framework integrates a finite-volume time-domain (FVTD) method with the adjoint-state method to enforce Maxwell's governing equations during optimization. We construct a generative network that combines convolutional neural network (CNN) with Transformer, achieving an effective fusion of local structure characterization and long-range dependencies. Moreover, a dynamic weighting strategy incorporating geological prior balances data misfit and prior constraint terms to constrain the solution space. Numerical experiments demonstrate that the proposed method achieves excellent imaging accuracy and stability across near-borehole, deep, cross-scale, and radial dual-anomaly scenarios without requiring any training dataset, significantly enhancing the reliability of TEM logging inversion.

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1. Introduction

The inversion of TEM logging aims to reconstruct the structure of electrically heterogeneous formations surrounding the borehole in the time domain, thereby accurately locating and quantifying the geometry and electrical properties of the anomalous body. This capability is of great significance in oil and gas exploration, coal resource assessment, as well as geothermal and groundwater investigations (Khan et al., 2020; Yuan et al., 2022; Wu et al., 2025; Xiong et al., 2025). The TEM logging data are typically time-

domain electromotive force (EMF). The diffusion characteristics of electromagnetic fields implies that signals at different times correspond to different detection depths. This makes TEM logging highly sensitive to one or multiple conductive anomalous bodies near the borehole, with particular advantages in depth detection (Fitterman and Stewart, 1986; Newman et al., 1986; Sharp et al., 1999; Auken et al., 2019). Therefore, it is essential to develop a high-precision time-domain inversion method to transform the measured transient responses into the subsurface conductivity distribution.

At present, TEM logging inversion methods can be broadly divided into two categories: smoke ring inversion and optimization-based inversion. The smoke ring inversion treats the induced eddy currents as expanding current loops over time. It uses the difference between early-time and late-time responses to estimate the position and approximate size of nearby anomalous

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body, providing a semi-quantitative “time–depth” correspondence (Nabighian, 1979). Although the smoke ring inversion method offers fast computation speeds, it only provides qualitative interpretations and cannot rigorously fit the observed data, limiting its accuracy for high-resolution TEM logging inversion. Optimization-based inversion method typically constructs an objective function consisting of a data misfit term and a regularization term. By introducing smoothness and geological priors under the constraint of the governing equations, this framework stabilizes the solution of the ill-posed inversion problem. The typical algorithms include steepest descent, Gauss–Newton, and Levenberg–Marquardt methods (Hoare and Ruijgrok, 1970; Gerstoft, 1995; Choi et al., 2005; Sugihara, 2011). The steepest descent method iteratively updates parameters along the negative gradient direction of the objective function. It only requires the calculation of the first-order derivatives in each iteration, resulting in simple implementation and low computational cost, making it suitable for the initial exploration of large-scale problems. However, in ill-conditioned or strongly nonlinear cases, the iteration trajectory may exhibit zigzag oscillations, leading to slow convergence. This typically requires an initial condition or adaptive step-size strategies to improve convergence (Widrow and McCool, 1976; Teschke and Borries, 2010; Ren, 2022). The Gauss–Newton method is a second-order optimization technique specifically designed for solving nonlinear least-squares problems. By approximating the Hessian matrix using the product of the Jacobian matrix, it leverages second-order information of the objective function to accurately estimate the descent direction. This method achieves rapid local convergence near the optimal solution or when the residuals are small. Its limitations lie in the fact that each iteration requires inverting the Jacobian matrix, resulting in high computational and storage costs. Additionally, this method may fail to converge or even diverge if the initial guess is too far from the solution (Pratt et al., 1998; Loke and Dahlin, 2002; Mojabi and LoVetri, 2010). The Levenberg–Marquardt method combines the advantages of the previous two approaches by introducing a damping factor to adaptively balance between the Gauss–Newton and gradient descent directions, achieving both stability and fast convergence (Pujol, 2007; Foroughi and Tenzer, 2014; Xu et al., 2023). However, the selection and updating strategy of the damping factor must be carefully designed for optimal performance. Despite reduced computational cost compared to Gauss–Newton, it still requires complex matrix operations at each iteration. Although these classical optimization algorithms have achieved notable success in TEM logging inversion, they still suffer from challenges such as high computational cost, slow convergence, and sensitivity to parameter selection (Nekut, 1987; Newman and Commer, 2005).

In recent years, with the advancement of computational power and the evolution of deep neural networks and optimization algorithms, deep learning methods have provided new perspectives for geophysical inversion problems (Zhang et al., 2022, 2025; Yan et al., 2023; Fu et al., 2025; Zhang and Yang, 2025). Noh et al. (2023) proposed a strategy to improve inversion accuracy under noisy measurement conditions. By incorporating noise into the training dataset and integrating noise layers within a deep residual neural network architecture, thereby enhancing resistivity inversion robustness in real-time geosteering. Pan et al. (2024) developed a neural network model for two-dimensional magnetotelluric (MT) inversion using deformable convolution technology (MT2DInv-Unet). They generated a large-scale synthetic MT dataset through forward modeling and trained a U-Net model to map observed data to resistivity parameters, achieving strong convergence and high inversion resolution for subsurface resistivity structures. Ma et al. (2025) introduced a collaborative

optimization framework combining physics-constrained data generation with an improved UNet++ architecture (R-BInVNet). This method utilizes a Markov chain perturbation strategy to generate formation samples with physical continuity and improves the network structure through dynamic fusion of multi-scale and directional features, providing an efficient solution for electromagnetic logging-while-drilling inversion. To overcome the limitations of pure CNNs in capturing global dependencies, hybrid CNN-Transformer architectures have been increasingly applied in supervised geophysical inversion. Ning et al. (2024) proposed a transformer and convolutional hybrid neural network (trans-CNN) trained on labeled well-log data to significantly improve seismic impedance inversion. Yang et al. (2025) introduced a hybrid convolutional transformer network (HCTNet) for prestack seismic inversion, effectively fusing local and global features to improve the robustness of elastic parameter estimation. Supervised network models can effectively learn the nonlinear mapping between observed data and subsurface parameters, demonstrating impressive performance in geophysical inversion. However, they still face several limitations. First, their performance strongly depends on large-scale, high-quality training datasets. In practice, especially under complex geological models, acquiring samples and constructing databases is costly and inefficient. Second, their generalization capability is often limited, leading to degraded performance on unknown data distributions. Furthermore, due to the lack of explicit physical constraints during training, their inversion results may violate the governing physical equations, reducing interpretability and credibility.

To overcome the limitations of traditional optimization methods, such as high computational cost, fragile convergence, and the dependency of deep learning on massive training datasets, this paper proposes an unsupervised time-domain inversion framework driven by dual constraints: physical and prior information. The FVTD method is employed as the forward modeling foundation, while a hybrid CNN-Transformer architecture is constructed to simultaneously capture local features and long-range dependencies. The Jacobian matrix is incorporated through adjoint-state method into the optimization process to ensure that update direction conforms to the physical laws of the governing equations. Geological prior information is integrated to regularize the solution space, and a dynamic weighting strategy is employed to form a “localization-first, refinement-later” solution approach, guaranteeing that the inversion updates achieve stable convergence under the dual constraints of physical principles and prior information. The proposed framework enables stable and high-precision reconstruction of the location, scale, and electrical parameters of anomalous body without relying on the training datasets, maintaining both interpretability and robustness.

2. Methodology

2.1. Inversion framework

The overall framework of the proposed “physics + prior” dual-driven inversion strategy is shown in Fig. 1. First, a FVTD model is established on a cylindrical coordinate grid. Then, the observed data is obtained through the discretization of the weak-form governing equation, followed by time discretization using the implicit Euler scheme. Subsequently, a generative model that couples a 3D-CNN with a Transformer model is employed to map low-dimensional random vectors into 3D conductivity model. In this architecture, the CNN extracts local features through multi-layer upsampling and 3D convolution layers, while the Transformer captures global radial context via horizontal pooling, positional encoding, Multi-Head Self-Attention, and a Feed-Forward

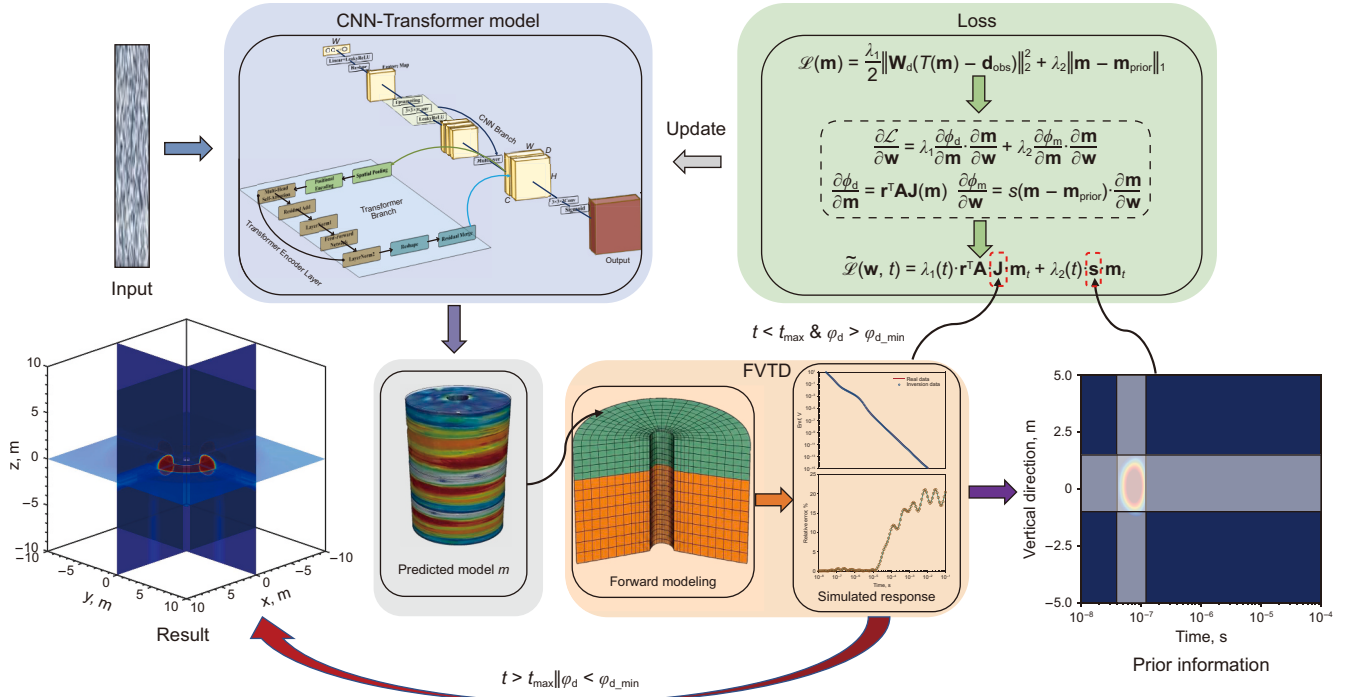


Fig. 1. Inversion framework.

Network. The encoded sequence is then reshaped and residually injected back into the CNN, enabling multi-scale integration of local convolutional features with long-range radial dependencies. In the optimization stage, the objective function incorporates both data misfit term and prior information term. At each iteration, the Jacobian matrix through adjoint-state method is introduced to ensure that the update direction remains consistent with Maxwell's equations. At the same time, geological prior information is incorporated into the loss function. A dynamic weighting strategy is employed to enforce constraints on both the smoothness of the solution and its geological plausibility during the inversion process. The network parameters are updated using the Adam optimizer, which dynamically adjusts the learning rate based on first-order and second-order moment estimates, thereby achieving a balance between fast convergence and training stability. Ultimately, driven by both physical constraints and prior information, the framework achieves stable, high-precision inversion of anomalous bodies without relying on large-scale training data. This workflow effectively balances the efficiency of deep learning with the reliability of physical modeling, offering a robust and interpretable inversion solution for TEM logging.

2.2. FVT method

In a homogeneous medium without an excitation source, and ignoring the displacement current, the differential form of Maxwell's equations is as follows (Buffa et al., 2000; Wolsky, 2015):

$$\begin{cases} \nabla \times \mathbf{H} = \sigma \mathbf{E} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{H} = 0 \end{cases} \quad (1)$$

where $\mathbf{E}$ is the electric field; $\mathbf{H}$ is the magnetic field; $\mathbf{B}$ is the magnetic induction; σ is the conductivity; t is the time.

Based on the natural boundary conditions, constitutive relations, and functional analysis theory, the weak-form governing equation can be derived (Hyman and Shashkov, 1999; Haber and Ruthotto, 2014):

$$\int_{\Omega} (\mu^{-1} \mathbf{B}) \cdot \nabla \times \mathbf{W} d\Omega = \int_{\Omega} \sigma \mathbf{E} \cdot \mathbf{W} d\Omega \quad (2)$$

$$\int_{\Omega} \nabla \times \mathbf{E} \cdot \mathbf{F} d\Omega = -\frac{\partial}{\partial t} \int_{\Omega} \mathbf{B} \cdot \mathbf{F} d\Omega \quad (3)$$

where $\mathbf{W}$ and $\mathbf{F}$ are vector fields that are in the same function space² as $\mathbf{E}$ and $\mathbf{B}$.

After eliminating $\mathbf{E}$, the matrix form of the spatial discretization for the source-free Maxwell governing equations can be obtained. By applying the implicit Euler scheme, the equations are subsequently discretized in the time domain:

$$\frac{\mathbf{B}^{n+1} - \mathbf{B}^n}{\Delta t} = -\mathbf{CURL} \mathbf{M}_{e\sigma}^{-1} \mathbf{CURL}^T \mathbf{M}_{f\mu} \mathbf{B}^{n+1} \quad (4)$$

where $\mathbf{CURL}$ is the matrix form of the curl operator; $\mathbf{M}_{e\sigma}$ is $\text{diag}[\mathbf{A}_e^T(\sigma \odot \mathbf{v})]$; $\mathbf{A}_e$ is a sparse matrix of $1/4$, $\mathbf{v}$ is the grid volume; $\mathbf{M}_{f\mu}$ is $\text{diag}[\mathbf{A}_f^T(\mu^{-1} \odot \mathbf{v})]$; $\mathbf{A}_f$ is a sparse matrix of $1/2$.

The initial magnetic field ($t = 0$) is computed from the analytical expression of the full-space magnetic vector potential, which is then solved by the direct solver PARDISO. Based on Faraday's law, the induced EMF in the receiver coil corresponds to the time derivative of the magnetic flux density:

$$\mathbf{V}(t) = -N \cdot S \cdot \frac{\partial \mathbf{B}}{\partial t} \quad (5)$$

where N is the number of coils and S is the coil area.

2.3. CNN + Transformer

The objective of TEM logging inversion is to reconstruct the three-dimensional (3D) conductivity distribution around the borehole. This paper constructs a generative network architecture that couples a 3D-CNN with a Transformer, which leverages the strengths in extracting local spatial feature and capturing long-range dependencies (Khan et al., 2023; Yuan et al., 2023). The network serves as a parametric mapping function $P_w(\xi) : \mathbb{R}^k \rightarrow \mathbb{R}^{N_r \times N_\phi \times N_z}$, which transforms a one-dimensional (1D) random vector ξ into a discretized 3D model in cylindrical coordinates, where w denotes the network parameters. The random vector ξ is first subjected to dimensional expansion and nonlinear transformation:

$$\mathbf{y}_0 = \text{LeakyReLU}(\mathbf{w}_0 \xi + \mathbf{b}_0) \quad (6)$$

where $\mathbf{w}_0$ and $\mathbf{b}_0$ represent the parameters and bias of the first layer, respectively. The subsequent layers consist of a sequence of “upsampling–3D convolution–activation function” operations. Denoting the input of the layer l as $\mathbf{X}^{(l)}$, the upsampling operation can be expressed as:

$$\tilde{\mathbf{X}}^{(l)} = \sum_{i=0}^1 \sum_{j=0}^1 \sum_{k=0}^1 \omega_r^{(i)} \omega_\phi^{(j)} \omega_z^{(k)} \mathbf{X}^{(l)} \quad (7)$$

where $\tilde{\mathbf{X}}^{(l)}$ denotes the upsampled tensor; $\omega_r^{(i)}, \omega_\phi^{(j)}, \omega_z^{(k)}$ represent the 1D linear interpolation weights. The upsampled result is then used as the input to the convolution operation, and the 3D convolution can be defined as:

$$\mathbf{X}^{(l+1)} = \text{LeakyReLU} \left(\sum_{c=1}^{C_l} \sum_{u=0}^{k_r-1} \sum_{v=0}^{k_\phi-1} \sum_{w=0}^{k_z-1} \mathbf{W}^{(l)} \tilde{\mathbf{X}}^{(l)} + \mathbf{b}^{(l)} \right) \quad (8)$$

where k_r, k_ϕ, k_z denote the kernel sizes in the radial, azimuthal, and axial directions, respectively; C_l represents the number of channels in the input layer; $\mathbf{W}^{(l)}$ and $\mathbf{b}^{(l)}$ denote the weights and biases of the convolution kernel, respectively.

The limited receptive field of local convolutions makes it difficult to capture long-range dependencies. Considering the availability of reliable depth-dependent prior information, this study introduces an attention module along the radial direction. The convolutional feature map $\mathbf{X}^{(m)}$ is first horizontally pooled along the (ϕ, z) dimensions to obtain a radial sequence $\mathbf{S}$, which effectively reduces computational complexity while focusing on long-range dependencies in the radial domain.

$$\mathbf{S} = \text{MeanPool}_{\phi, z}(\mathbf{X}^{(m)}) \quad (9)$$

The sequence $\mathbf{S}$ is then fed into an encoder composed of multi-head self-attention and a feed-forward network, together with layer normalization and dropout operations. This process yields the encoded radial sequence $\mathbf{S}'$, which enhances the structural consistency and long-range correlations along the radial direction.

$$\begin{aligned} \mathbf{Q} &= \mathbf{S} \mathbf{W}^Q, \mathbf{K} = \mathbf{S} \mathbf{W}^K, \mathbf{V} = \mathbf{S} \mathbf{W}^V, \\ \text{head}_h &= \text{Softmax} \left(\frac{\mathbf{Q}_h \mathbf{K}_h^\top}{\sqrt{d_k}} \right) \mathbf{V}_h, h = 1, \dots, H_{\text{attn}}, \\ \text{MHA}(\mathbf{S}) &= \text{Concat}(\text{head}_1, \dots, \text{head}_{H_{\text{attn}}}) \mathbf{W}^O, \\ \mathbf{Z} &= \text{LN}(\mathbf{S} + \text{Dropout}(\text{MHA}(\mathbf{S} + \mathbf{PE}))), \\ \mathbf{S}' &= \text{LN}(\mathbf{Z} + \text{Dropout}(\mathbf{W}_2 \text{ReLU}(\mathbf{W}_1 \mathbf{Z} + \mathbf{b}_1) + \mathbf{b}_2)). \end{aligned} \quad (10)$$

where $\mathbf{W}^Q, \mathbf{W}^K, \mathbf{W}^V$ are the projection matrices that generate the $\mathbf{Q}, \mathbf{K}, \mathbf{V}$ sequences, respectively; d_k denotes the dimensions of $\mathbf{Q}$ and $\mathbf{K}$; head_h represents the output of the h th attention head; H_{attn} is the total number of attention heads; $\text{Concat}(\cdot)$ denotes the concatenation and reorganization; $\mathbf{Z}$ represents the combination of the attention output and the original input; $\text{LN}(\cdot)$ refers to the layer normalization operation; $\mathbf{PE}$ is the positional encoding vector; $\mathbf{W}^O$ is the output projection matrix; $\mathbf{W}_1$ and $\mathbf{W}_2$ denote the linear mapping of the feedforward network; $\mathbf{b}_1$ and $\mathbf{b}_2$ represent the bias terms of the feedforward network.

The radial sequence is then mapped back to the 3D convolutional space and fused with the convolutional feature map $\mathbf{X}^{(m)}$, enabling multiscale integration of local convolutional features and global radial information.

$$\bar{\mathbf{X}}^{(m)} = \mathbf{X}^{(m)} + \gamma \text{Reshape}(\mathbf{S}') \quad (11)$$

where γ is the fusion coefficient; $\text{Reshape}(\cdot)$ adjusts the radial sequence $\mathbf{S}'$ to the dimensions of $\mathbf{X}^{(m)}$.

2.4. Loss and optimization

To ensure that the FVTD method imposes effective physical constraints on the deep learning model, such that the forward responses remain consistent with the observed data while incorporating prior information, the following objective function is defined:

$$\mathcal{L}(\mathbf{m}) = \frac{\lambda_1}{2} \|\mathbf{W}_d(T(\mathbf{m}) - \mathbf{d}_{\text{obs}})\|_2^2 + \lambda_2 \|\mathbf{m} - \mathbf{m}_{\text{prior}}\|_1 \quad (12)$$

where $T(\mathbf{m})$ denotes the forward time-domain response, and $\mathbf{m}$ represents the inversion result, $\mathbf{m} = P_w(\xi)$, $\mathbf{m} = [m_1, m_2, \dots, m_p]^\top$, p is the total number of forward-modeling grids; $\mathbf{m}_{\text{prior}}$ denotes the prior model; $\mathbf{d}_{\text{obs}} = [d_1, d_2, \dots, d_n]^\top$ represents the observed data, n is the number of data points; $\mathbf{W}_d$ is the diagonal matrix, $\mathbf{W}_d = \text{diag}(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \dots, \frac{1}{\alpha_n})$; λ_1 and λ_2 are weighting functions used to dynamically balance the data misfit term and the prior information term, respectively, and their expressions are given as follows:

$$\begin{aligned} \lambda_1 &= 1 - \left(1 + \frac{t}{\beta_1} \right)^{-1} \\ \lambda_2 &= \left(1 + \frac{t}{\beta_2} \right)^{-1} \end{aligned} \quad (13)$$

where t denotes the number of iterations; β_1 and β_2 represent the control coefficients, which are set to 1000 and 300 respectively in this paper.

For time-domain signals, the early-time responses are several orders of magnitude larger than the late-time signals, making the latter more susceptible to background noise. To prevent overfitting to late-time noise, we set a baseline value ϵ to control diagonal matrix via $\alpha = q\% \times \mathbf{d}_{\text{obs}} + \epsilon$.

The subsurface conductivity model is generated by the CNN–Transformer model, where the network weights determine the mapping from the low-dimensional latent vector ξ to the model domain. Therefore, the model update is essentially achieved by adjusting the network weights. To ensure that the optimization direction not only satisfies the constraints of the forward algorithm but also reflects the regulation of prior information, it is

necessary to derive the partial derivative of the objective function with respect to $\mathbf{w}$ (Xu and Heagy, 2024).

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \lambda_1 \frac{\partial \phi_d}{\partial \mathbf{w}} + \lambda_2 \frac{\partial \phi_m}{\partial \mathbf{w}} \quad (14)$$

where $\phi_d = \frac{1}{2} \|\mathbf{W}_d(T(\mathbf{m}) - \mathbf{d}_{\text{obs}})\|_2^2$ denotes data misfit term; $\phi_m = \|\mathbf{m} - \mathbf{m}_{\text{prior}}\|_1$ denotes prior information term.

According to the chain rule:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \lambda_1 \frac{\partial \phi_d}{\partial \mathbf{m}} \frac{\partial \mathbf{m}}{\partial \mathbf{w}} + \lambda_2 \frac{\partial \phi_m}{\partial \mathbf{m}} \frac{\partial \mathbf{m}}{\partial \mathbf{w}} \quad (15)$$

Let $\mathbf{r} = T(\mathbf{m}) - \mathbf{d}_{\text{obs}}$, then we obtain:

$$\phi_d = \frac{1}{2} \mathbf{r}^T \mathbf{A} \mathbf{r} \quad (16)$$

where $\mathbf{A} = \mathbf{W}_d^T \mathbf{W}_d$ is a symmetric matrix; $\mathbf{r}$ is a function of $\mathbf{m}$.

Therefore, according to the chain rule:

$$\frac{\partial \phi_d}{\partial \mathbf{m}} = \mathbf{r}^T \mathbf{A} \mathbf{J}(\mathbf{m}) \quad (17)$$

where $\mathbf{J}(\mathbf{m}) = \frac{\partial T(\mathbf{m})}{\partial \mathbf{m}}$ represents the Jacobian matrix.

Next, we process the prior information term to obtain:

$$\frac{\partial \phi_m}{\partial \mathbf{w}} = s(\mathbf{m} - \mathbf{m}_{\text{prior}}) \frac{\partial \mathbf{m}}{\partial \mathbf{w}} \quad (18)$$

$$s(x) = \begin{cases} +1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (19)$$

Eq. (15) can be rearranged as:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \lambda_1 \mathbf{r}^T \mathbf{A} \mathbf{J}(\mathbf{m}) \frac{\partial \mathbf{m}}{\partial \mathbf{w}} + \lambda_2 s(\mathbf{m} - \mathbf{m}_{\text{prior}}) \frac{\partial \mathbf{m}}{\partial \mathbf{w}} \quad (20)$$

Subsequently, a loss function $\tilde{\mathcal{L}}$ is constructed to ensure that the gradient obtained through the deep learning model is consistent with that of the original objective function. Meanwhile, based on the current iteration model $\mathbf{m}_t$, we compute $\mathbf{g} = \mathbf{r}^T \mathbf{A} \mathbf{J}(\mathbf{m}_t)$ via adjoint-state method, without storing or computing $\mathbf{J}(\mathbf{m}_t)$. This strategy ensures that the optimization trajectory at each iteration remains governed by the FVTD method while effectively reducing implementation complexity and computational cost.

$$\tilde{\mathcal{L}}(\mathbf{w}, t) = \lambda_1(t) \cdot \mathbf{r}^T \mathbf{A} \mathbf{J} \cdot \mathbf{m}_t + \lambda_2(t) \cdot \mathbf{s} \cdot \mathbf{m}_t \quad (21)$$

This framework ensures that the results obtained by the optimizer when updating network weights satisfy both the physical laws governed by Maxwell's equations and the constraints imposed by geological prior information. As a result, the proposed method achieves high-precision inversion of TEM logging without relying on any training data. The inversion workflow is shown in Table 1.

3. Prior information analysis

3.1. Principles of TEM logging

The principle of TEM logging involves deploying transmitter and receiver coils within the borehole (Fig. 2(a)). The transmitter coil typically using a bipolar pulse current (Fig. 2(b)), generating a primary electromagnetic field within the formation during the current-on period. When the current is abruptly turned off, the primary field collapses, inducing eddy currents that diffuse outward with time. These induced eddy currents gradually decay

under the influence of the formation's electrical properties, producing a secondary electromagnetic field. By measuring the EMF generated by this secondary field at the receiver coil, the electrical conductivity distribution of the formation can be determined. As the detection depth of TEM logging increases with time, the early-time response primarily reflects near-borehole properties, while the late-time response contains information about deeper formation, enabling the characterization of subsurface resistivity variations. Compared with frequency-domain methods, TEM logging exhibits stronger penetration and better adaptability in formations with high conductivity contrasts or deep-target detection, offering distinct advantages in complex geological environments (Li et al., 2026a, 2026b).

3.2. Responses and field distribution of anomalous body

A single-transmitter and single-receiver coil configuration is employed in this study. When the size of the transmitter coil is relatively small, it can be approximated as a magnetic dipole located at the origin along the longitudinal axis. The magnetic dipole source emits step waveform pulses in the z -direction, with transmitter-receiver spacing of 0.3 m. The anomalous body is positioned radially between 2 and 4 m, longitudinally from -1 to 1 m, and spans the full azimuth (0 to 2π). The borehole radius is 10 cm, the mud resistivity is $1 \Omega \cdot \text{m}$, and the formation resistivity is $10 \Omega \cdot \text{m}$. The resistivities of the anomalous body are set to 1, 2, 5, 10, 20 and $50 \Omega \cdot \text{m}$, respectively. The effect of the anomalous body resistivity on the TEM logging response is numerically investigated.

As shown in Fig. 3, when the resistivity of the anomalous body is lower than that of the background formation, the logging curves exhibit pronounced differences in the time range of $10^{-7} \text{s} - 10^{-4} \text{s}$. A lower anomalous body resistivity leads to a greater separation amplitude and a delayed convergence with the background response. Thus, both the curve morphology and amplitude can qualitatively indicate the resistivity of the anomalous body. However, when the anomalous body resistivity exceeds the background formation, the curve separation becomes less distinct, with only slight deviations appearing at mid-time responses. This indicates that the TEM logging response exhibits low sensitivity to high-resistivity anomalous body. Therefore, additional data processing and network adjustments are required to achieve accurate inversion for such cases. This study focuses primarily on low-resistivity anomalous body, while the investigation of inversion of high-resistivity targets will be addressed in future work.

Fig. 4 shows the TEM logging responses under different mud resistivities. The resistivity of the anomalous body is fixed at $1 \Omega \cdot \text{m}$, while other parameters remain unchanged. The mud resistivities are set to 1, 2, 5, 10, 20 and $50 \Omega \cdot \text{m}$. It can be seen that mud resistivity only affects the early-time responses. After approximately 10^{-7}s , all curves nearly converge, indicating that the mud resistivity has negligible influence on the response associated with the anomalous body. This finding suggests that in practical inversion applications, the influence of mud resistivity can be largely neglected, or borehole parameters can be treated as known conditions by referring to other logging measurements.

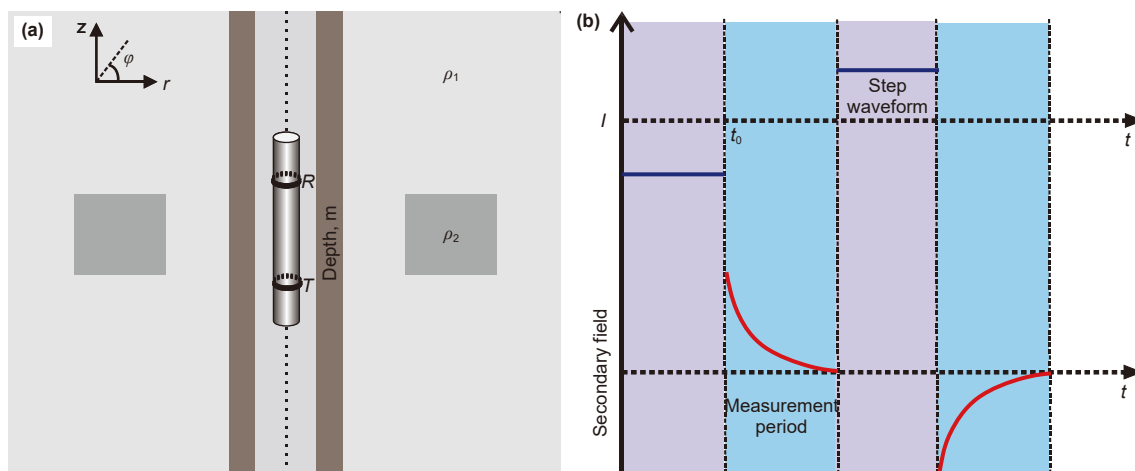
Fig. 5 shows the TEM logging responses under different azimuthal distributions of the anomalous body, with other parameters kept constant. The azimuthal distributions are set to $0-\pi/2$, $\pi/2-\pi$, $0-\pi$ and $\pi-2\pi$, while the black dashed line represents the response in a homogeneous formation. The results demonstrate that the TEM responses for quarter-ring or half-ring anomalous bodies are nearly identical. This is because the zz -component is inherently insensitive to azimuthal distribution and cannot distinguish asymmetric anomalous body in the azimuthal

Table 1
Inversion Process.**Algorithm 1:** Inversion process under Physical–Prior Constraints**Input:** $t_{\max}$, $\mathbf{m}_{\text{prior}}$, $\mathbf{d}_{\text{obs}}$, I_r , ξ , ε , q , $\varphi_{d_{\min}}$, β_1 , β_2 **Output:** $\mathbf{m}$

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1:  $t = 0$ 
2: Compute  $\mathbf{W}_d = \text{diag}(1 / (q\% \times \mathbf{d}_{\text{obs}}) + \varepsilon)$ 
3: while ( $t < t_{\max}$ ) do
4:   Compute  $\mathbf{m}_t = P_w(\xi)$ 
5:   Compute  $\lambda_1 = 1 - (1 + t/\beta_1)^{-1}$ ;  $\lambda_2 = (1 + t/\beta_2)^{-1}$ 
6:   Compute  $\mathbf{r} = (T(\mathbf{m}_t) - \mathbf{d}_{\text{obs}})$ 
7:   Compute  $\mathbf{A} = \mathbf{W}_d^T \mathbf{W}_d$ 
8:   Compute  $\mathbf{s} = \mathbf{s}(\mathbf{m} - \mathbf{m}_{\text{prior}})$ 
9:   Compute  $\text{Loss} = \lambda_1(t) \cdot \mathbf{r}^T \mathbf{A} \cdot \mathbf{r} + \lambda_2(t) \cdot \mathbf{s} \cdot \mathbf{m}_t$ 
10:  Compute  $\varphi_d = 0.5 \|\mathbf{W}_d(T(\mathbf{m}_t) - \mathbf{d}_{\text{obs}})\|^2$ 
11:  if ( $\varphi_d < \varphi_{d_{\min}}$ ) then
12:    break;
13:  end if
14:  Update  $\mathbf{w}_{t+1} = \text{Adam}(\mathbf{w}_t, \text{Loss}, I_r)$ 
15: end while
16: return  $\mathbf{m} = [m_1, m_2, \dots, m_p]^T$ 

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**Fig. 2.** Basic principle of a TEM logging system. (a) Well logging environment. (b) Transmit current and received signal.

direction. Therefore, for azimuthally inhomogeneous anomalous body, the single zz -component signal is insufficient for effective identification. It is necessary to combine cross-component signals to enhance sensitivity to such asymmetric structures.

Fig. 6 shows the TEM logging responses under different radial depths of the anomalous body, while keeping the longitudinal position and other parameters constant. The radial depths are set to 0–2 m, 2–4 m, 4–6 m, 6–8 m, and 8–10 m, respectively. The green dashed line represents the response of the homogeneous formation. It can be seen that anomalous bodies closer to the transmitter coil produce significantly higher early-time responses compared to the background response, and this deviation gradually weakens as the radial depth increases. As the radial depth of the anomalous body increases, the sensitivity of the response diminishes, leading to a greater challenge for inversion. Overall, the closer the anomalous body is to the source, the stronger the influence on the early-time responses. After approximately 40 μs , all curves gradually converge and eventually coincide with the background response.

Nabighian (1979) proposed the “smoke ring” theory, indicating that the intensity of the induced eddy currents is greatest at their center. This central position represents the current investigation depth of the tool and serves as the primary source of the EMF in the receiver coil. As shown in Fig. 7, the eddy current center initially concentrates near the transmitter coil, and the logging response at this stage is unaffected by the anomalous body (Fig. 7(a)). As time progresses, the eddy currents begin to be affected by the anomalous body, leading to a localized enhancement in current density (Fig. 7(b)). At 10^{-6} s, the current density within the cross section of the anomalous body continues to increase relative to the background, indicating that the logging response during this time interval is strongly influenced by the anomalous body. At 10^{-4} s, the center of the eddy currents in the background formation has diffused to deeper formation, while the induced eddy currents within the anomalous body gradually weaken. This results in a significant reduction in the sensitivity of the late-time responses to the anomalous body.

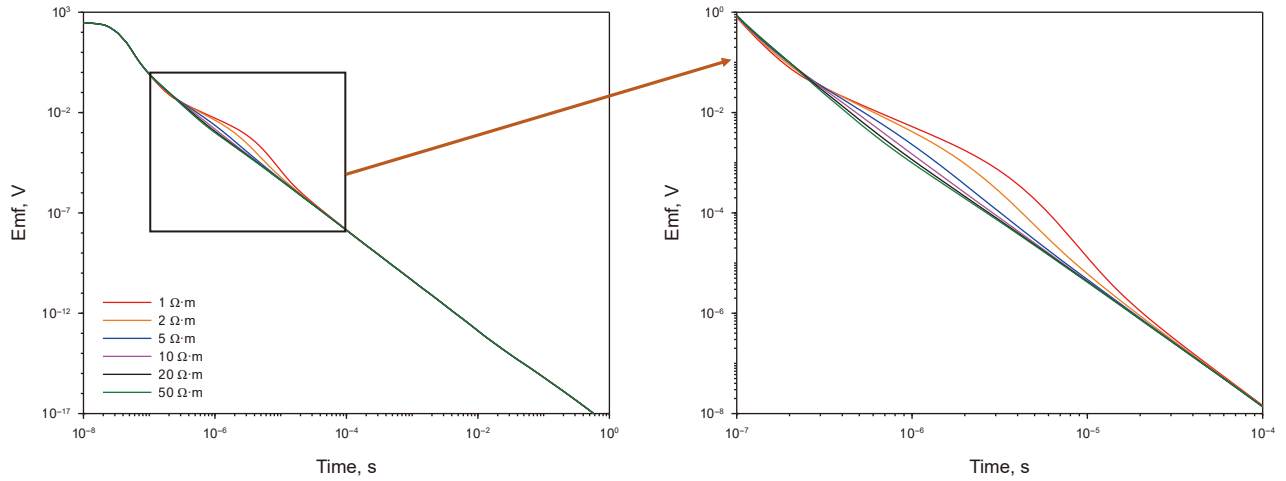


Fig. 3. Responses with different anomalous body resistivity. Emf denotes electromotive force.

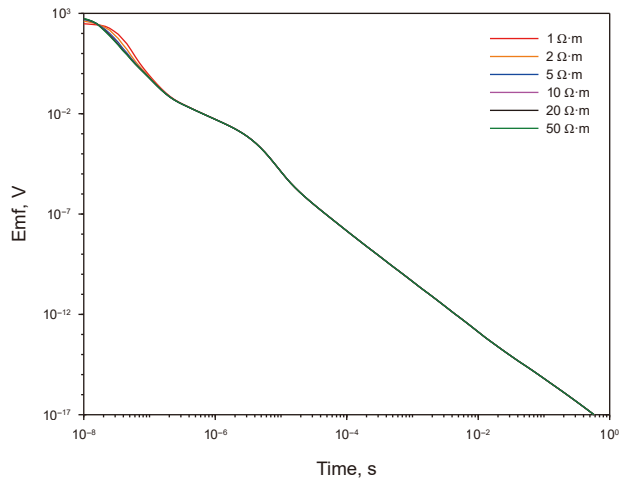


Fig. 4. Responses with different mud resistivity.

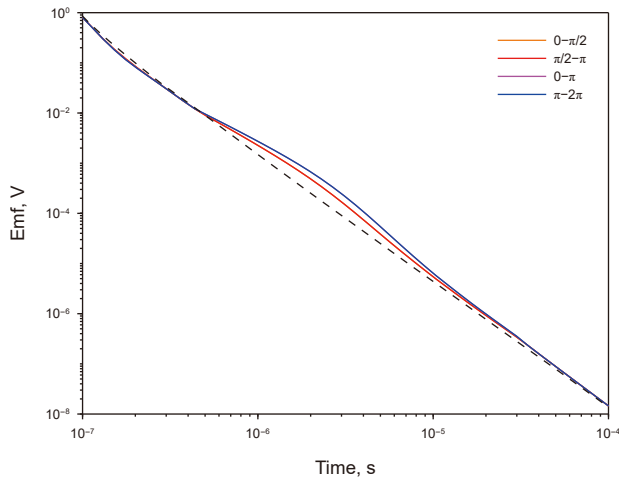


Fig. 5. Responses with different anomalous body azimuthal distributions.

To quantitatively analyze the influence range of the anomalous body on the logging response, the relative difference (RD) between the total field containing the anomalous body and the background

field in a homogeneous formation is defined based on the electromagnetic scattering.

$$RD = \frac{V_{zz_total}(t) - V_{zz_background}(t)}{V_{zz_total}(t)} \quad (22)$$

As shown in Fig. 8, the RD curve begins to vary at approximately 5×10^{-8} s, indicating that the logging response starts to reflect the influence of the anomalous body. Subsequently, the RD curve increases rapidly and reaches its maximum around 4×10^{-6} s, suggesting that the logging response during this stage is primarily dominated by the anomalous body. As the eddy currents diffuse to distant formation, the contribution of the anomalous body to the logging response decays rapidly. After approximately 1 ms, the RD curve gradually stabilizes, indicating that the late-time response becomes nearly insensitive to the anomalous body. During the inversion process, priority should be given to fitting the data within the anomaly-dominated time window, while the weighting of late-time responses should be correspondingly reduced. This adjustment can be effectively controlled through the introduction of a baseline value ϵ .

3.3. Prior information processing

The transmitter coil served as the measurement point, and the tool is simulated to move along the borehole from -5 m to 5 m, with a sampling interval of 0.2 m. The measurement time is taken as the horizontal axis, and the measurement point position as the vertical axis. The field in the presence of the anomalous body is treated as the total field, and its difference from the homogeneous formation response constitutes the secondary field response. The secondary field effectively reflects information about the anomalous body, the formula is:

$$V_{zz_sec}(z_i, t_j) = V_{zz_total}(z_i, t_j) - V_{zz_background}(z_i, t_j) \quad (23)$$

where the position of the i th measurement point is z_i , with the measurement positions satisfying $z_1 < z_2 < \dots < z_N$, and the j th time channel is t_j .

Fig. 9 shows the two-dimensional (2D) imaging results of the anomalous body's response. In the total-field imaging, an anomaly along the longitudinal direction can be observed around 10^{-6} s, but its position and size cannot be clearly identified. In contrast, the secondary field image (Fig. 10) clearly reveals both the longitudinal

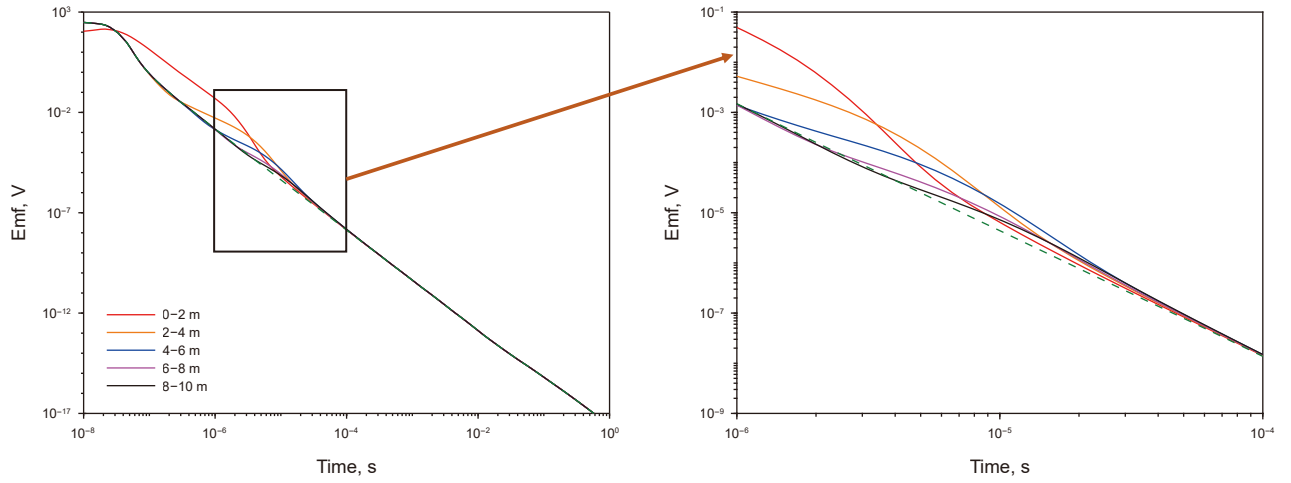


Fig. 6. Responses with different radial depths of the anomalous body.

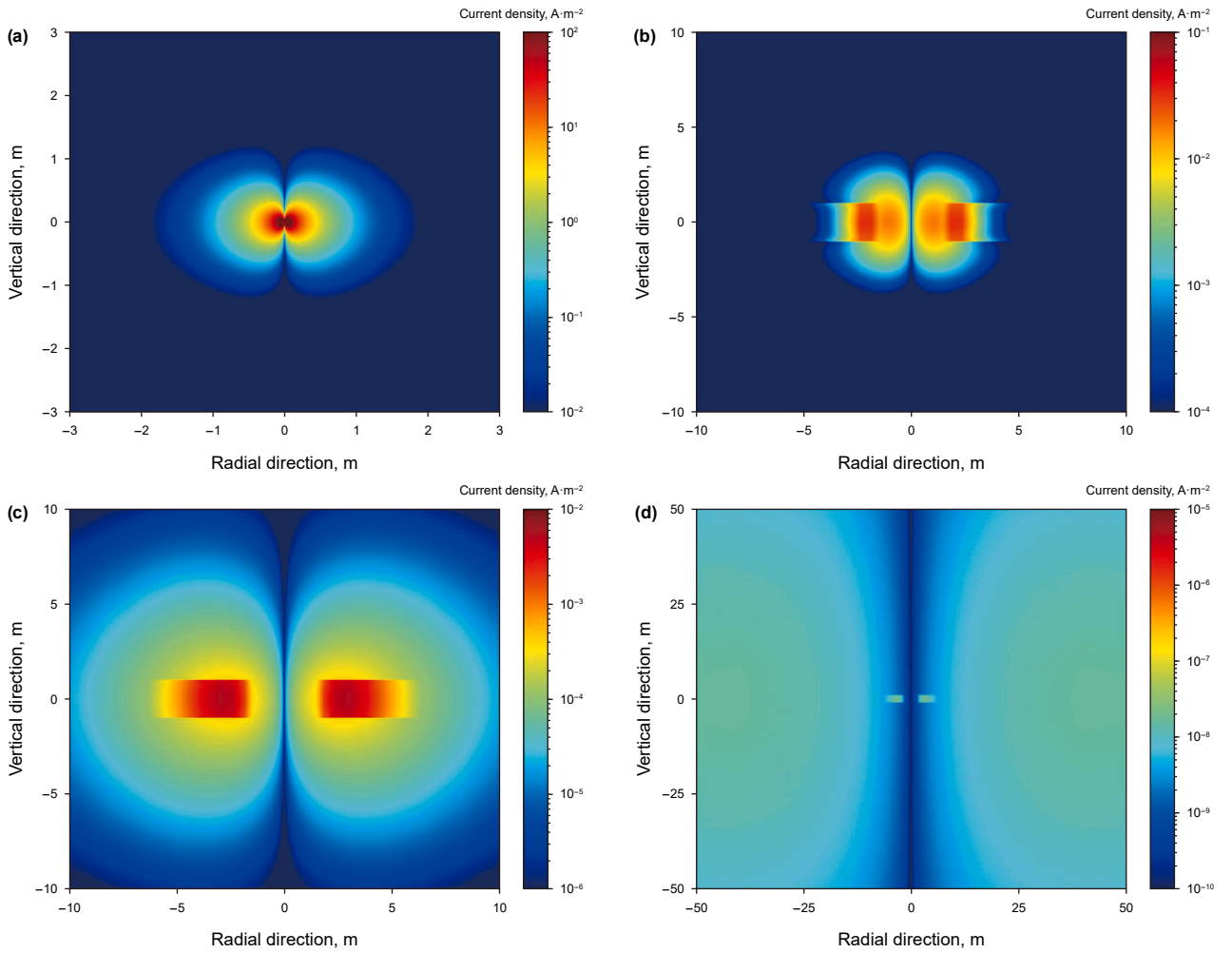


Fig. 7. Distribution characteristics of TEM Fields in the anomalous body. (a) $t = 10^{-8}$ s, (b) $t = 10^{-7}$ s, (c) $t = 10^{-6}$ s, (d) $t = 10^{-4}$ s.

position and extent of the anomalous body at approximately 10^{-7} s. We extract the secondary field data from the time window exhibiting distinct longitudinal features as prior information for the inversion. The 1D prior data along the longitudinal position is defined as:

$$p(z_i) = \frac{1}{N_T} \sum_{j \in J} v_{zz_sec}(z_i, t_j) \quad (24)$$

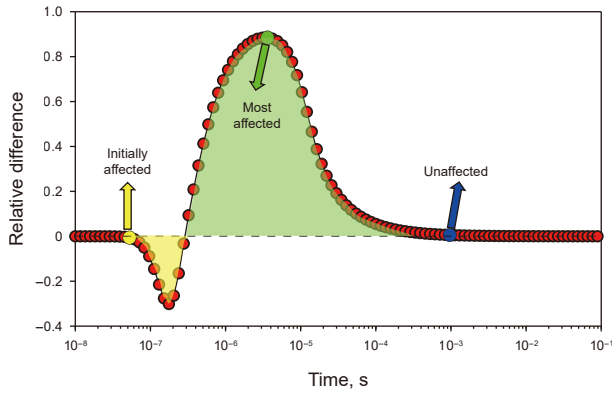


Fig. 8. RD curve of anomalous body.

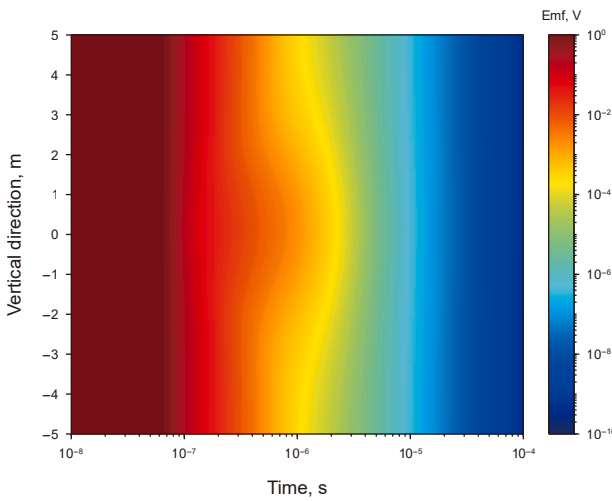


Fig. 9. Total field image.

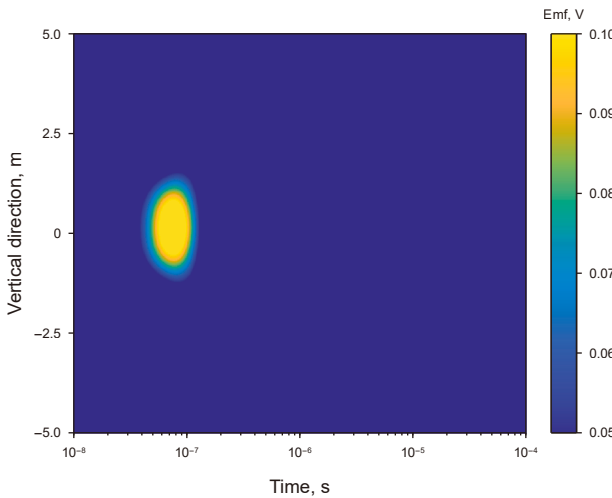


Fig. 10. Secondary field image.

where $J_T = \{j \mid t_a \leq t_j \leq t_b\}$, $N_T = |J_T|$, $T = [t_a, t_b]$ represents a time window with pronounced longitudinal features. Specifically, t_a is the onset time where the vertical features of the anomalous body become distinguishable, t_b is the cut-off time when the vertical features begin to blur.

Since the measurement points typically do not coincide with the boundaries of the non-uniform axial grid used in the inversion, a rigorous mapping strategy is required to transform the observation-derived prior into the model space. The processing workflow involves three key steps: continuous function construction, grid discretization, and spatial normalization. First, to bridge the gap between measurement points and the inversion grid, we construct a continuous 1D prior function $p'(z_i)$ using piecewise-linear interpolation based on the extracted secondary field features $p(z_i)$. For regions outside the measurement interval $[z_1, z_N]$, nearest-neighbor extrapolation is applied to ensure coverage across the entire vertical domain. The specific formula is as follows:

$$p'(z_i) = \begin{cases} p(z_1), & z < z_1 \\ (1 - \lambda(z))p(z_i) + \lambda(z)p(z_{i+1}), & z \in [z_1, z_N] \\ p(z_N), & z > z_N \end{cases} \quad (25)$$

$$\lambda(z) = \frac{z - z_i}{z_{i+1} - z_i} \quad (26)$$

Second, we map this continuous function onto the non-uniform axial grid of the forward modeling domain. We employ a cell-average projection method and the discretized prior value for the k th axial grid is calculated as:

$$p_k = \frac{1}{h_z(k)} \int_{z(k)}^{z(k+1)} p'(z) dz \quad (27)$$

where k is index of the axial grid, $h_z(k)$ is the thickness of the k th axial grid, $z(k)$ and $z(k+1)$ are the lower and upper axial coordinates of the boundaries of the k th grid, respectively.

Third, since the prior is derived from the vertical variation of the secondary field, it primarily constrains the longitudinal boundaries of the anomalous body. To apply this constraint to the 3D model, we perform a lateral expansion, replicating the 1D vertical profile across the radial and azimuthal dimensions, thereby forming the 3D prior information.

$$\mathbf{m}_{\text{prior}}(r, \phi, z_k) = p_k \quad (28)$$

Finally, to eliminate the magnitude discrepancy between the secondary field and the formation conductivity, the prior data are normalized alongside the inversion model $\mathbf{m}$ in Eq. (20). This process provides an effective support to enhance the reliability of TEM logging inversion.

4. Inversion results

In the fields of oil and gas exploration, coal resource assessment, and groundwater and geothermal investigation, low-resistivity annular anomalous bodies are often associated with hydrocarbon reservoirs, coal seams, or aquifers, and thus hold significant diagnostic importance. Specifically, in shale oil and gas exploration, such anomalous body typically indicate the presence of reservoirs. In coal exploration, they may correspond to organic-rich coal seams, and in groundwater or geothermal prospecting, they are usually related to aquifers or geothermal reservoirs, enabling effective identification of water-bearing zones or delineation of geothermal reservoir distributions (Ohwoghere-Asuma et al., 2020; Spacapan et al., 2020; Alzahrani et al., 2022; Ma et al., 2022; Chen et al., 2023). To evaluate the performance and adaptability of the proposed inversion framework, four typical models are designed, including a near-borehole anomalous body, a deep anomalous body, a cross-scale anomalous body, and radially dual anomalous bodies. These scenarios are employed to verify the

algorithm's capability in near-borehole imaging, deep target detection, multiscale adaptation, and the separation of multiple anomalous targets, demonstrating the stability and robustness of the proposed method.

4.1. Near-borehole anomalous body

4.1.1. CNN–transformer model inversion results

Fig. 11 presents the conductivity inversion results from TEM logging. The formation conductivity is 0.1 S/m, containing a low-resistivity annular anomalous body with a conductivity of 1 S/m, located radially between 2 m and 4 m and longitudinally between -1 m and 1 m. Fig. 11(a) shows the 3D inversion result. A distinct high-conductivity ring is observed in the azimuthal direction, with different azimuthal slices showing consistent results. This indicates that the CNN–Transformer model effectively captures the position and geometry of the anomalous body in the azimuthal direction. Consequently, for other annular models, there is no need to refine the grid in the azimuth direction, which can improve inversion efficiency. Fig. 11(b) shows the 2D cross-section at $z = 0$. The inversion result clearly reveals the annular distribution of the near-borehole anomalous body, showing a strong contrast with the background formation. This cross-section also demonstrates the continuity in the azimuthal direction, further confirming the model's superiority in reconstructing near-borehole anomalous body. Fig. 11(c) shows the 2D cross-section at $\phi = 0$, where the green frame outlines the true anomaly boundaries. The high-conductivity region almost perfectly overlaps with this reference frame, validating the high accuracy of the proposed method in delineating near-borehole targets.

4.1.2. Comparison with CNN model

Fig. 12 presents the inversion results using the CNN model. Although this model can indicate the presence of the anomaly, the reconstructed boundaries are significantly blurred compared to those in Fig. 11. This degradation is particularly evident in the

radial direction, where the contour distorts into a triangular shape, leading to a substantial reduction in spatial fidelity. While the prior constraints help the CNN model maintain a reasonable vertical resolution, its radial characterization remains inadequate. Overall, the inversion results of the CNN model demonstrate noticeable deficiencies in both accuracy and morphological fidelity when compared with the CNN–Transformer model.

4.1.3. Comparison with inversion without prior information

Fig. 13 shows the inversion results of the combined model without prior information. Although the inversion result generally indicates the presence of a near-borehole anomalous body, there are notable deficiencies in both accuracy and morphology. First, regarding the position of the anomalous body, the inversion results are generally shifted downward, and the radial positioning is inaccurate. This causes portions of the anomalous body within the green frame to be inadequately identified, revealing significant positional deviation. Second, as shown from the cross-section, the recovered result exhibits an irregular shape, which differs considerably from the actual geometry. Additionally, a prominent high-conductivity zone appears in the background formation, suggesting the presence of substantial inversion errors. These results highlight the critical role of incorporating prior constraints in TEM logging inversion, particularly in enhancing model accuracy, improving inversion stability, and restoring realistic geometries.

Fig. 14 shows the EMF fitting result and the corresponding relative error for the combined model. Within the time range of 10^{-8} to 10^{-5} s, the inverted response almost completely overlaps with the true data, and the relative error remains consistently low. This demonstrates that the proposed inversion framework can stably fit the decay pattern of the EMF. As time increases, although the EMF decreases rapidly, the inversion data remains consistent with the true trend. The relative error begins to increase after 10^{-5} s, which corresponds to the baseline adjustment strategy designed in the weighting scheme. This strategy avoids over-emphasizing the late-time signals and assigns higher weights to the time interval

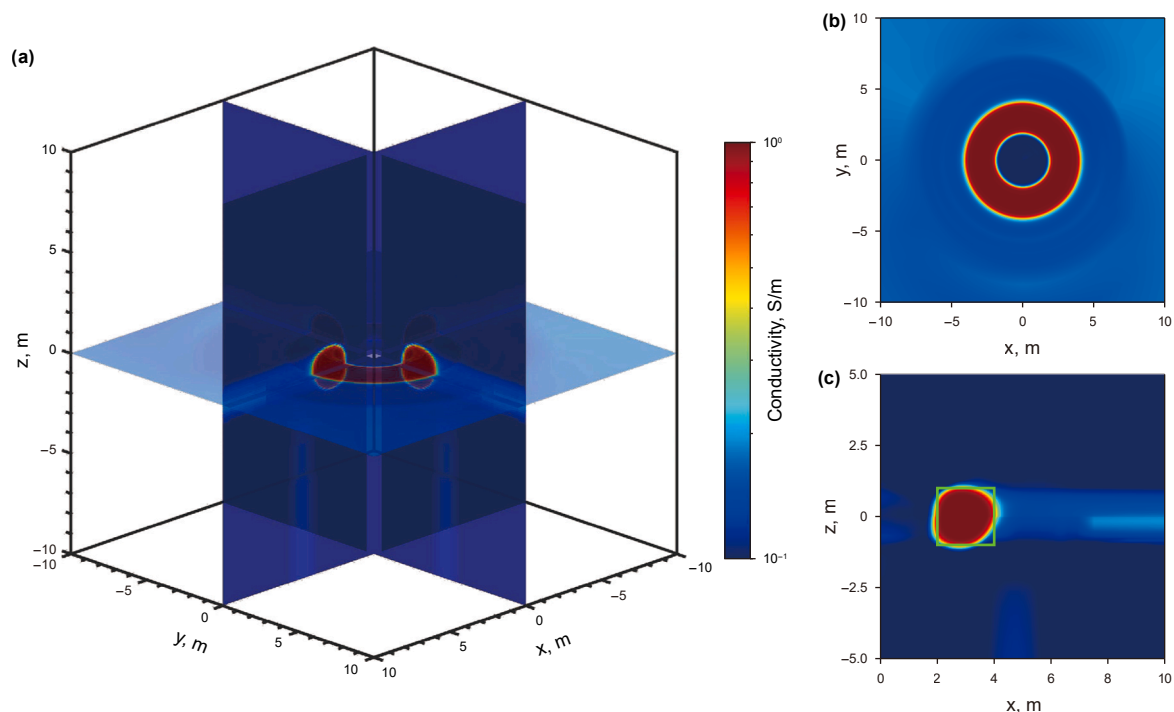


Fig. 11. CNN–Transformer model inversion results. (a) 3D result, (b) XOY cross-section, (c) XOZ cross-section.

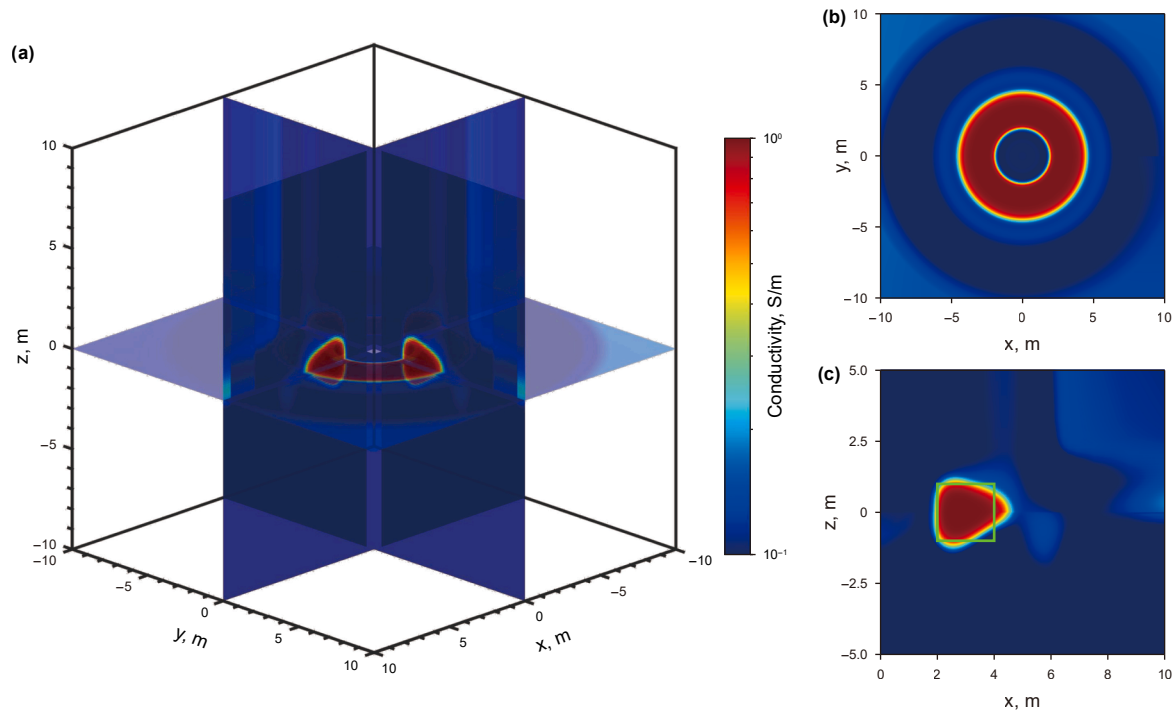


Fig. 12. CNN model inversion results. (a) 3D result, (b) XOY cross-section, (c) XOZ cross-section.

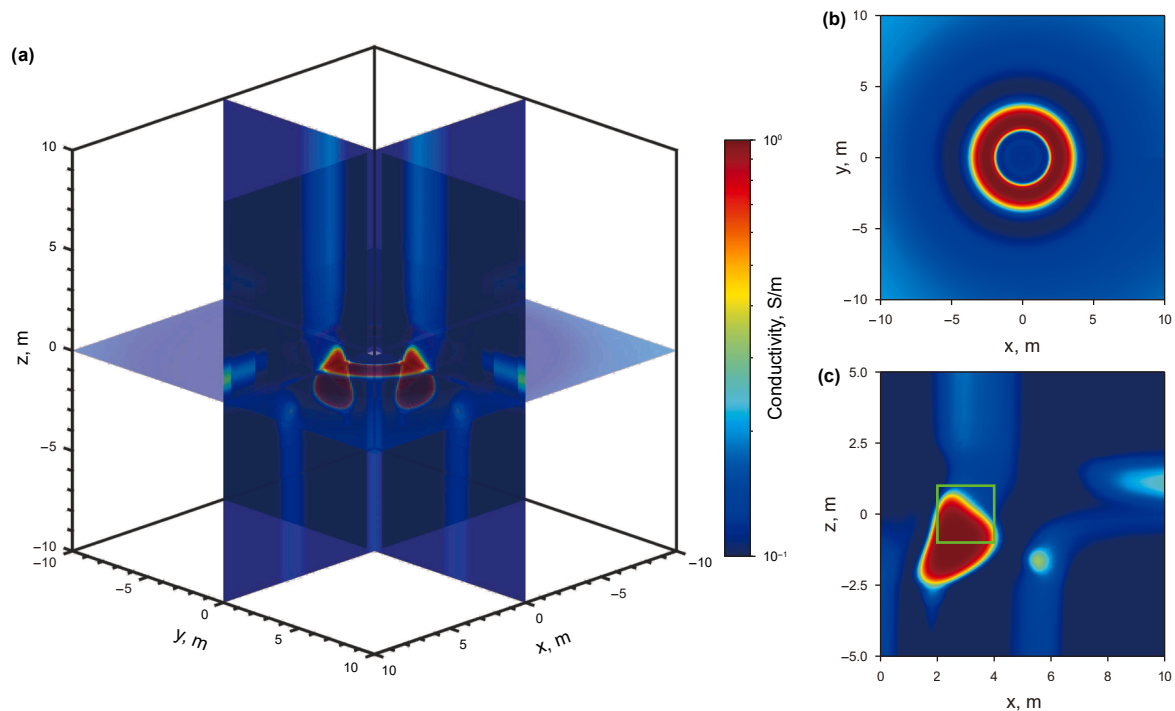


Fig. 13. Inversion results of CNN-Transformer without prior information. (a) 3D result, (b) XOY cross-section, (c) XOZ cross-section.

that contains stronger information from the anomalous body. Consequently, a larger relative error at late times is expected, while the response still follows the correct overall decay trend.

4.1.4. Comparison with conventional inversion method

As shown in Fig. 15(b) and (c), the Gauss–Newton method achieved a good fit to the time-domain response. However, the reconstructed conductive body in the ROZ cross-section (Fig. 15(a)) exhibits

pronounced lateral spreading and blurred boundaries, showing clear deviation from the true structure. This indicates that, the conventional method suffers from pronounced non-uniqueness, making it difficult to recover the actual geometry of the anomalous body.

4.1.5. Robustness analysis under noise conditions

To evaluate robustness under realistic conditions, we superimposed 5% Gaussian random noise onto the synthetic data. As

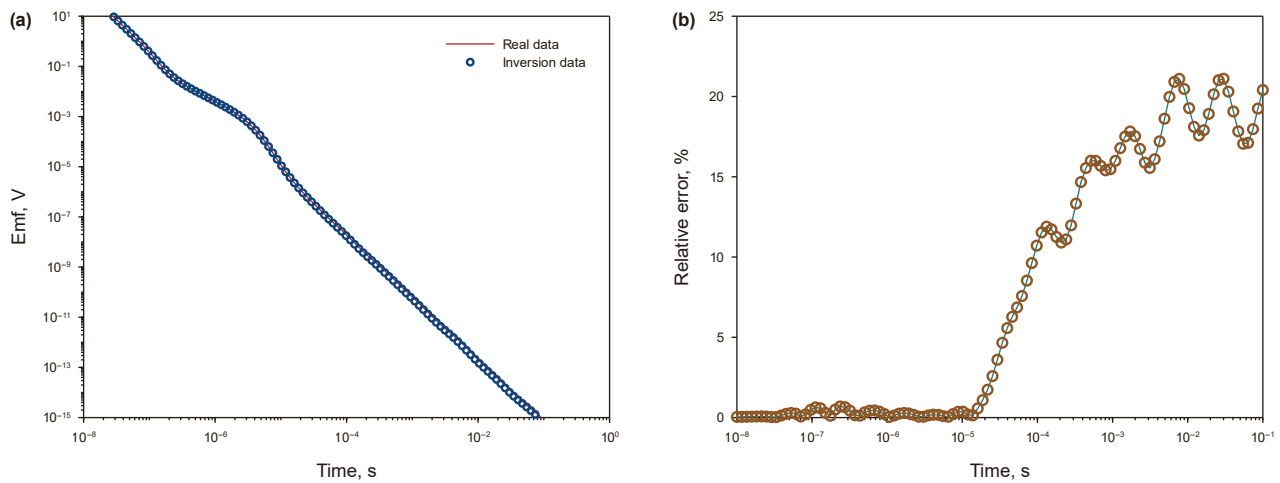


Fig. 14. Response fitting result for near-borehole anomalous body. (a) EMF fitting curve, (b) relative error.

shown in Fig. 16, the inversion accurately reconstructs the anomalous body's geometry and fits the noisy data well, confirming the method's stability. While field environments involve complex noise sources, advanced denoising techniques such as PC-BiLSTMNet have been shown to effectively reduce relative errors to below 3%, with nearly zero error before 10^{-3} s (Cheng and Wu, 2025). This suggests that by integrating such pre-processing strategies, this method can reliably handle field data even with significant measurement uncertainties.

4.2. Deep anomalous body

Fig. 17 shows the inversion result of the proposed framework for a deep target located between 10 and 12 m in the radial direction and -1 to 1 m in the longitudinal direction. In terms of position, the inversion successfully identifies the low-resistivity anomalous body beyond a 10 m radial distance. Although the inversion result exhibits some deviation in the radial direction, the longitudinal direction shows better inversion performance due to the influence of prior constraints. In terms of morphology, the result's shape shows certain discrepancies, exhibiting irregular radial stretching compared to the ideal shape, and the square structure is not fully recovered. Nevertheless, the result still captures the main spatial distribution of the deep anomalous body. The proposed model can effectively identify low-resistivity deep anomalous body located more than 10 m away from the transmitter coil, demonstrating its capability for accurate detection over considerable distances. This result also highlights the advantage of TEM logging in deep-detection applications.

As shown in Fig. 18(a), the true EMF curve exhibits an almost straight-line trend in the log-log coordinate system, indicating that the influence of the deep anomalous body located at a radial depth of 10–12 m on the logging response is extremely weak. This indicates that, during diffusive propagation, TEM fields have relatively weak sensitivity to deep targets, and the target-induced response can be easily masked by the background. Although the inverted responses can still fit the true data, the imaging results exhibit more pronounced errors compared with those of the near-borehole anomalous body. In future work, joint inversion with other logging data can be considered to improve the imaging

accuracy. It is worth noting that to more accurately reconstruct the weak responses from deep anomalous body, a smaller baseline value must be set to ensure that weak responses are not overlooked due to an excessively large baseline setting that prevents effective fitting.

4.3. Cross-scale anomalous body

Fig. 19 shows the inversion result of the proposed framework for a cross-scale anomalous body located between 2 and 2.2 m in the radial direction and -1.5 to 1.5 m in the longitudinal direction. In terms of scale, the anomalous body has a radial thickness of only 20 cm, which poses a significant challenge in practical exploration. The inversion result demonstrate that the proposed method can successfully identify the position of the anomalous body even at such a small radial extent, exhibiting high spatial resolution. Although localized bright spots appear in the result, the high conductivity values are primarily distributed within the black frame. Overall, it can accurately identify cross-scale anomalous body near the boundaries of the actual model. These findings indicate that the proposed framework maintains high accuracy and robustness when dealing with cross-scale anomalous and effectively overcomes the limitations of conventional inversion methods in small-scale anomalous body.

Due to the smaller radial length of the anomalous body, the curve in Fig. 20(a) is affected by the anomalous body for a shorter duration and exhibits lower amplitude compared to the inversion result in Fig. 14(a). Nevertheless, the proposed method still fits the real data well, with the relative error remaining stable until 10^{-5} s, demonstrating good fitting accuracy. This indicates that the method can reliably recover the trend of logging response variations when dealing with cross-scale anomalous body.

4.4. Radially dual anomalous bodies

Fig. 21 shows the inversion result of the proposed framework for radially dual anomalous bodies. The first anomalous body located between 1 and 2 m in the radial direction and -1 to 1 m in the longitudinal direction, while the second anomalous body located between 4 and 5 m in the radial direction and -1 to 1 m in

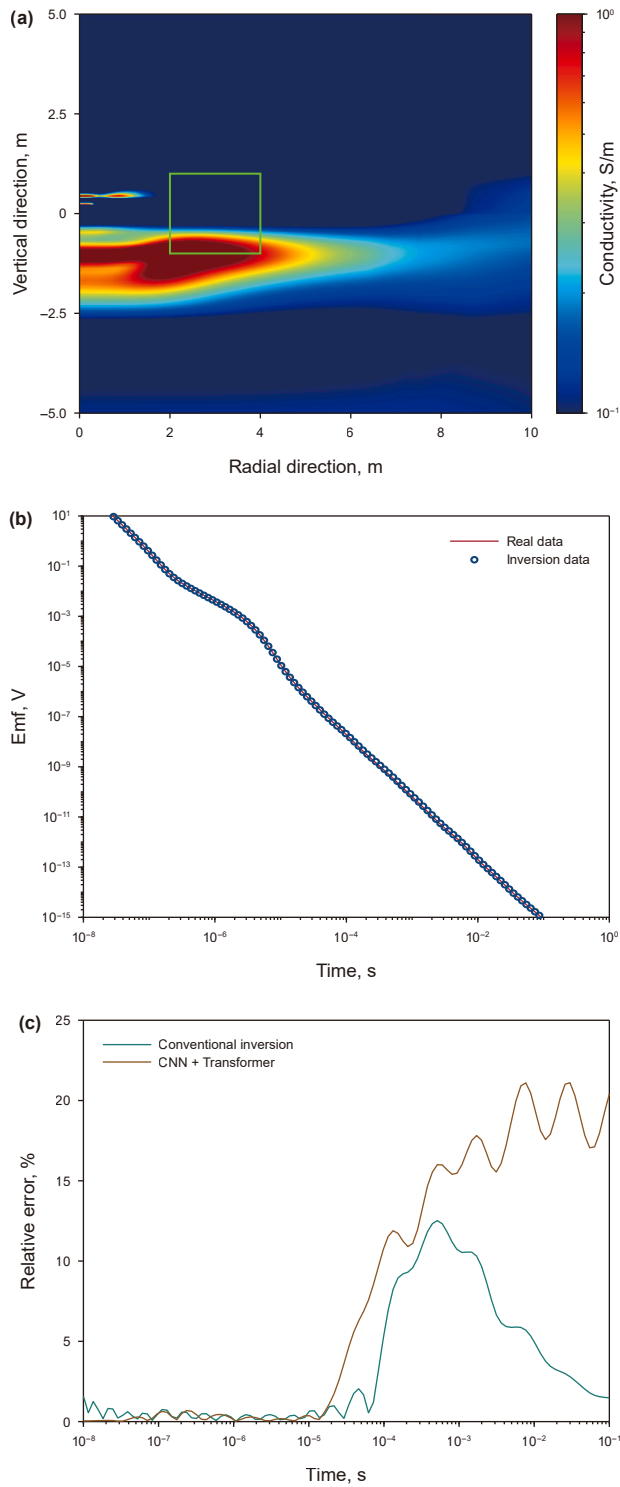


Fig. 15. Conventional inversion results. (a) ROZ cross-section, (b) EMF fitting curve, (c) relative error.

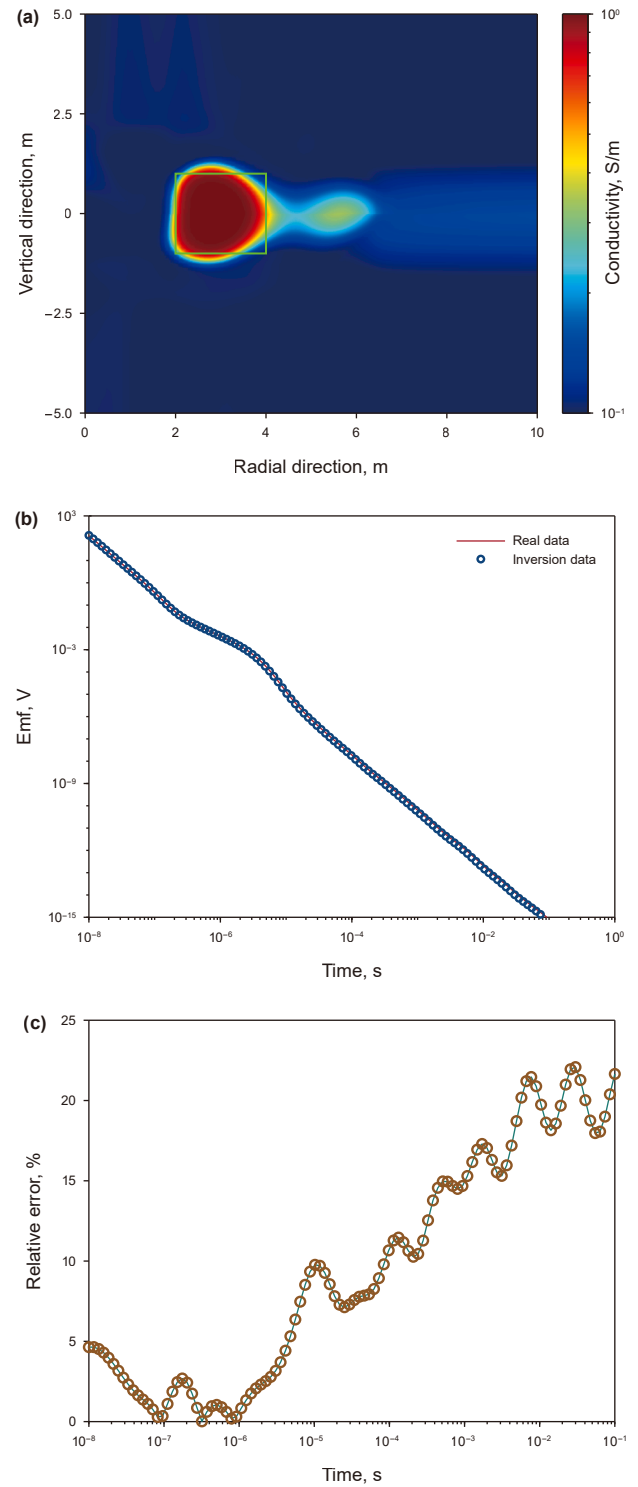


Fig. 16. Inversion results with 5% Gaussian random noise. (a) ROZ cross-section, (b) EMF fitting curve, (c) relative error.

the longitudinal direction. The black frame indicates the boundaries of the true anomalous bodies. For the first anomalous body, the result demonstrates high accuracy, with its location largely consistent with the target area. Meanwhile, the contours of the inversion result are relatively clear, and the area covered by the black frame accurately reflects the boundaries of the first

anomalous bodies. This demonstrates the model's exceptional performance in identifying near-borehole anomalous body. For the second anomalous body, although the boundary of the reconstructed structure is relatively blurred and the radial dimension deviates from the true model, the high-conductivity values remain within the target region. This indicates that the model retains

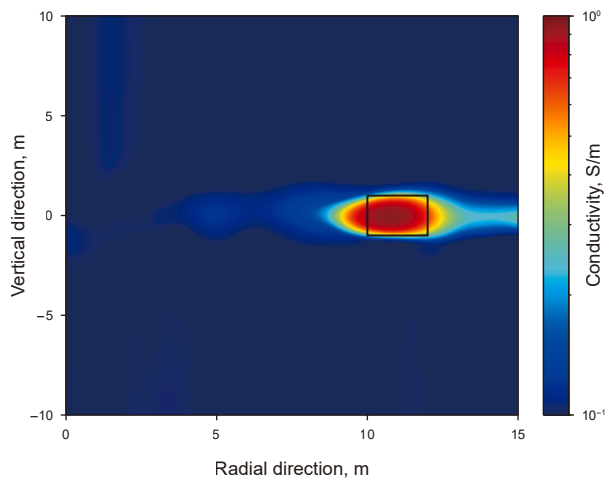


Fig. 17. Inversion results of deep anomalous body.

strong capability in detecting the presence and approximate location of radially multiple anomalous bodies. Furthermore, the method demonstrates robust performance by achieving high-quality inversion results across multiple models without relying on large training datasets, thereby enhancing its practical value for engineering applications.

Due to the combined influence of the two anomalous bodies, their responses are more likely to be coupled under a single-transmitter single-receiver configuration. As a result, the anomalous time segment in the response curve of Fig. 22(a) is prolonged, making it difficult to distinguish the presence of two distinct bodies from the curve alone. Consequently, the responses associated with the second anomalous body are harder to separate from the influence of the first one, leading to stronger non-uniqueness in localizing the second target and thus a higher likelihood of localization bias. Nevertheless, the proposed method successfully identifies the existence of the two anomalous bodies, further demonstrating its high reliability and robustness when handling complex problems.

5. Discussions

The proposed framework is currently time-consuming because it relies on numerical forward modeling, whose computational

cost is inherently much higher than that of analytical or semi-analytical approaches. Moreover, during iterative inversion, both the data-misfit evaluation and the adjoint-state Jacobian computation require repeated forward and adjoint simulations, which further increase the overall runtime. Future work will therefore focus on GPU-parallel implementation of the FVTD solver, parameterized or reduced-order forward modeling, and more efficient multi-resolution or cached evaluation strategies to further shorten the inversion time.

It should be emphasized that the current inversion framework is primarily optimized for conductive targets. Due to the shielding effect of low-resistivity formations, the response generated by high-resistivity targets relative to the background formation is inherently weak, making it challenging for the current inversion framework to recover high-resistivity targets with high accuracy. Therefore, in future work, we will further improve the framework for high-resistivity targets by incorporating multi-component measurements and exploring alternative preprocessing or response-enhancement strategies. Meanwhile, through multi-component coil design, the framework will be extended to detect and characterize azimuthally asymmetric structures, such as blocky anomalies.

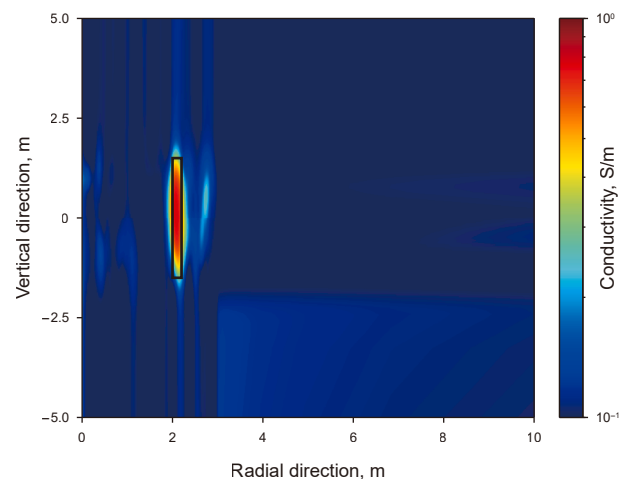


Fig. 19. Inversion results of cross-scale anomalous body.

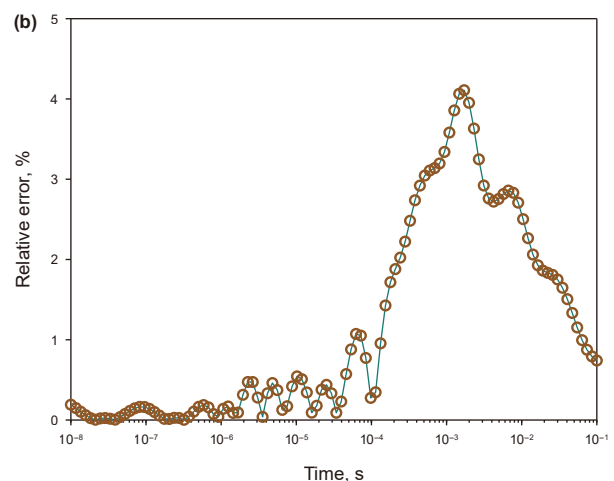
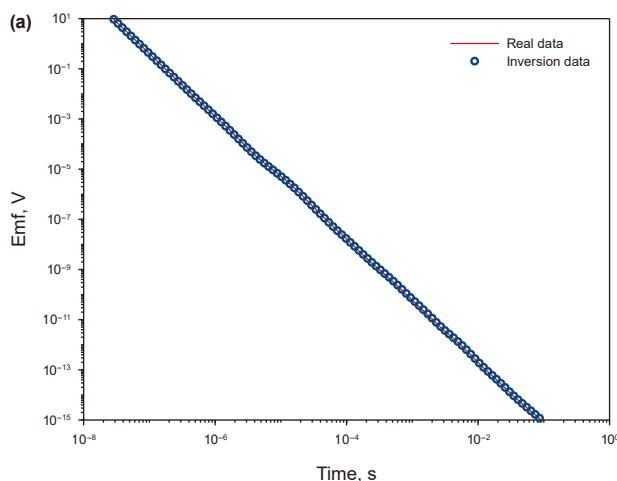


Fig. 18. Response fitting result for deep anomalous body. (a) EMF fitting curve, (b) relative error.

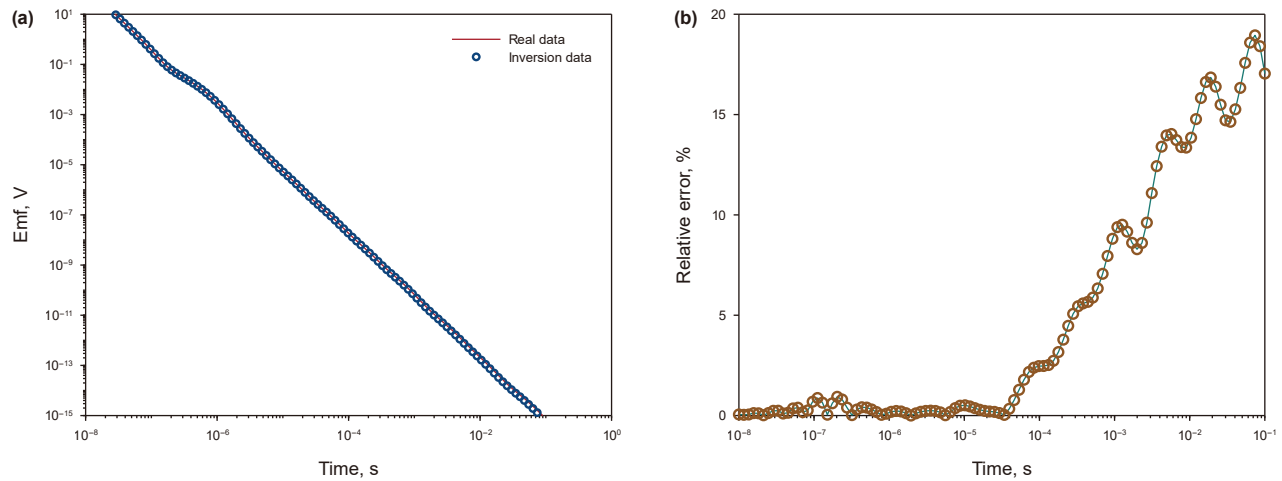


Fig. 20. Response fitting result for cross-scale anomalous body. (a) EMF fitting curve, (b) relative error.

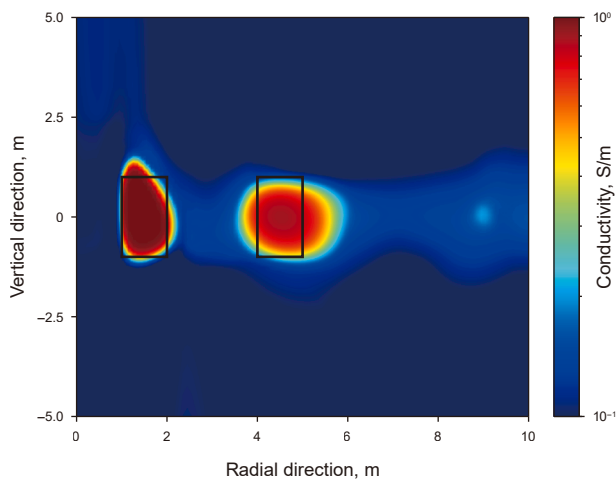


Fig. 21. Inversion results of radially dual anomalous bodies.

6. Conclusions

This study presents an unsupervised inversion framework for TEM logging that integrates physical constraints with geological prior information. The framework is built upon a CNN–Transformer generative model, in which the CNN model extracts local spatial features of borehole conductivity structures through multi-layer upsampling and 3D convolution, while the Transformer captures long-range dependencies via multi-head self-attention, achieving multi-scale fusion of local and global features. By incorporating the Jacobian matrix through adjoint-state method into the optimization process, the update direction of network parameters strictly adheres to the physical laws governed by the Maxwell's governing equations, ensuring effective coupling between the deep learning model and the FVTD method. In addition, the framework incorporates geological prior information and employs a weighting strategy to adaptively balance the data misfit term and the prior information prior during inversion process. In the early stages of optimization, the strategy

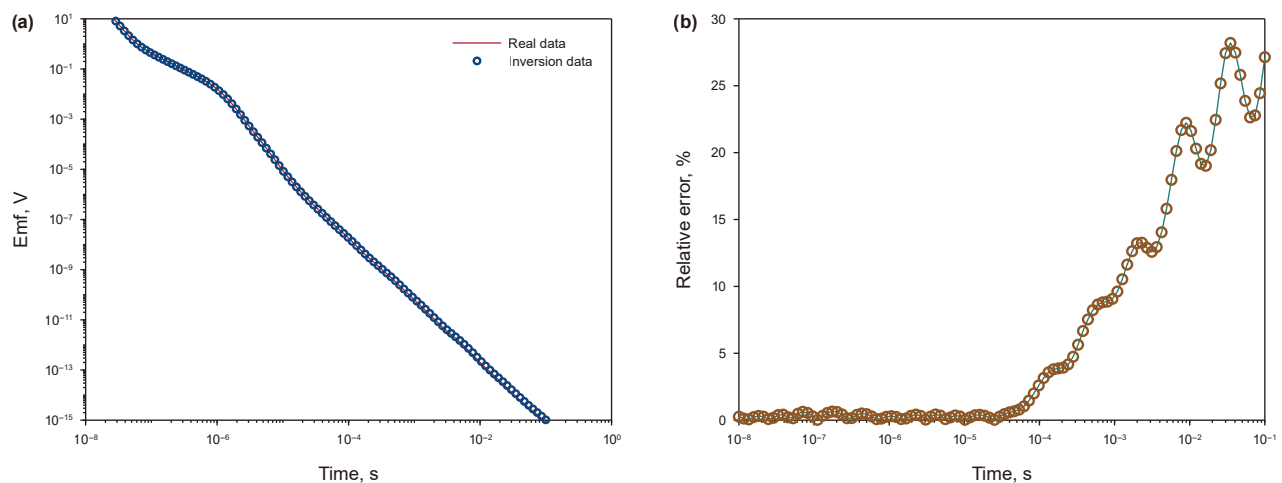


Fig. 22. Response fitting result for radially dual anomalous bodies. (a) EMF fitting curve, (b) relative error.

enhances the prior constraints to guide the model toward a geologically reasonable solution space. As iterations progress, it gradually increases the emphasis on the data misfit term to improve the fitting accuracy of the observed responses. The proposed framework operates without reliance on training datasets, effectively suppresses non-uniqueness, and improves boundary recovery accuracy. It achieves stable convergence under the dual constraints of the governing physical equations and geological priors. Numerical experiments demonstrate that this method exhibits outstanding imaging accuracy and convergence stability in models featuring near-borehole anomalous body, deep anomalous body, cross-scale anomalous body, and radially dual anomalous bodies. This work provides a novel approach to high-precision and robust inversion for TEM logging.

CRediT authorship contribution statement

Zhi-Jun Li: Writing – original draft, Methodology, Conceptualization. **Shao-Gui Deng:** Supervision, Project administration, Funding acquisition. **Yu-Zhen Hong:** Investigation, Formal analysis. **Yue-Qin Guo:** Writing – review & editing. **Xi-Yong Yuan:** Funding acquisition, Data curation. **Lian-Yun Cai:** Software, Formal analysis. **Zhou-Tuo Wei:** Resources, Funding acquisition. **Wen-Hao Chen:** Validation. **Chang-Sheng Liu:** Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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