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Joint PP–PS multi-objective inversion based on exact reflection coefficients in VTI media

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ABSTRACT

With the rapid progress of unconventional oil and gas exploration, detailed characterization of reservoir anisotropy has emerged as a key focus in seismic inversion studies. Traditional pre-stack inversion approaches that rely on approximate reflection coefficient equations often yield biased results at wide incidence angles, limiting their ability to provide high-precision parameter estimates for complex reservoirs. In this work, we develop a joint PP–PS multi-objective inversion strategy based on the exact reflection coefficient formulation for vertical transverse isotropy (VTI) media. To overcome the heavy reliance on weighting factors in joint inversion, we adopt improved Strength Pareto Evolutionary Algorithm (SPEA-II) into multi-objective evolutionary optimization algorithm. This allows simultaneous optimization of PP and PS objective functions without predefined weights, significantly mitigating solution non-uniqueness. Synthetic experiments confirm that the method reliably recovers model parameters under both noise-free and low signal-to-noise ratio (SNR) conditions. The inclusion of PS-wave data greatly enhances inversion accuracy, with notable improvements in thin-bed scenarios and structurally complex models. Tests on the 2D Marmousi-VTI model further validate the method's superiority in preserving lateral continuity and improving resolution. Application to field data shows that the proposed approach yields more accurate estimates of both elastic and anisotropic parameters compared with approximate-equation-based methods. The proposed PP–PS joint inversion method, grounded in the exact reflection coefficient formulation for VTI media, exhibits both theoretical rigor and practical applicability, providing a promising pathway for high-resolution exploration of unconventional reservoirs.

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1. Introduction

High-precision estimation of subsurface geophysical parameters has emerged as a key research focus nowadays (Chen et al., 2017; Pan et al., 2017; Guo et al., 2021; Wang et al., 2022). Compared with post-stack data, pre-stack seismic gathers preserve rich amplitude variation with offset (AVO) information, which serves as a crucial element for lithology discrimination, fluid prediction, and anisotropy estimation (Huang et al., 2022; Liu et al., 2023b; Zhou et al., 2023). Pre-stack seismic data typically exhibits low signal-to-noise ratios, which complicates the

inversion of formation parameters and poses a common challenge for both academic researchers and industry engineers (Cao et al., 2024). Nevertheless, pre-stack AVO inversion can offer high efficiency and great flexibility in practical parameter prediction (Yin et al., 2018; Stovas et al., 2021; Ding et al., 2022), including P- and S-wave velocities, density, fluid indicators, rock mechanical properties and anisotropic parameters. As a result, pre-stack AVO inversion has become a core technology in modern oil and gas exploration and development (Li et al., 2015; Cheng et al., 2022; Liu et al., 2024).

Traditional pre-stack inversion techniques are mostly based on the assumption of isotropic media, and have achieved remarkable success in conventional sandstone reservoir exploration (Wang et al., 2020; Zidan et al., 2022; Yang et al., 2024). However, as unconventional energy resources expand—particularly with the rising significance of shale oil and gas—the limitations of the

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isotropy assumption have become increasingly evident. Shale reservoirs commonly display pronounced anisotropy, most notably vertical transverse isotropy (VTI), characterized by a vertical symmetry axis (Zhou et al., 2020; Wang et al., 2025a). The VTI property mainly arises from horizontal bedding and the preferential alignment of mineral grains, and therefore is widely present in most shale gas and oil reservoirs (Chen and Zong, 2022). It is evident that isotropy-based pre-stack inversion methods fall short in accurately characterizing VTI reservoirs, highlighting the urgent need for more suitable anisotropic AVO inversion strategies (Yang et al., 2024). Researches on reflection coefficients in VTI media have progressed through several key stages (Cao et al., 2024; Zong et al., 2024). Thomsen proposed the well-known Thomsen parameters under the weak anisotropy assumption and derived approximate expressions for PP-wave reflection coefficients (Thomsen, 1986). Graebner derived exact reflection and transmission coefficients for interfaces between TI formations (Graebner, 1992). Rüger refined formulations of Thomsen and Graebner and derived multi-wave approximations for reflection coefficients applicable to both PP and PS waves (Rüger, 1997). Zhang and Li (2013) proposed higher-order approximations under the weak impedance contrast assumption. Zhou et al. (2020) derived simplified but more accurate reflection coefficient formulas directly from Graebner's exact equations. Recently, two main approaches have emerged for VTI inversion: one relies on simplified approximations offering higher computational efficiency, the other employs exact reflection coefficient equations for higher prediction results. Luo et al. (2022) proposed a stepwise inversion approach based on exact coefficients. Chen and Zong (2022) incorporated the Zoeppritz equation into Rüger's approximations to derive improved VTI reflection expressions, enabling joint PP–PS inversion. Fu (2024) developed a new approximation for VTI media by linking reflection coefficients with P- and S-wave impedances. In summary, although approximate methods remain appealing for their simplicity, inversion strategies employing exact reflection coefficients are increasingly recognized as a vital pathway for meeting the precision demands of modern hydrocarbon exploration.

PP-wave inversion based on exact VTI reflection equation has attracted considerable attention and has achieved meaningful results (Rüger, 1998; Sondergeld and Rai, 2011; Zhou et al., 2017; Deng and Zong, 2023; Ma et al., 2024). However, seismic inversion problems are generally ill-posed and exhibit nonuniqueness, making it difficult to obtain reliable density estimates by solely inverting PP-wave seismic data. Compared with PP-wave, PS-wave is less sensitive to fluids but contains richer density information (Wang et al., 2025b). Therefore, combining PP-wave and PS-wave data for joint inversion can not only effectively improve inversion accuracy, but also reduce non-uniqueness, in reservoir interpretation (Yin et al., 2018; Li et al., 2021). In PP–PS joint inversion, a joint objective function should be firstly constructed by weighting and summing the PP- and PS-wave objective functions. However, this approach is high weight-dependent, and weight selection is determined by empirical choices or trial-and-error, which may even result in entirely different inversion outcomes (Lim et al., 2019; Song et al., 2023). In contrast, multi-objective intelligent optimization circumvents the need for preset weights by simultaneously optimizing multiple objective functions, ultimately yielding a set of non-dominated Pareto-optimal solutions.

Bayesian framework has been widely adopted in geophysical inversion due to its unique strength in parameter uncertainty quantification (Zhou et al., 2021; Liu et al., 2023a). It allows researchers to introduce complex prior distributions through incorporating geological background and geophysical principles into the inversion process, thereby enhancing the accuracy and

stability of the results (Pan et al., 2019; Huang et al., 2022; Chen et al., 2023; Song et al., 2023). Alemie and Sacchi employ tri-variate Cauchy prior to exploit parameter correlations, which significantly improves AVO inversion stability and resolution (Alemie and Sacchi, 2011). Moreover, the heavy-tailed characteristics of Cauchy distribution can promote sparsity of the solution, thereby enhancing the inversion resolution. Yin and Zhang (2014) proposed a pre-stack AVO inversion constrained by a T-distribution prior. Zhou et al. (2023) incorporated blocky constraints governed by a differential Laplace prior, effectively reducing side lobes and sharpening reservoir boundary delineation. This study extends earlier work to a five-variable formulation by generalizing the tri-variate Cauchy prior for better adapting multi-parameter and high-resolution characteristics joint inversion of elastic and anisotropic parameters in VTI media.

This study proposes a Bayesian-based PP–PS joint inversion framework grounded in the exact reflection coefficient formulation for VTI media. Compared with conventional inversion methods based on approximate reflection coefficient equations, the proposed approach fully preserves the physical accuracy of anisotropic wave propagation, thereby improving parameter estimation reliability, particularly at large incidence angles. Based on the exact VTI reflection coefficients, a multi-objective inversion function is constructed to simultaneously estimate P-wave velocity, S-wave velocity, density, and two anisotropic parameters. To overcome the strong dependence on empirical weighting factors in conventional multi-wave joint inversion, an enhanced Strength Pareto Evolutionary Algorithm (SPEA-II) is introduced to solve the multi-objective nonlinear optimization problem without pre-defined weights, effectively reducing solution non-uniqueness and improving inversion stability. Furthermore, a Bayesian inversion framework incorporating prior constraints is employed to enhance robustness under noisy conditions. Synthetic experiments and field data applications demonstrate that the proposed method provides more accurate and stable recovery of both elastic and anisotropic parameters compared with traditional approximate-equation-based inversion approaches, confirming the effectiveness and practical applicability of the proposed joint inversion strategy.

2. Methodology

2.1. Derivation of exact reflection coefficient formulations for VTI media

VTI is the simplest anisotropic case in subsurface. In VTI media, the vertical axis serves as the symmetry axis, and the elastic behavior can be fully characterized by five independent stiffness coefficients. The stiffness tensor of a VTI medium can be represented in matrix form as follows (Schoenberg, 1980):

$$c_{ij} = \begin{bmatrix} c_{11} & c_{11} - c_{66} & c_{13} & & \\ & c_{11} & c_{13} & & \\ & & c_{33} & & \\ & & & c_{44} & \\ & & & & c_{44} \\ & & & & & c_{66} \end{bmatrix} \quad (1)$$

To clarify the physical significance of VTI media, Thomsen characterized the medium using two vertical velocities and three dimensionless anisotropy parameters instead of the five independent stiffness coefficients (Thomsen, 1986). The relationship among them can be expressed as follows:

Table 1
Parameters of the model.

Layer	V_{P0} , km/s	V_{S0} , km/s	ρ , g/cm ³	ϵ	δ
Sandstone	3.368	1.829	2.50	0.110	−0.035
Shale	4.529	2.703	2.52	0.034	0.211

$$V_{P0} = \sqrt{C_{33}/\rho}, V_{S0} = \sqrt{C_{44}/\rho}, \gamma = \frac{C_{66} - C_{44}}{2C_{44}},$$

$$\epsilon = \frac{C_{11} - C_{33}}{2C_{33}}, \delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \quad (2)$$

where V_{P0} and V_{S0} denote the vertical P-wave and S-wave velocities, respectively; ρ is the density of the medium; and C_{ij} represents the elastic stiffness coefficients of a transversely isotropic medium. The parameters ϵ and δ are the Thomsen anisotropy parameters, which describe the strength of seismic anisotropy.

Graebner (1992) derived exact analytical formulations for the reflection and transmission coefficients in VTI media:

$$\mathbf{r} = \mathbf{G}^{-1} \mathbf{b} \quad (3)$$

The specific form is as follows:

$$\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \begin{bmatrix} l_\alpha^u & n_\beta^u & -l_\alpha^d & -n_\beta^d \\ n_\alpha^u & -l_\beta^u & n_\alpha^d & -l_\beta^d \\ a^u & b^u & a^d & b^d \\ d^u & e^u & -d^d & -e^d \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad (4)$$

$$a^u = c_{55}^u (q_\alpha^u l_\alpha^u + p n_\alpha^u), \quad b^u = c_{55}^u (q_\beta^u l_\beta^u - p l_\beta^u),$$

$$a^d = c_{55}^d (q_\alpha^d l_\alpha^d + p n_\alpha^d), \quad b^d = c_{55}^d (q_\beta^d l_\beta^d - p l_\beta^d),$$

$$d^u = p l_\alpha^u c_{13}^u + q_\alpha^u n_\alpha^u c_{33}^u, \quad e^u = p n_\beta^u c_{13}^u - q_\beta^u n_\beta^u c_{33}^u,$$

$$-d^d = -(p l_\alpha^d c_{13}^d + q_\alpha^d n_\alpha^d c_{33}^d), \quad -e^d = -(p n_\beta^d c_{13}^d - q_\beta^d n_\beta^d c_{33}^d) \quad (5)$$

In the above formulas, the superscript u denotes parameters of the overlying medium, while the superscript d corresponds to those of the underlying medium. The terms l_α , n_α , l_β and n_β denote the cosine components of the polarization vectors, which can be explicitly expressed as (Zhou et al., 2020):

$$l_\alpha = \sqrt{\frac{c_{33} q_\alpha^2 + c_{55} p^2 - \rho}{c_{55} q_\alpha^2 + c_{11} p^2 - \rho + c_{33} q_\alpha^2 + c_{55} p^2 - \rho}} \quad (6)$$

$$n_\alpha = \sqrt{\frac{c_{55} q_\alpha^2 + c_{11} p^2 - \rho}{c_{55} q_\alpha^2 + c_{11} p^2 - \rho + c_{33} q_\alpha^2 + c_{55} p^2 - \rho}} \quad (7)$$

$$l_\beta = \sqrt{\frac{c_{55} q_\beta^2 + c_{11} p^2 - \rho}{c_{55} q_\beta^2 + c_{11} p^2 - \rho + c_{33} q_\beta^2 + c_{55} p^2 - \rho}} \quad (8)$$

$$n_\beta = \sqrt{\frac{c_{33} q_\beta^2 + c_{55} p^2 - \rho}{c_{55} q_\beta^2 + c_{11} p^2 - \rho + c_{33} q_\beta^2 + c_{55} p^2 - \rho}} \quad (9)$$

The detailed calculation procedures for the symbols used in Eqs. (5)–(9) are provided in Appendix A. Here, p represents the ray parameter. For VTI media, Thomsen (1986) presented a simplified expression.

$$p = \frac{\sin \theta}{V_{P0} (1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta)} \quad (10)$$

In Eq. (4), $\mathbf{r}$ represents reflection and transmission coefficients at the interface:

$$\mathbf{r} = (R_{PP} \ R_{PS} \ T_{PP} \ T_{PS})^T \quad (11)$$

The symbol $\mathbf{b}$ represents a computation vector related to wave elastic and anisotropic parameters:

$$\mathbf{b} = [-G_{11} \ G_{21} \ G_{31} \ -G_{41}]^T \quad (12)$$

To evaluate the differences between the exact reflection coefficients and Rüger approximate reflection coefficient equations for VTI media (Rüger, 1997), we employed the two models in Table 1 and analyzed results over incident angles ranging from 1° to 60°. The comparison is illustrated in Fig. 1. Fig. 1(a) displays the PP-wave reflection coefficient, Fig. 1(b) presents the PS-wave reflection coefficient. The results indicate that both Rüger approximate VTI reflection coefficients and the exact Zoeppritz solution under the isotropic assumption gradually deviate from the exact VTI reflection coefficients with incident angles increasing.

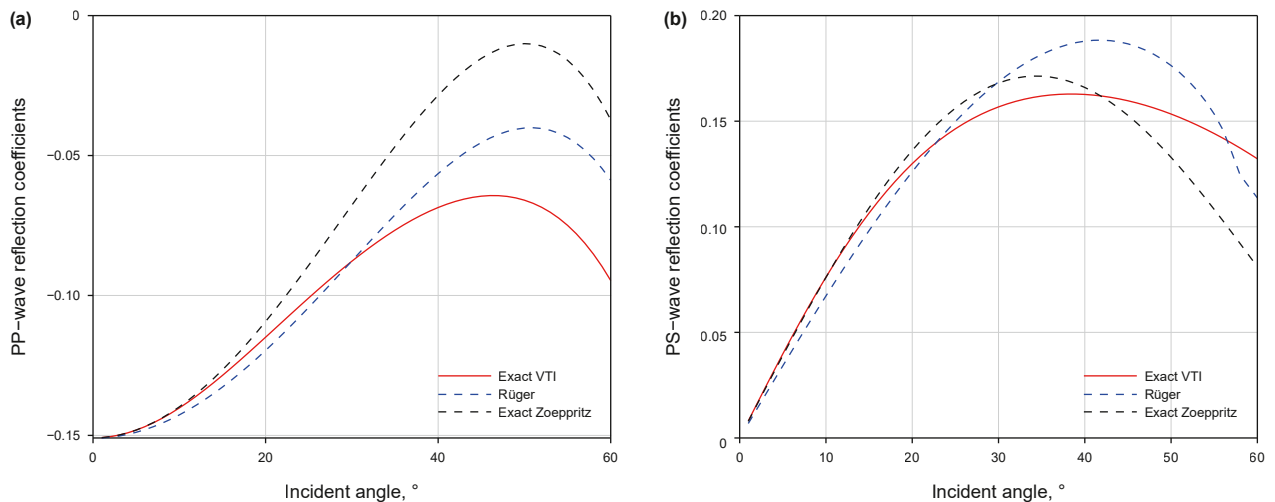


Fig. 1. Comparison of exact and approximate reflection coefficients: (a) PP-wave reflection coefficient, (b) PS-wave reflection coefficient.

2.2. Construction of the inversion objective function under the bayesian framework

The inversion work based on the exact reflection coefficient equations of VTI media can be described as obtaining elastic and anisotropic parameters from PP- and PS-wave seismic records. Pre-stack angle gather seismic is obtained through the convolution of a seismic wavelet with the reflection coefficient series, and the linkage between model parameters and pre-stack angle gathers can be expressed as:

$$\mathbf{d} = G(\mathbf{m}) \quad (13)$$

where $\mathbf{d}$ refers to seismic angle gather data, G denotes the nonlinear forward modeling operator, and $\mathbf{m}$ corresponds to the model elastic and anisotropic parameters. Constructing a PP–PS joint inversion objective function based on Bayesian framework not only aims to find the optimal solution, but also analyzes the uncertainty of inversion. The key difference of statistical theory lies in the introduction of prior information. The posterior probability can be obtained from the prior information and the likelihood function, and shown as:

$$P(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{m})P(\mathbf{m})}{P(\mathbf{d})} \propto P(\mathbf{d}|\mathbf{m})P(\mathbf{m}) \quad (14)$$

where $P(\mathbf{m}|\mathbf{d})$ is the posterior probability distribution function, $P(\mathbf{d}|\mathbf{m})$ is the likelihood function, $P(\mathbf{m})$ is the estimated prior probability distribution, and $P(\mathbf{d})$ is the marginal probability distribution, a normalization constant, aims to ensure that the sum of the posterior distribution probabilities integrates to 1. Assuming the likelihood function follows a Gaussian distribution:

$$P(\mathbf{d}|\mathbf{m}) = \frac{1}{\sqrt{(2\pi)^{n_d} |\mathbf{C}_d|}} \exp\left(-\frac{1}{2}(\mathbf{d} - G(\mathbf{m}))^T \mathbf{C}_d^{-1} (\mathbf{d} - G(\mathbf{m}))\right) \quad (15)$$

In the above equation, n_d represents the dimension of the data, and $\mathbf{C}_d$ is the noise covariance matrix, which is usually reduced to a diagonal matrix. Bayesian inversion, being inherently probabilistic, often yields solutions with uncertainty. To address this, prior distributions of model parameters are introduced to provide additional constraints and enhance the well-posedness of the inversion. Alemie and Sacchi (2011) introduced a trivariate Cauchy distribution to retain the heavy-tailed characteristic of the Cauchy distribution while accounting for statistical correlations among elastic parameters, thus enabling efficient recovery of subsurface characteristics. In this work, we extend it to a five-variable modified Cauchy distribution:

$$P(\mathbf{m}) = \frac{1}{\pi(2n_d) |\mathbf{C}_m|^{n_d/2}} \exp\left(-\sum_{i=1}^{n_d} \ln(1 + \mathbf{m}^T \Phi^i \mathbf{m})\right) \quad (16)$$

where $\Phi^i = \mathbf{D}_i^T \chi^{-1} \mathbf{D}_i$, where $\mathbf{D}_i$ is a $5 \times 5n_d$ matrix with a specific form of

$$\mathbf{D}_{xy}^i = \begin{cases} 1 & \text{if } x = 1 \text{ and } y = i \\ 1 & \text{if } x = 2 \text{ and } y = i + \mathbf{n}_d \\ 1 & \text{if } x = 3 \text{ and } y = i + 2\mathbf{n}_d \\ 1 & \text{if } x = 4 \text{ and } y = i + 3\mathbf{n}_d \\ 1 & \text{if } x = 5 \text{ and } y = i + 4\mathbf{n}_d \\ 0 & \text{else} \end{cases} \quad (17)$$

In conventional PP–PS joint inversion, PP and PS objective functions are merged into a single-objective problem via

weighted summation, simplifying the formulation and making it easier to solve. However, such weighted combinations do not ensure that the final solution is simultaneously optimal for both objectives. Moreover, the subjective choice of weights inevitably introduces human bias into the inversion outcome. To overcome this limitation, researchers employed multi-objective optimization algorithms to simultaneously optimize both functions, leading to a new objective function of the PP–PS joint inversion:

$$\begin{cases} \mathbf{J}_{PP}(\mathbf{m}) = \frac{1}{2}(\mathbf{d}_{PP} - G_{PP}(\mathbf{m}))^T (\mathbf{d}_{PP} - G_{PP}(\mathbf{m})) + \lambda \Gamma(\mathbf{m}) \\ \mathbf{J}_{PS}(\mathbf{m}) = \frac{1}{2}(\mathbf{d}_{PS} - G_{PS}(\mathbf{m}))^T (\mathbf{d}_{PS} - G_{PS}(\mathbf{m})) + \lambda \Gamma(\mathbf{m}) \end{cases} \quad (18)$$

where $\Gamma(\mathbf{m})$ denotes the prior constraint, λ specifies its associated weighting factor.

2.3. SPEA-II

SPEA-II is a multi-objective evolutionary optimization method. Zitzler and Thiele (1999) first introduced SPEA in 1999, where individual fitness values were defined as “strength”. Zitzler et al. (2001) refined the approach and developed SPEA-II to overcome its limitations. PP–PS joint inversion is an inherently high nonlinear problem, where the non-uniqueness of solution is theoretically unavoidable. Conventional approaches usually merge multiple objectives into a single function via weight assignment, but this inevitably introduces subjectivity where different weight settings can result in entirely different solutions. By contrast, SPEA-II, as a representative multi-objective optimization algorithm, simultaneously optimizes multiple objectives without preset weights, yielding a set of trade-off solutions that balance competing objectives. In single-objective optimization, solutions can be directly ranked by their objective function values. In multi-objective optimization, however, objectives often conflict, making direct comparison between solutions impossible. To address this, the concept of Pareto non-dominated sorting is employed, enabling rational ranking of solutions under multiple objectives and producing a set of non-dominated optimal solutions. The essence of SPEA-II is to integrate the neighborhood search mechanism of genetic algorithms with Pareto-based non-dominated sorting, thereby generating solution sets that approximate the global optimum. The workflow of multi-objective optimization is outlined in Fig. 2 and consists of the following main steps.

- (1) Initialization: Generate the initial population P_0 under model constraints. Each individual consists of randomly selected P-wave velocity, S-wave velocity, and density values within the permissible range. The population size is set to N , and an external archive Q_0 of size M is created simultaneously.
- (2) Fitness evaluation: Using the initialized parameters, evaluate the two objective functions and compute the fitness values for all individuals in P_t and Q_t , where t representing the iteration index.
- (3) Environmental selection: Following the Pareto ranking strategy, all non-dominated individuals in P_t and Q_t are preserved in Q_{t+1} . If the archive size exceeds M , surplus individuals are truncated; if the size of Q_{t+1} is less than M , dominated individuals from P_t and Q_t are chosen to fill Q_{t+1} .
- (4) Termination: If $t \geq T$ (maximum iterations) or other stopping criteria is met, all nondominated individuals in Q_{t+1} are taken as the final solution set.

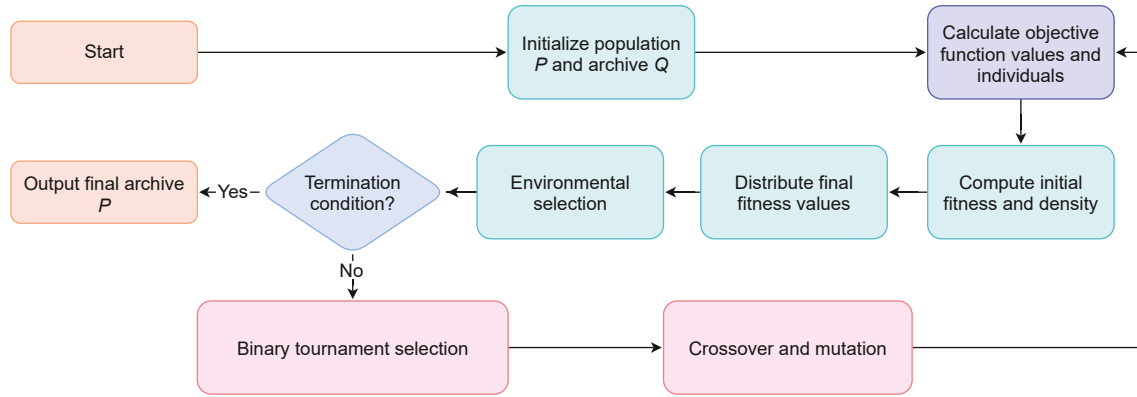


Fig. 2. Flowchart of SPEA-II.

In SPEA-II, fitness is calculated as Eq. (19). The strength of each solution is calculated and defined as the number of other solutions it dominates:

$$R(i) = \sum_{j \in P+Q, j > i} S(j) \quad (19)$$

In SPEA-II, the final fitness of each individual accounts for both its raw fitness $R(i)$ and a density estimation term $D(i)$, which is introduced to differentiate solutions with comparable fitness levels, defined as:

$$D(i) = \frac{1}{\sigma_i^k + 2} \quad (20)$$

where

$$k = \sqrt{|P| + |Q|} \quad (21)$$

where σ_i^k denotes the distance from individual i to its k -th nearest neighbor, while P and Q correspond to the sizes of the population and the archive, respectively.

By integrating raw fitness with density estimation, SPEA-II evaluates not only the dominance ranking of individuals but

also their similarity. Accordingly, the final fitness of individual i can be expressed as:

$$F(i) = D(i) + R(i) \quad (22)$$

3. Synthetic test

3.1. Application on single-well model data

In this section, the proposed nonlinear inversion approach, derived from the exact VTI reflection coefficient formulation, is validated using synthetic pre-stack angle gather data. The validation focuses on assessing inversion accuracy and robustness against noise. First, a theoretical model is built from real well-log data, with its parameters illustrated in Fig. 3. Subsequently, within the incidence angle range of 5° – 45° , PP- and PS-wave reflection coefficients are calculated using the re-derived exact VTI reflection coefficient equations, and convolved with a 30 Hz Ricker wavelet to generate synthetic multi-wave seismic records. To examine robustness of the method, Gaussian random noise with signal-noise-ratio (SNR) of 5 is superimposed on the noise-free synthetic

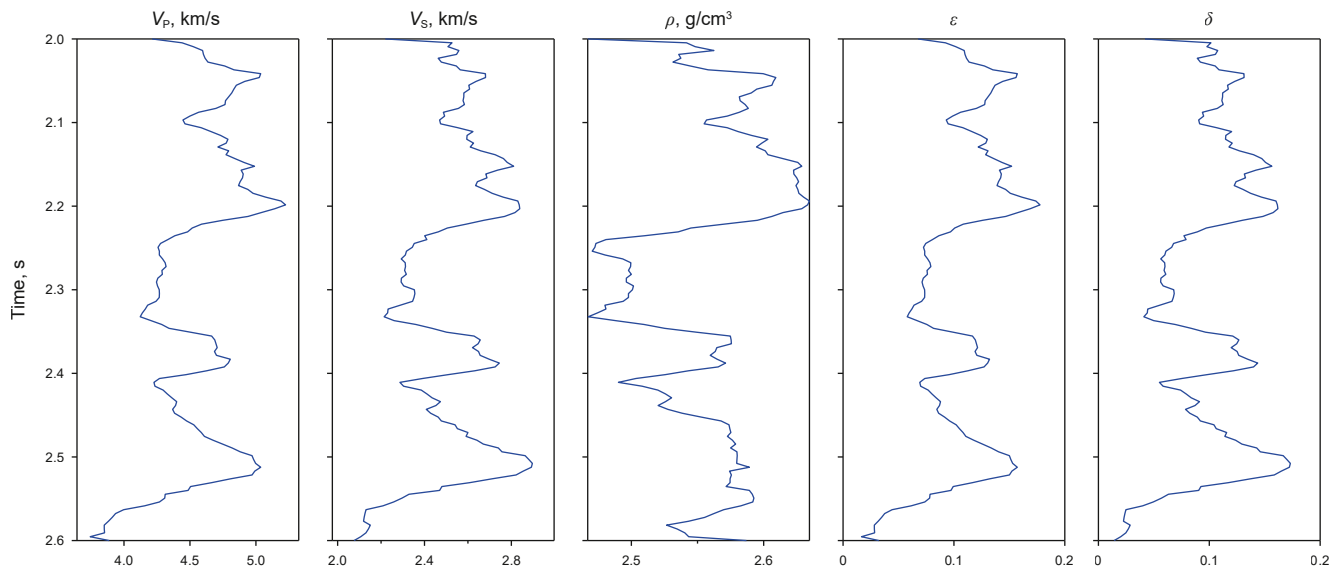


Fig. 3. Real well logs data. P-wave velocity (V_p), S-wave velocity (V_s), density (ρ), Thomsen anisotropy parameters ϵ and δ are shown from left to right.

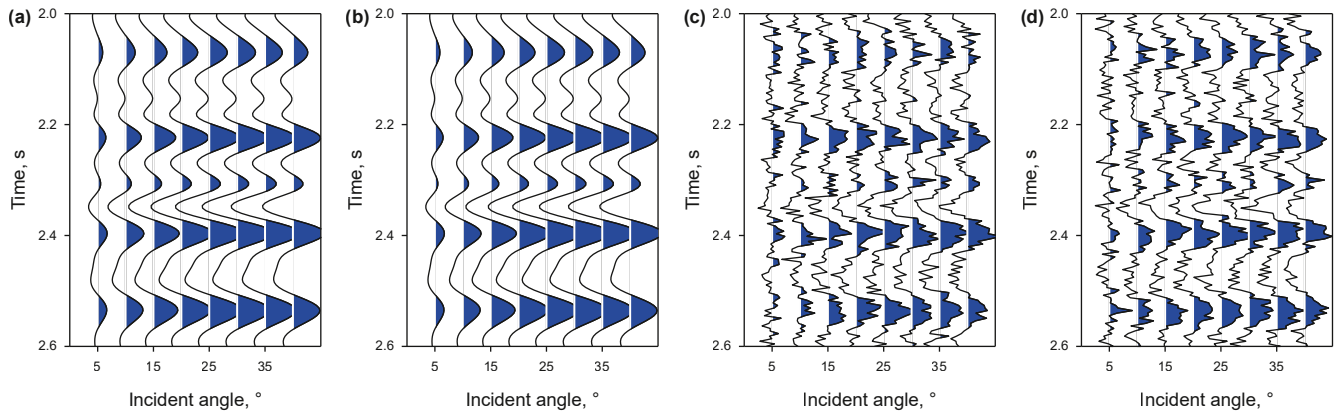


Fig. 4. Synthetic seismic data. (a) PP-wave record without noise. (b) PS-wave record without noise. (c) PP-wave record with SNR = 5. (d) PS-wave record with SNR = 5.

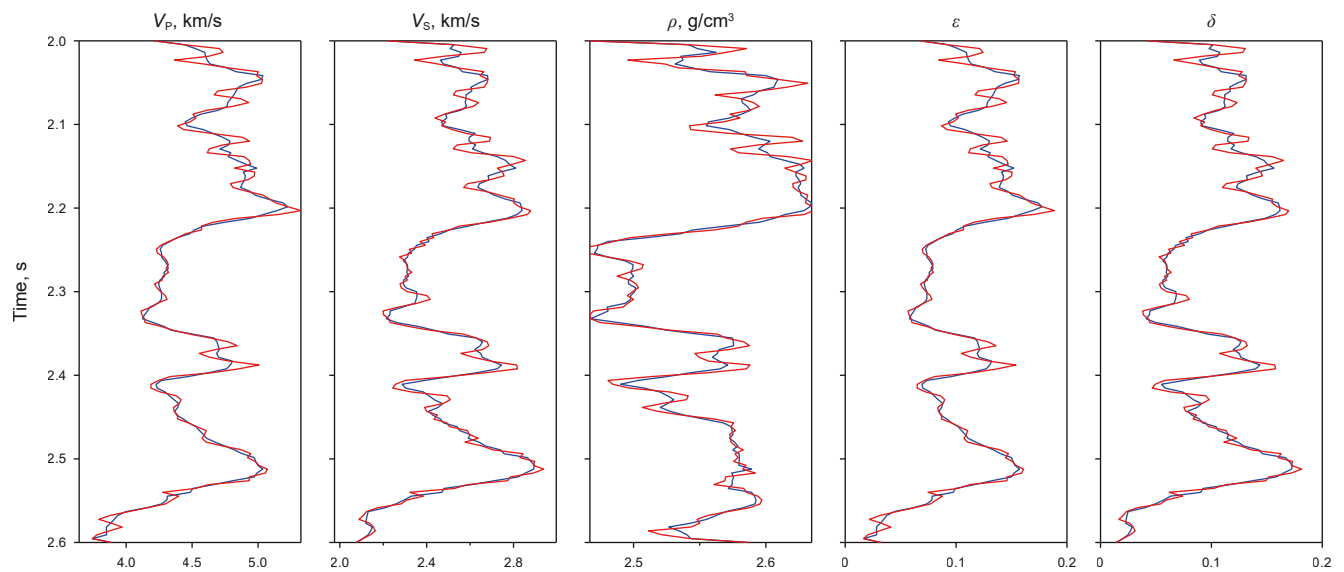


Fig. 5. PP-PS inversion results using the proposed method based on the exact VTI reflection coefficients in the case of noise free. The red solid line denotes the inversion results, the blue solid line indicates true model.

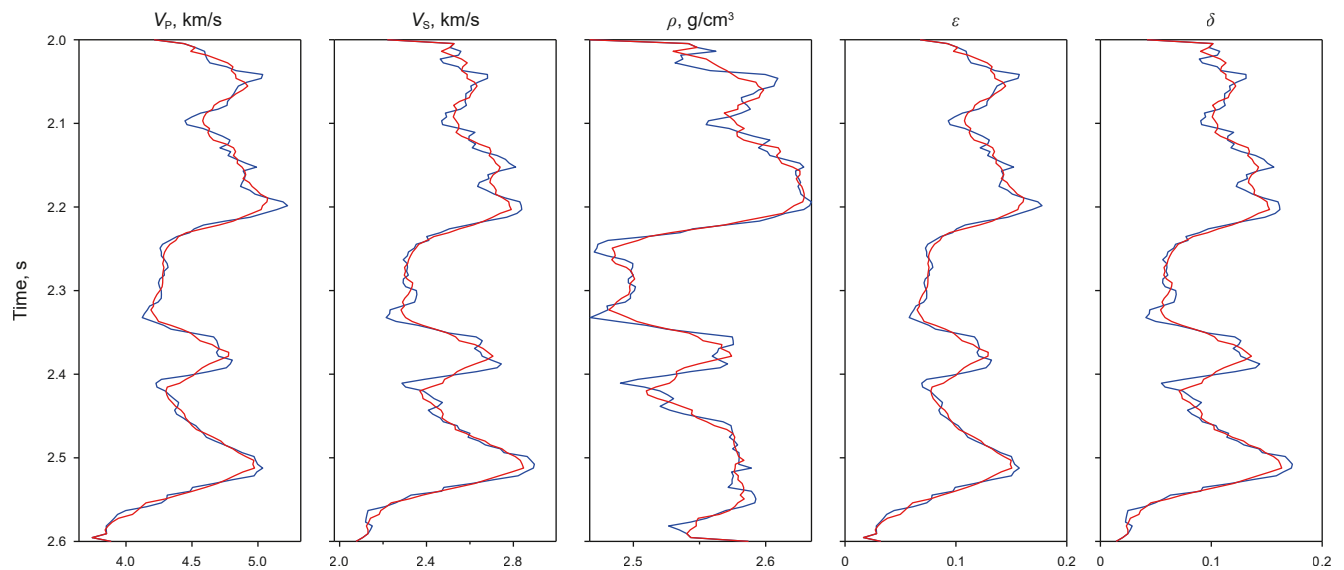


Fig. 6. PP inversion results using the proposed method based on the exact VTI reflection coefficient in the case of noise free. The red solid line denotes the inversion results, the blue solid line indicates true model.

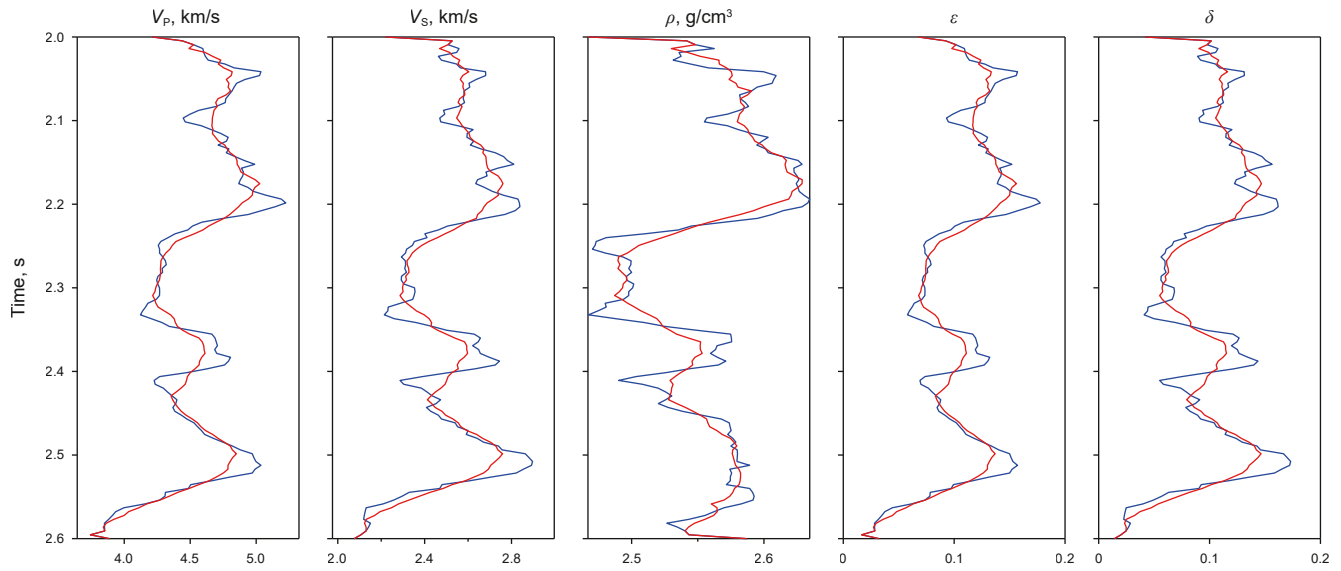


Fig. 7. PP inversion results using based on Rüger approximation. The red solid line denotes the inversion results, the blue solid line indicates true model.

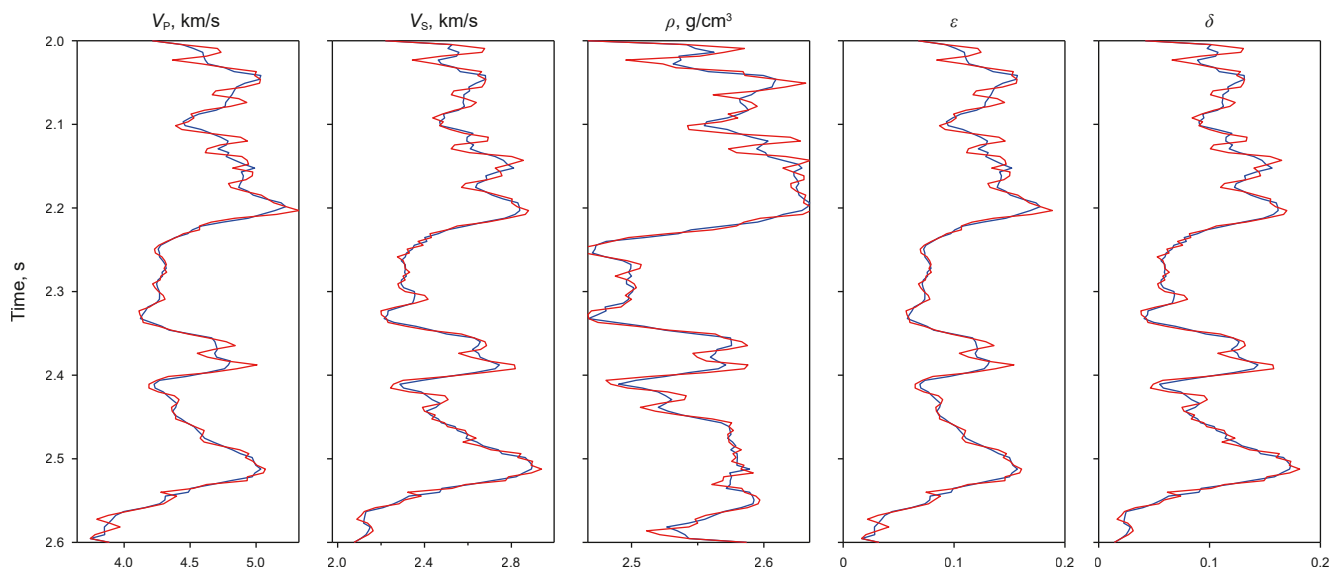


Fig. 8. PP-PS inversion results using the proposed method based on the exact VTI reflection coefficient in the case of SNR = 5. The red solid line denotes the inversion results, the blue solid line indicates true model.

records, generating noisy datasets. The resulting synthetic seismic records are displayed in Fig. 4. Figs. 5–9 show the inversion results obtained using different datasets and methods. Pearson correlation coefficients and mean relative errors of the inversion results are then calculated in Table 2.

3.2. Application on 2-D Marmousi-VTI model

In this section, the proposed PP-PS nonlinear inversion scheme based on the exact VTI reflection coefficients is employed to the Marmousi-VTI model to evaluate the performance. The tests are conducted on a sparsified version of the

Marmousi-VTI model. The model contains 150 traces laterally and 200 sampling points vertically, with a time sampling rate of 2 ms. First, synthetic seismic gathers are generated using the exact VTI reflection coefficient formulation. In the synthetic data, a 35 Hz Ricker wavelet is used and the incidence angles vary from 5° to 40° with 5° increments. SPEA-II is applied in the inversion with a population size of 200, 800 iterations, and parameter search ranges constrained to $\pm 30\%$ of the true values. Parallel multi-trace computation is also employed to reduce computation time.

The sparsified Marmousi-VTI model is displayed in Fig. 10. Fig. 11 presents the inversion results from noise-free synthetic

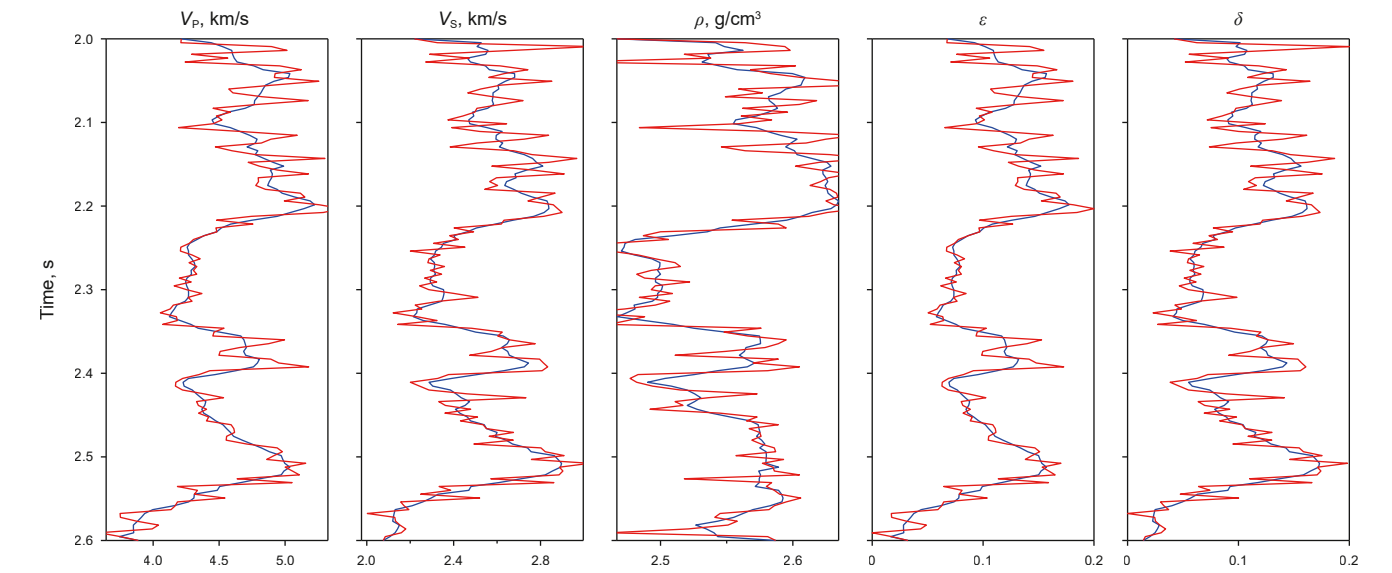


Fig. 9. PP-PS inversion results using the proposed method based on Rüger approximation in the case of SNR = 5. The red solid line denotes the inversion results, the blue solid line indicates true model.

Table 2
Pearson correlation coefficients (PCC) and mean relative errors (MRE, in %) of the inversion results.

SNR	Method	P-wave velocity (V_P)		S-wave velocity (V_S)		Density (ρ)		Thomsen anisotropy parameter (ϵ)		Thomsen anisotropy parameter (δ)	
		PCC	MRE, %	PCC	MRE, %	PCC	MRE, %	PCC	MRE, %	PCC	MRE, %
Noise free	Proposed	0.98	0.05	0.98	0.06	0.97	0.06	0.97	0.09	0.98	0.09
	PP wave	0.95	0.75	0.95	0.65	0.95	0.97	0.93	1.02	0.93	1.07
	Rüger	0.92	1.22	0.92	0.98	0.92	1.37	0.90	2.89	0.90	2.57
SNR = 5	Proposed	0.93	0.17	0.92	0.19	0.93	0.24	0.91	0.69	0.91	0.84
	Rüger	0.89	3.56	0.88	3.46	0.87	4.46	0.85	6.46	0.85	6.28

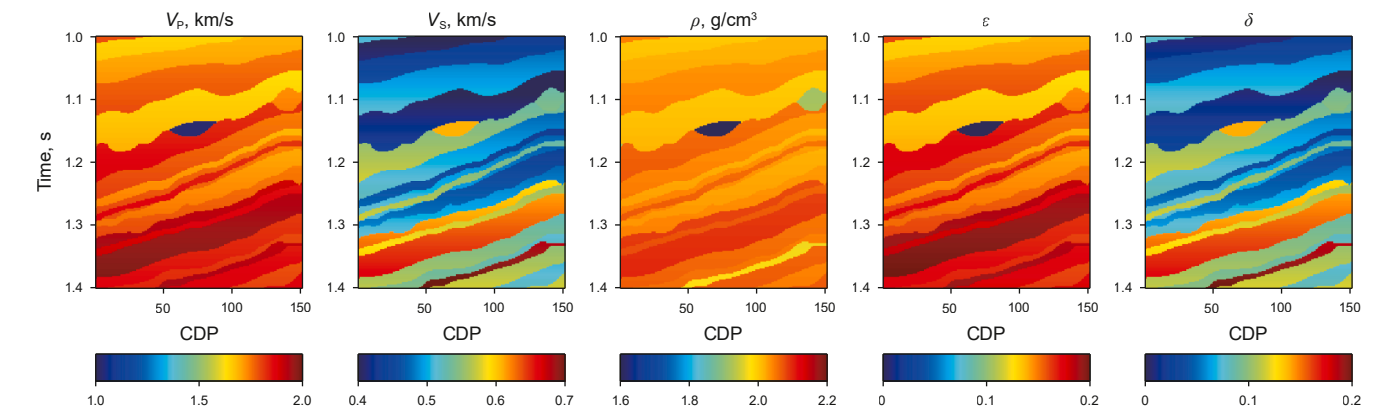


Fig. 10. Marmousi-VTI model.

data. Fig. 12 presents the inversion results of multi-wave based on approximate equations in the case of noise free. Fig. 13 illustrates the inversion results in the case of SNR = 5.

It can be seen from Fig. 11 that the proposed inversion method can well reflect the structural features of the true model. In the noise-free case, the correlation coefficient between the inverted results and the true model exceeds 0.95,

demonstrating excellent agreement. Notably, the inversion accuracy of elastic parameters is generally higher than that of anisotropic parameters. This is attributed to the greater sensitivity of anisotropic parameters to large-incidence-angle data, whereas the incidence angle range used in this study provides relatively limited constraints. Inversion results verify that the method can invert high resolution results in thin-bed

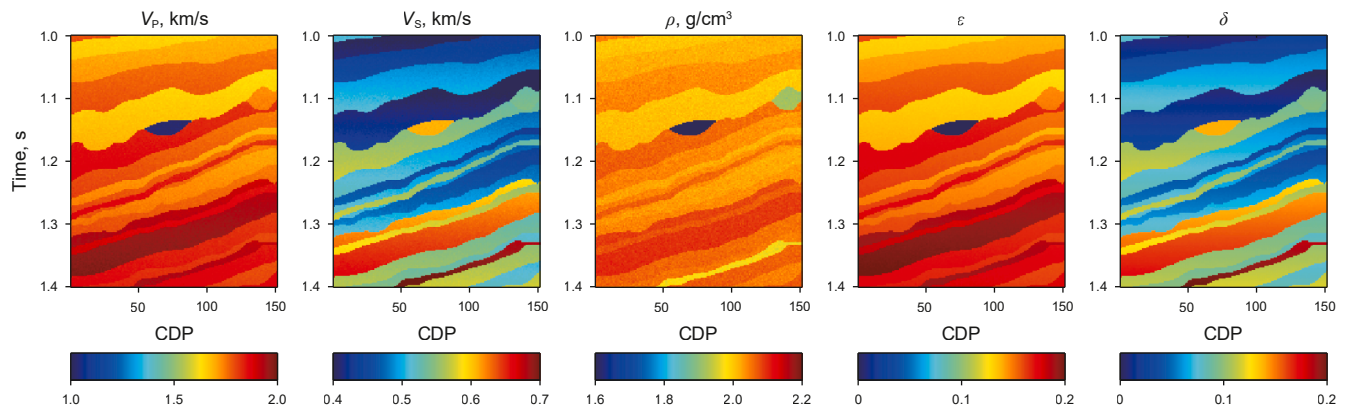


Fig. 11. Inversion results using the proposed method in the case of noise free.

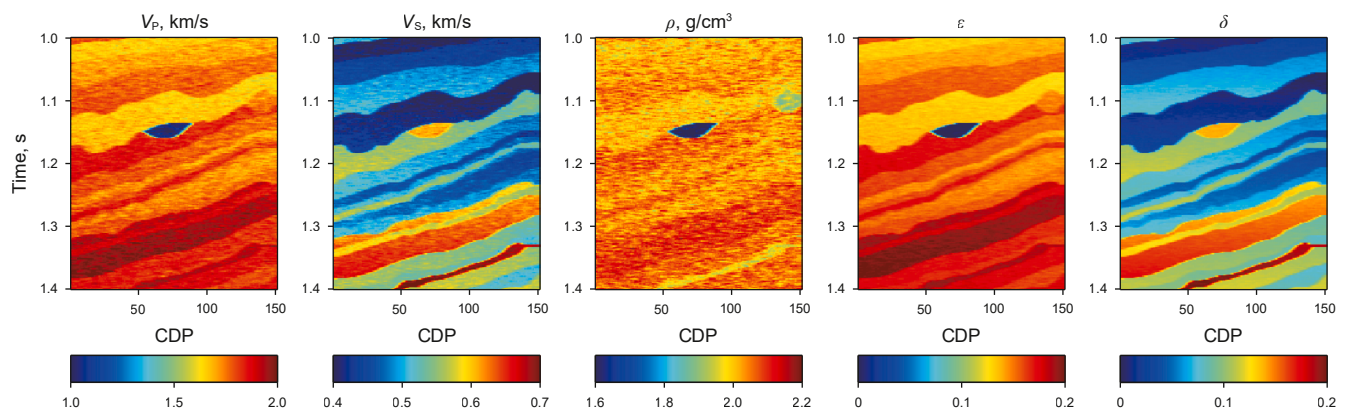


Fig. 12. Multi-wave inversion results based on approximate equations in the case of noise free.

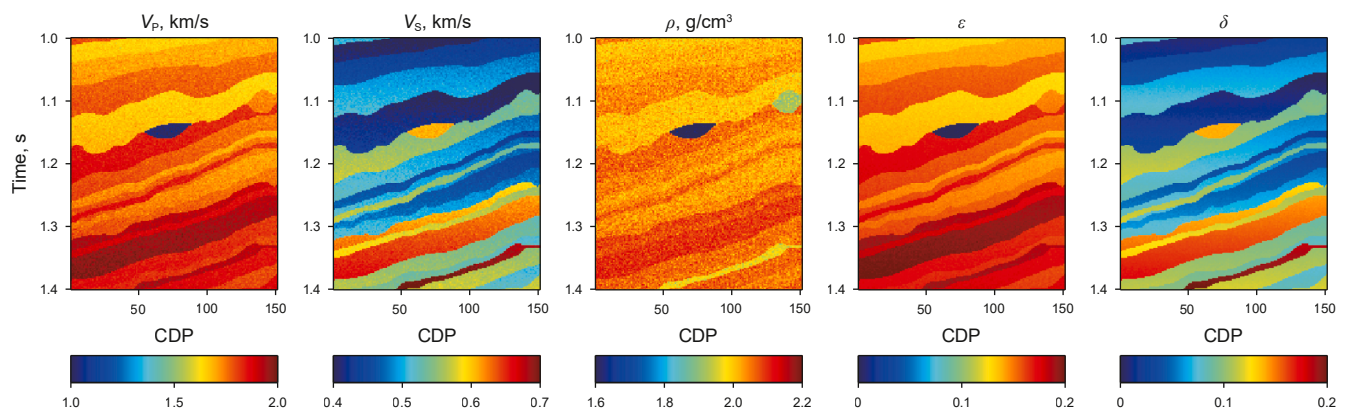


Fig. 13. Inversion results using the proposed method in the case of SNR = 5.

formations. The improvement is mainly come from the inclusion of PS-wave data, which enhances both the accuracy and stability of the inversion. In the case of SNR = 5, although the inversion accuracy declines (lower correlation coefficients), the method can still reliably reconstruct the overall trends and primary

structural features of the model. In the inverted anisotropic parameter profiles, local structural variations remain identifiable even under the noisy condition.

Compared with the noise-free scenario, the inversion maintains good lateral continuity even when moderate noise is present in the

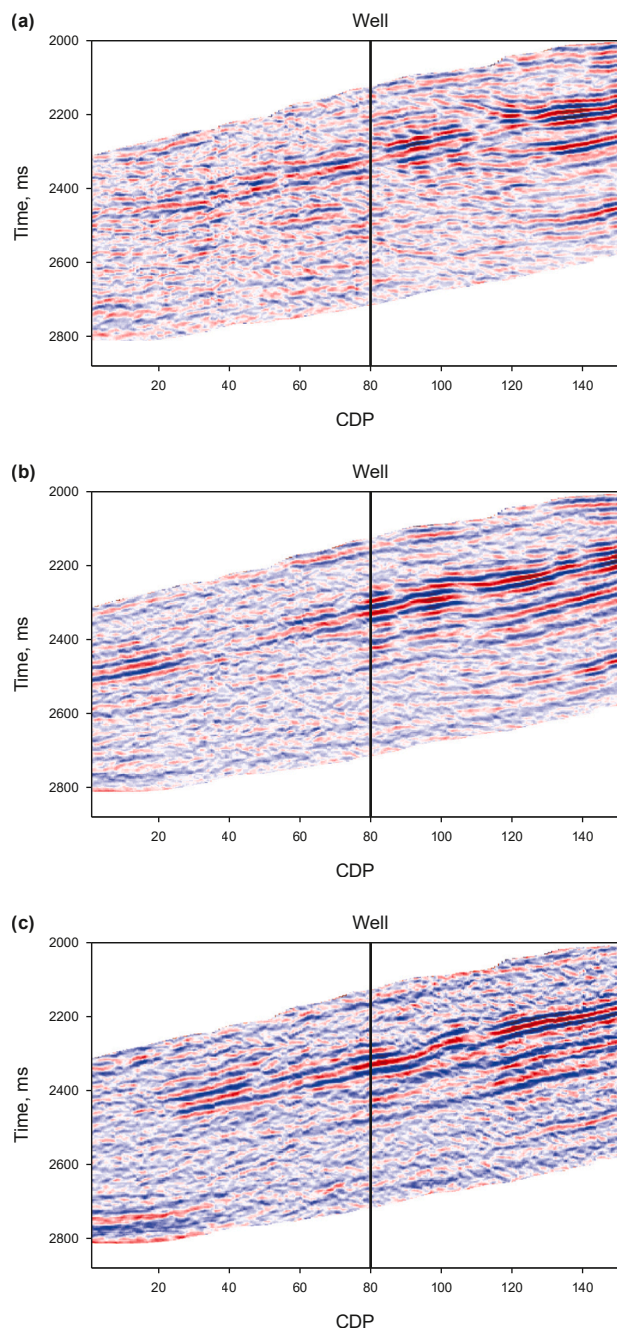


Fig. 14. Partial stacked seismic records of field data: (a) incidence angle of 5°, (b) incidence angle of 15°, (c) incidence angle of 25°.

synthetic data. While the resolution of thin beds decreases slightly, the method can still discern thin-bed anomalies. Overall, the 2D modeling results demonstrate that the proposed method excels in both lateral continuity and vertical resolution, confirming its potential and practical value for inversion in complex 2D models.

4. Field application

The proposed nonlinear inversion method based on the exact VTI reflection coefficient equations is used to field data for verifying the feasibility and stability in practical application. It is important to note that seismic acquisition in the study area employed single-component recording, which precluded PS-wave data acquisition. Consequently, the testing is conducted solely on PP-wave seismic data. The dataset originates from a shale gas production zone in the Sichuan Basin, southwestern China. The test interval is dominated by limestone and shale formations. Geological information indicates that shale reservoir of this region is thick, overpressured with simple structure buried deeper than 3000 m. The field data was processed using a standard pre-stack workflow, including amplitude compensation and correction, deconvolution, noise suppression, NMO correction, interbed multiple suppression, and pre-stack time migration. From the processed pre-stack angle gathers, partial angle stacks at 5°, 15°, and 25° were extracted, as displayed in Fig. 14, with the black line indicating the well location. For the well data, in the absence of image logs, anisotropic parameter curves were predicted using rock-physics modeling, which has been extensively validated in previous studies and proven reliable in practice, thereby providing reasonable parameter constraints for subsequent inversion. Given the limited lateral variation of the formation, the parameter search ranges in SPEA-II inversion are constrained within $\pm 50\%$ of the well-log values. Moreover, well-log information is incorporated as a prior constraint in the objective function, improving both accuracy and stability of the inversion. During the inversion, SPEA-II is run for 500 iterations with an initial population of 200 individuals, and multi-trace parallel computation is employed to reduce run-time and improve efficiency.

As shown in Fig. 15, the inversion results of elastic and anisotropic parameters are obtained through applying the proposed nonlinear inversion scheme based on the exact VTI reflection coefficient equations. For further comparison, an inversion using traditional method based on Rüger approximate VTI reflection coefficient equations was also conducted, and the results are shown in Fig. 16. By comparing Figs. 15 and 16, it can be observed that the proposed method outperforms traditional method based on Rüger approximation in terms of overall inversion performance with higher lateral resolution, clearer structural features, and better overall accuracy. Fig. 17 shows the inversion results and the logging curves at the well location.

The results indicate that the proposed nonlinear inversion method based on the exact VTI reflection coefficients can achieve strong agreement with well-log data, particularly demonstrating excellent matching within the target reservoir. By contrast, the linear inversion method based on approximate reflection coefficients captures only the general trend of the well logs, but fails to provide sufficient accuracy near the target reservoir. Overall, this method can provide reliable anisotropic parameter estimates and is suitable for shale oil and gas reservoir inversion. It can offer key insights for reservoir enhancement and oil and gas reservoir evaluation, thereby reducing the risks in horizontal well trajectory design and deployment.

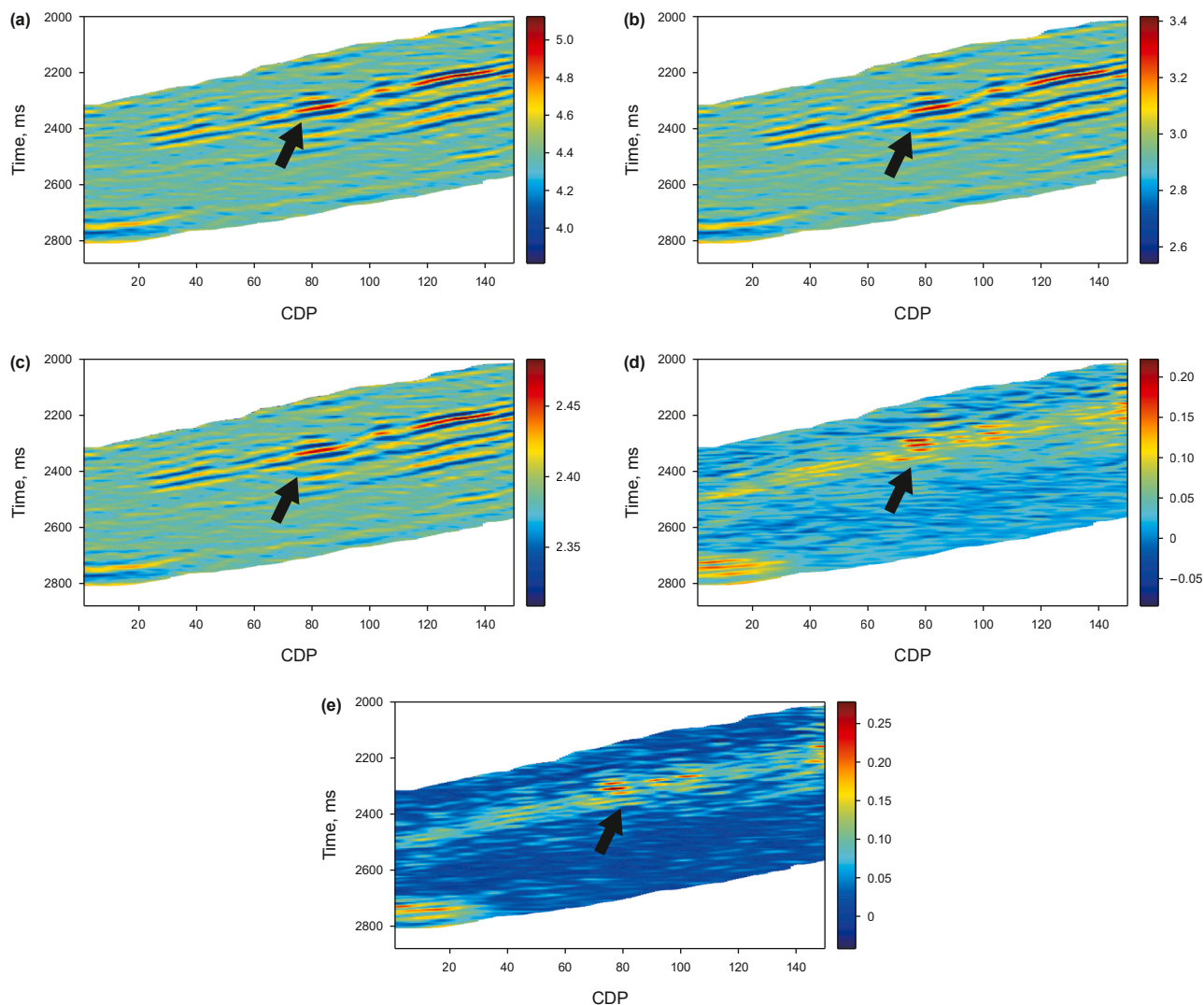


Fig. 15. Inversion profile using the proposed method.

5. Discussion

The proposed PP–PS joint inversion method based on the exact VTI reflection coefficients shows strong stability and accuracy across theoretical derivations, synthetic experiments, and field applications. Compared with conventional inversion strategies relying on approximate reflection coefficients, the proposed method based on the exact VTI reflection coefficients achieves accurate estimation of subsurface parameters at far offsets and large incidence angles. These findings suggest that adopting the exact reflection coefficients can well reflect the underlying physics and provide great practical value in real seismic processing. From a theoretical perspective, conventional AVO/AVA inversion relies on weak-anisotropy assumptions and approximate reflection coefficients, which leads to an effective inversion while biases at large incidence angles, resulting in unstable parameter estimates.

By incorporating Graebner exact formulation into the joint inversion model, this study overcomes the limitations of

approximate expressions in complex reservoirs and highlights the critical role of exact coefficients in enhancing inversion accuracy. Moreover, a multi-objective optimization framework is introduced to eliminate the strong dependence on weight selection inherent in conventional joint inversion. SPEA-II enables simultaneous optimization of multiple objectives without preset weights, yielding a Pareto-optimal solution set and reducing uncertainties introduced by subjective weight assignments. Coupled with Bayesian inference and sparse prior constraints, the method excels in mitigating non-uniqueness and improving inversion resolution.

Both synthetic tests and field application confirm that the method reliably reconstructs structure and key parameters under moderate SNR conditions. In field application, the proposed method shows superior performance, achieving strong agreement with well logs at well location and outperforming conventional approaches in lateral resolution and structure delineation. These results are significant for shale gas exploration, offering reliable anisotropy information for reservoir stimulation and providing

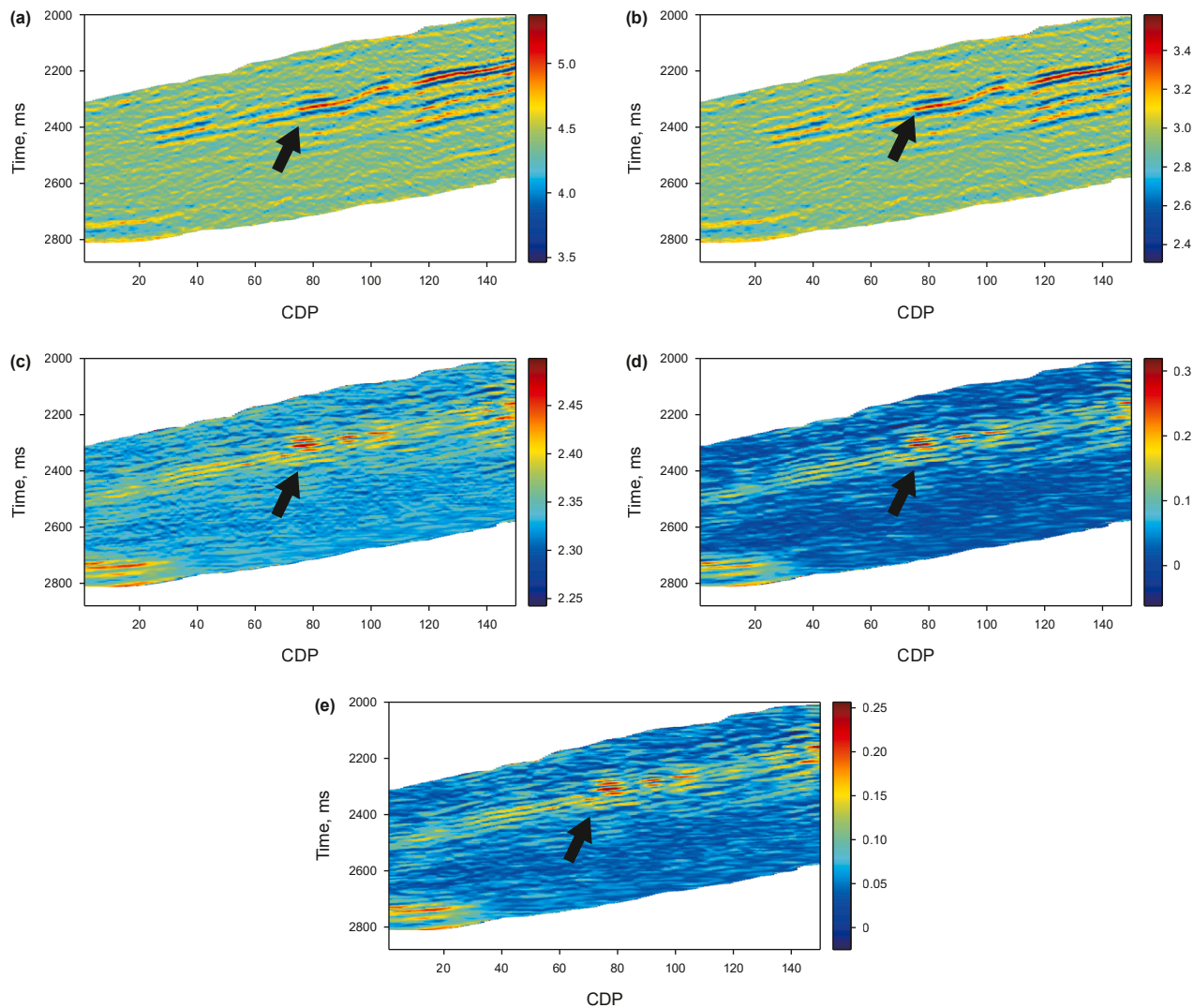


Fig. 16. Inversion results using traditional method based on Rüger approximate formula.

valuable guidance for horizontal well placement and trajectory design.

It should be noted that the field dataset used in this study was acquired using single-component seismic recording, which limits the availability of converted PS-wave information. Therefore, the field-data application presented in this work only validates the PP-wave component of the proposed inversion framework rather than a fully implemented PP-PS joint inversion. Although the results still demonstrate improved inversion stability and parameter estimation accuracy, the absence of PS-wave data restricts the full exploitation of multi-wave complementary information. Future work will focus on applying the proposed method to multicomponent seismic datasets to fully evaluate the performance and practical advantages of true PP-PS joint inversion in field applications. Despite multi-objective optimization method and prior constraints are involved in the inversion, computational complexity remains high in high dimension

parameter spaces, demanding substantial computational resources. Future studies could leverage parallel computing and emerging techniques such as deep learning to improve efficiency. In practice, the selection of priors and the tuning of constraint strength depend heavily on geological data and expert judgment, which may bias results. Developing automated data-driven approaches for prior modeling remains an important avenue for future research.

Despite the promising performance of the proposed method, several limitations should be acknowledged. Anisotropic parameters are generally more difficult to estimate than elastic parameters due to their weaker sensitivity to seismic amplitudes and stronger parameter coupling. When incidence-angle coverage is limited or the signal-to-noise ratio is low, inversion stability may decrease and parameter trade-offs may increase. Although SPEA-II multi-objective optimization improves solution diversity, it may still converge to local Pareto-optimal

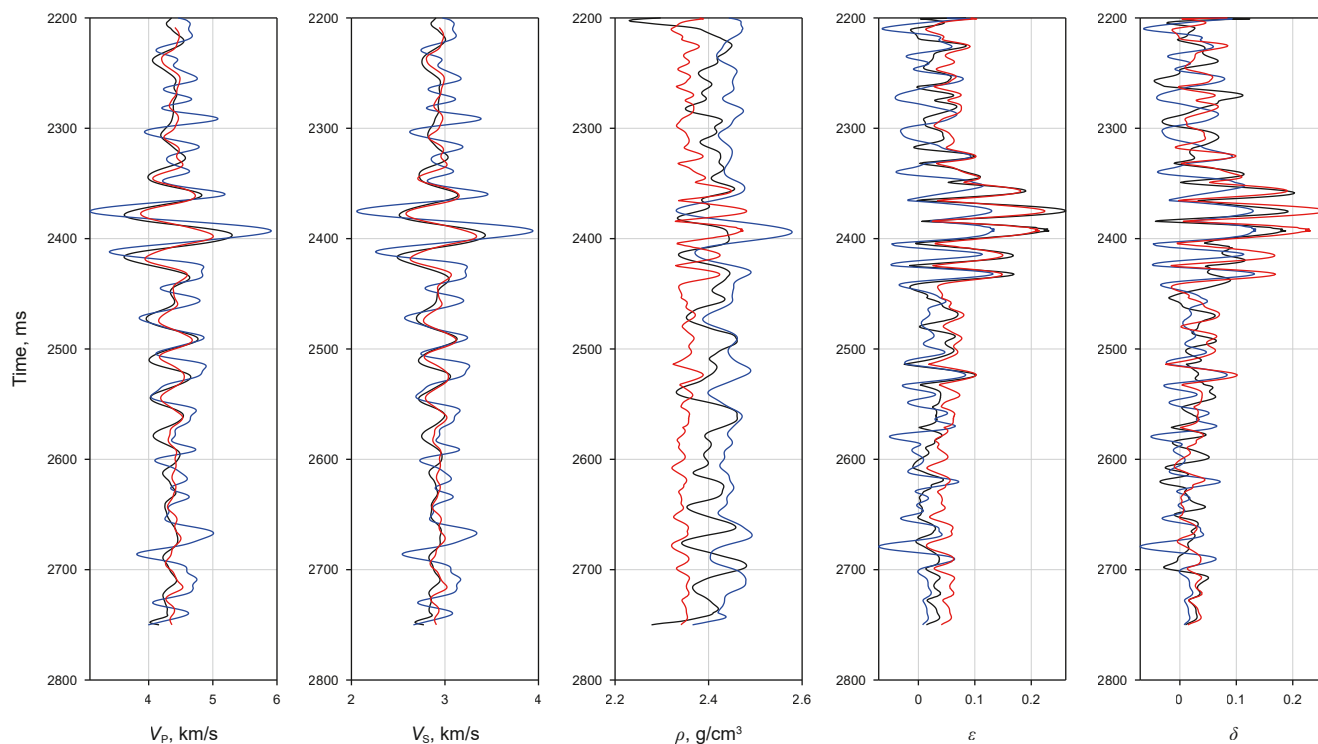


Fig. 17. Inversion results using different methods at well location. The black solid line denotes the well-log data, the red line corresponds to the proposed method, and the blue line indicates the traditional method.

regions in highly nonlinear inversion problems. The Bayesian prior constraints help improve stability, but inaccurate prior information may introduce bias. Future work will focus on incorporating multi-component seismic data, expanding incidence-angle coverage, and improving optimization strategies to enhance inversion robustness.

In summary, the proposed PP-PS joint inversion method, integrating the exact formulations, Bayesian inference, and multi-objective optimization, has been validated as an effective tool for parameter estimation in complex reservoirs. With broader application across diverse reservoirs and integration high-performance computing with multi-source data fusion, the method holds promise as a valuable tool for unconventional oil and gas exploration and development.

6. Conclusion

This paper proposes a multi-objective PP-PS joint inversion method based on the exact reflection coefficient equations for VTI media. By integrating SPEA-II, the method overcomes the strong weight dependence and non-uniqueness problems of conventional weighted joint inversion. Synthetic single-well tests demonstrate that the method reliably recovers P- and S-wave velocities, density, and anisotropy parameters under noise-free and medium SNR conditions, where PS-wave information markedly enhancing accuracy. 2D Marmousi-VTI modeling further confirms the strength of method in capturing lateral continuity and structure details. Application to a shale gas field demonstrates that proposed method surpasses approximate equation-based

inversion in accuracy, delivering reliable anisotropy estimates that support reservoir evaluation, fracturing design, and horizontal well planning. In summary, the proposed joint inversion method integrates theoretical soundness with engineering practicality, demonstrating strong potential for application in complex reservoir settings. Future work may integrate multi-component seismic data, high-performance computing, and data-driven prior modeling to broaden the applicability of the method in unconventional oil and gas exploration and development.

CRedit authorship contribution statement

Ji-Wei Cheng: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Ke-Xin Zhang:** Software, Funding acquisition, Data curation. **Chao-Qian Zhang:** Software, Project administration. **Jia-Ming Li:** Software, Resources. **Zhong-Zhen Ma:** Writing – original draft, Software. **Xiang-Lin Xu:** Writing – original draft. **Yu-Xing Chen:** Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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APPENDIX A

$$\eta_{\alpha} = V_{S0}^2 q_{\alpha}^2 + (2\varepsilon + 1) V_{P0}^2 p^2 + V_{P0}^2 q_{\alpha}^2 + V_{S0}^2 p^2 - 2 \quad (A1)$$

$$\eta_{\beta} = V_{S0}^2 q_{\beta}^2 + (2\varepsilon + 1) V_{P0}^2 p^2 + V_{P0}^2 q_{\beta}^2 + V_{S0}^2 p^2 - 2 \quad (A2)$$

$$q_{\alpha} = \frac{1}{\sqrt{2}} \sqrt{K_1 - \sqrt{K_1^2 - 4K_2K_3}} \quad (A3)$$

$$q_{\beta} = \frac{1}{\sqrt{2}} \sqrt{K_1 + \sqrt{K_1^2 - 4K_2K_3}} \quad (A4)$$

$$K_1 = \frac{1}{A_{33}} + \frac{1}{A_{55}} - \left(\frac{A_{11}}{A_{55}} + \frac{A_{55}}{A_{33}} - \frac{(A_{13} + A_{55})^2}{A_{33}A_{55}} \right) p^2 \quad (A5)$$

$$K_2 = \frac{A_{11}}{A_{33}} p^2 - \frac{1}{A_{33}} \quad (A6)$$

$$K_3 = p^2 - \frac{1}{A_{55}} \quad (A7)$$

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