

## Original Paper

# The energy-concentrated transform for gas-bearing reservoir identification

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## ABSTRACT

Identifying gas-bearing reservoirs remains challenging in seismic exploration because the time-frequency (TF) response of seismic signals is often complex and difficult to interpret. Newly developed TF post-processing methods enhance the energy concentration of traditional TF spectra, yielding a peak-shaped TF representation. However, such a representation may overlook the inherent time-width and bandwidth of seismic wavelets, limiting a comprehensive characterization of abnormal TF responses. To address this issue, this paper proposes an energy-concentrated transform (ECT) within the Wigner-Ville distribution framework. The ECT uses the second-order central moments of the local energy distribution to concentrate the diffused energy into the relevant TF interval, rather than reassigning the energy to the centroid as existing TF post-processing methods do. This approach not only concentrates the diffuse TF energy but also preserves essential time-width and bandwidth features important for geological interpretation. Its theoretical foundation is validated, demonstrating consistency with the Morlet wavelet spectrum. Numerical tests show that the ECT exhibits low sensitivity to window length and has strong noise robustness. Furthermore, field seismic data indicate that the ECT provides a clearer TF distribution, enabling the reliable identification of gas-bearing reservoirs.

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## 1. Introduction

Reliable identification of gas-bearing reservoirs remains a key goal and persistent challenge in seismic exploration. Extensive studies show that, as seismic waves propagate through fluid-saturated porous rocks, intrinsic absorption causes high-frequency components to attenuate more rapidly than low-frequency ones. This high-frequency attenuation is widely regarded as an important indicator of fluid-related anomalies in subsurface formations (Winkler and Nur, 1982; Wu et al., 2023; Xue et al., 2014). Clearly characterizing these subtle spectral signatures is therefore crucial for improving reservoir prediction reliability and informing exploration decisions.

In this context, time-frequency (TF) analysis serves as an effective tool for extracting these spectral characteristics from seismic data (Chen et al., 2026; Tary et al., 2014). Classical linear TF analysis methods, including the short-time Fourier transform (STFT) and the continuous wavelet transform, have been widely used owing to their stability and interpretability. However, these methods exhibit a trade-off between time and frequency localization, resulting in spectral smearing.

To overcome this limitation, a class of TF post-processing methods has been developed by synchrosqueezing or synchroextracting TF coefficients based on estimated ridges. The core idea of TF post-processing originates from the reassignment method (Auger and Flandrin, 1995), which reassigns TF coefficients toward their local centers of gravity. Although reassignment significantly sharpens the TF representation (TFR), it does not preserve phase information, thereby limiting signal reconstruction and its applicability in seismic analysis.

A significant development is the introduction of the synchrosqueezed wavelet transform by Daubechies et al. (2011). The synchrosqueezing transform (SST) sharpens the TFR by

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squeezing TF coefficients along the estimated instantaneous frequency (IF) ridges while preserving invertibility, thereby enabling signal reconstruction. This property has attracted considerable attention and has motivated extensive subsequent studies and applications (Iatsenko et al., 2015; Yang and Ying, 2014). In contrast to the SST, Yu et al. (2017) proposed the synchroextracting transform (SET), which extracts TF coefficients using its estimator, achieving precise localization for signals with linear frequency modulation while preserving the original TF energy relationship. To further improve TF concentration, various extensions of the SST and SET have been proposed. These include multi-SST based on iterative refinement (Yu et al., 2019), high-order SST that incorporates Taylor expansions to characterize nonlinear frequency modulation (Chen et al., 2021; Pham and Meignen, 2017), as well as other related methods (Li et al., 2020; Mao, 2022; Zhu et al., 2025b).

Subsequent studies show that when TF energy diffusion primarily occurs along the time direction, post-processing methods that distribute energy along the frequency axis are less effective. To address this issue, group delay (GD) estimators have been introduced to reassign TF coefficients along the time direction. Representative methods include the time-reassigned synchrosqueezing transform (He et al., 2019; Tu et al., 2020) and the transient-extracting transform (TET) (Yu, 2020), as well as their extensions (Song et al., 2023; Zhu et al., 2025a).

Despite their effectiveness, the above post-processing methods share a common limitation: they concentrate TF energy strictly along the estimated ridges. Such energy concentration strategies are suitable for harmonic or pulsed signals, but may be less appropriate for seismic signals with inherent time-width and bandwidth. Consequently, although these methods enhance time or frequency localization, their highly concentrated TFRs make it difficult to observe spectral features other than IF or GD and may not fully preserve the physical characteristics of seismic signals. Therefore, it is crucial to develop a new TF analysis method that balances high energy concentration with the preservation of intrinsic spectral characteristics, rather than concentrating energy only on estimated ridges (Chen et al., 2026).

The recently proposed energy-concentrated transform (ECT) (Wang et al., 2025) offers a promising alternative to address this issue. Although ECT shows potential, several aspects of the original equations remain insufficiently clarified. In particular, the real part introduced in the construction of the ECT operator is adopted empirically, and its theoretical interpretation is not explicitly discussed. Furthermore, there is no in-depth discussion beyond the analysis of white noise.

Motivated by the limitations above, this study revisits ECT within the Wigner-Ville distribution (WVD) framework and provides a rigorous theoretical interpretation of its operator construction. Within this framework, both the real and imaginary parts of the ECT operator arise naturally from the analytical expression of the local energy distribution, rather than being introduced empirically. Based on this theory, a previously implicit assumption in ECT analysis is validated: that the TFR of ECT does not introduce additional TF broadening beyond the signal's inherent time-width and bandwidth. It also demonstrates that the resulting energy-concentrated spectrum is consistent with the corresponding Morlet wavelet in terms of time-width and bandwidth. Therefore, the TFR of ECT can be interpreted as physically meaningful rather than merely artificially enhanced.

## 2. Methods

### 2.1. Review of synchro-extracting methods

For a seismic signal  $x(t)$ , its STFT with a window function  $g(t)$  is defined as:

$$S^g(t, \omega) = \int_{-\infty}^{+\infty} x(\tau) \overline{g(t - \tau)} e^{-j\omega\tau} d\tau \quad (1)$$

where  $j = \sqrt{-1}$ ,  $\tau$  denotes the time variable, and  $\bar{g}$  is the complex conjugate of  $g$ . Multiplying Eq. (1) by a phase shift factor  $e^{j\omega t}$  to obtain the modified STFT form

$$S_e^g(t, \omega) = e^{j\omega t} S^g(t, \omega) \quad (2)$$

According to Parseval's theorem, the STFT is expressed in the frequency domain as:

$$S^g(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\xi) \overline{G(\omega - \xi)} e^{j(\xi - \omega)t} d\xi \quad (3)$$

where  $\xi$ ,  $X$ , and  $G$  denote the frequency variable, the FT result of signal  $x$  and the FT result of window function  $g$ .

The spectrogram  $P(t, \omega)$  is defined as

$$P(t, \omega) = |S^g(t, \omega)|^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \quad (4)$$

where  $W_g(t, \omega)$  and  $W_x(t, \omega)$  are the WVD results of the window function  $g$  and the signal  $x$ , respectively.  $W_x(t, \omega)$  and  $W_g(t, \omega)$  are defined as follows:

$$W_x(t, \omega) = \int_{-\infty}^{+\infty} x\left(t + \frac{\tau}{2}\right) \overline{x\left(t - \frac{\tau}{2}\right)} e^{-j\omega\tau} d\tau \quad (5)$$

$$W_g(t, \omega) = \int_{-\infty}^{+\infty} g\left(t + \frac{\tau}{2}\right) \overline{g\left(t - \frac{\tau}{2}\right)} e^{-j\omega\tau} d\tau \quad (6)$$

The time and frequency centroids are defined as follows (Auger and Flandrin, 1995):

$$t_c(t, \omega) = t - \frac{P_t(t, \omega)}{P(t, \omega)} \quad (7)$$

$$\omega_c(t, \omega) = \omega - \frac{P_\omega(t, \omega)}{P(t, \omega)} \quad (8)$$

with

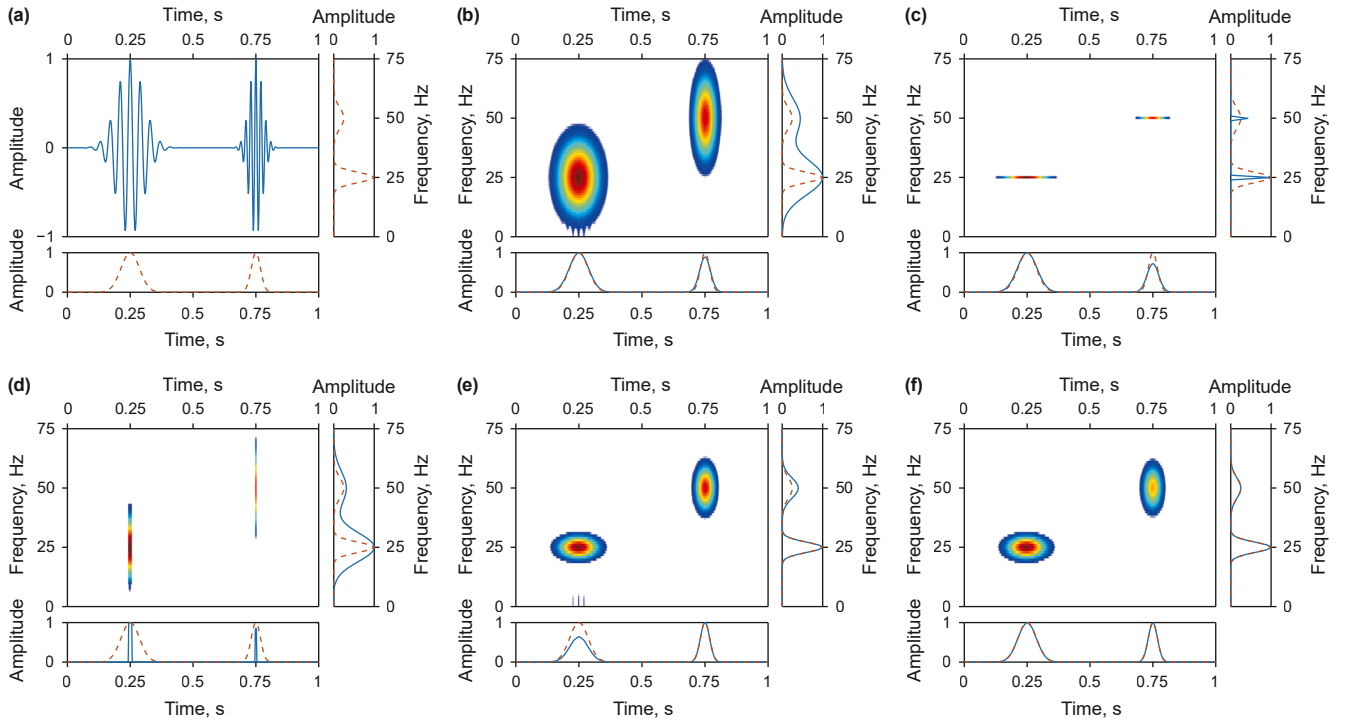
$$P_t(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \quad (9)$$

$$P_\omega(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \quad (10)$$

Subsequently, SET and TET, which are widely used, are defined as:

$$Se(t, \omega) = S_e^g(t, \omega) \delta(\omega - \omega_c(t, \omega))$$

$$= S_e^g(t, \omega) \lim_{\lambda \rightarrow 0^+} \frac{1}{\lambda} h\left(\frac{\omega - \omega_c(t, \omega)}{\lambda}\right)$$



**Fig. 1.** (a) The waveform. TFRs of (b) STFT, (c) SET, (d) TET, (e) ESM, (f) ECT.

$$= S_e^g(t, \omega) \lim_{\lambda \rightarrow 0^+} \frac{1}{\lambda \sqrt{2\pi}} e^{-\frac{(\omega - \omega_c(t, \omega))^2}{2\lambda^2}} \quad (11)$$

$$Te(t, \omega) = S^g(t, \omega) \delta(t - t_c(t, \omega))$$

$$= S^g(t, \omega) \lim_{\lambda \rightarrow 0^+} \frac{1}{\lambda} h\left(\frac{t - t_c(t, \omega)}{\lambda}\right) \quad (12)$$

$$= S^g(t, \omega) \lim_{\lambda \rightarrow 0^+} \frac{1}{\lambda \sqrt{2\pi}} e^{-\frac{(t - t_c(t, \omega))^2}{2\lambda^2}}$$

where  $h(x)$  is an  $L_1$ -normalized positive function and  $\lambda$  is a parameter controlling the accuracy.

Here, we use Morlet wavelets with dominant frequencies of 25 Hz and 50 Hz to evaluate the performance of the STFT, SET, and TET. Fig. 1(a) shows the signal waveform. As shown in Fig. 1(b), the TFR of the STFT exhibits energy spreading, which limits the accurate characterization of seismic signals. Fig. 1(c) and (d) show the TFRs of SET and TET. SET uses a frequency extractor that enhances

frequency localization, while TET uses a time extractor that improves time localization. However, the highly focused TFRs lose bandwidth information in SET and time-width information in TET, which hinders a comprehensive characterization of the seismic signal's TF features.

## 2.2. The proposed ECT

In SET and TET, due to the selectivity of the  $\delta(\cdot)$  function, the spectra retain energy only at the estimated centers  $t_c$  or  $\omega_c$ . As a result, this behavior explains the loss of time-width and bandwidth information in the TFRs of SET and TET. To address this issue, we propose replacing the  $\lambda \rightarrow 0$  with a finite parameter that can better preserve the TF characteristics of the signal. Motivated by the observation that the centroid corresponds to the first-order moment of the local kernel  $W_g(\tau, \xi)W_x(t - \tau, \omega - \xi)$ , we further introduce its second-order central moments to concentrate TF energy, as defined by:

$$D_t(t, \omega) = \frac{\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\tau - (t - t_c(t, \omega)))^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi}{P(t, \omega)}$$

$$= \frac{P_{tt}(t, \omega)}{P(t, \omega)} - \left( \frac{P_t(t, \omega)}{P(t, \omega)} \right)^2 \quad (13)$$

$$D_\omega(t, \omega) = \frac{\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\xi - (\omega - \omega_c(t, \omega)))^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi}{P(t, \omega)}$$

$$= \frac{P_{\omega\omega}(t, \omega)}{P(t, \omega)} - \left( \frac{P_\omega(t, \omega)}{P(t, \omega)} \right)^2 \quad (14)$$

with

$$P_{tt}(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \quad (15)$$

$$P_{\omega\omega}(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \quad (16)$$

where  $D_t(t, \omega)$  and  $D_\omega(t, \omega)$  are the second-order central moments of the local kernel  $W_g(\tau, \xi) W_x(t - \tau, \omega - \xi)$  and characterize the degree of signal diffusion along the time and frequency directions, calculated as:

$$D_t(t, \omega) = \frac{1}{2} \Re \left\{ \frac{S^{tg}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{tg}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\} \quad (17)$$

$$D_\omega(t, \omega) = \frac{1}{2} \Re \left\{ \left( \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right)^2 - \frac{S^{g''}(t, \omega)}{S^g(t, \omega)} \right\} \quad (18)$$

where  $S^{tg}$  means the STFT under window function  $tg(t)$ .  $S^{g'}$  and  $S^{g''}$  represent the STFT result under  $g'(t)$  and  $g''(t)$ .

Based on Eq. (13) and Eq. (14), we can construct the energy suppression operator (ESO) of WVD that is similar to Eq. (11) and Eq. (12):

$$ESO_{wvd}(t, \omega) = e^{-\left( \frac{(t-t_c(t, \omega))^2}{2D_t(t, \omega)} + \frac{(\omega-\omega_c(t, \omega))^2}{2D_\omega(t, \omega)} \right)} \quad (19)$$

Note that  $W_x$  is a quadratic representation of the signal. Therefore, ESO( $t, \omega$ ) for the STFT spectrum  $S^g(t, \omega)$  is defined as (see Appendix A for the derivation):

$$ESO(t, \omega) = \sqrt{ESO_{wvd}(t, \omega)} = e^{-\left( \frac{(t-t_c(t, \omega))^2}{4D_t(t, \omega)} + \frac{(\omega-\omega_c(t, \omega))^2}{4D_\omega(t, \omega)} \right)} \quad (20)$$

Therefore, the energy suppression method (ESM) is defined as:

$$ESM(t, \omega) = S^g(t, \omega) ESO(t, \omega) \quad (21)$$

As shown in Fig. 1(e), ESM suppresses the diffuse spectral energy to a range that matches the signal's analytical time-width and bandwidth. This is achieved by extracting TF coefficients within an adaptive neighborhood whose extent is determined by the second-order central moments of each TF point (see Appendix B for the derivation). However, since the operator extracts only some of the TF coefficients, the amplitudes of the ESM's marginal distributions are inconsistent with analytical values.

To ensure energy preservation of the TFR, we construct the following energy recovery operator (ERO), which is defined as:

$$ERO(t, \omega) = \sqrt{\frac{|S^g(t, \omega)|^2}{|ESM(t, \omega)|^2 * W_g(t, \omega)}} * W_g(-t, -\omega) \quad (22)$$

where  $*$  means convolution.  $ERO(t, \omega)$  is defined as a TF-dependent compensation factor that recovers the spectral energy ratio, ensuring that the spectral energy of the obtained spectrum is preserved relative to the original STFT spectrum.

Therefore, ECT can be defined as:

$$ECT(t, \omega) = ESM(t, \omega) ERO(t, \omega) \quad (23)$$

As shown in Fig. 1(f), the proposed ECT retains the energy concentration achieved by ESM while correcting inconsistencies in

marginal amplitudes, resulting in time and frequency marginal distributions that are closely aligned with the analytical ones.

### 3. Numerical analysis

#### 3.1. Window length sensitivity analysis

The selection of the window length is a critical factor that influences the results of windowed TF analysis methods. In this study, a Gaussian window  $g(t) = e^{-\frac{t^2}{2\sigma_1^2}}$  is adopted, where the window length is controlled by the parameter  $\sigma_1$ . To evaluate the impact of this parameter on the proposed ECT, we construct a synthetic signal consisting of a Morlet wavelet with a dominant frequency of 35 Hz, a broadband Gaussian wavelet, and a Ricker wavelet centered at 45 Hz. The signal is then analyzed using three window lengths ( $\sigma_1 = 0.03, 0.05, \text{ and } 0.07$ ) to examine how each method responds to parameter variations.

Fig. 2 presents the TFRs of STFT, SET, TET, and ECT. The first three columns of Fig. 2 show that the STFT, SET, and TET all vary with window length, where larger window lengths improve frequency localization at the expense of time localization. By contrast, the proposed ECT produces a TFR with consistently concentrated energy and preserved features across all parameter values, suggesting that it is less sensitive to window length, as shown in the fourth column.

Fig. 3 further compares the time and frequency marginal distributions obtained under the conditions described above. For the STFT, increasing the window length enhances frequency localization and brings the frequency marginal distribution closer to the analytical reference distribution. However, this improvement is achieved at the expense of time localization, as indicated by the purple curves. The SET yields strong frequency localization but limited bandwidth coverage, as shown by the green dashed curves. The TET, which emphasizes time localization, produces the time marginal distribution concentrated at a few discrete points, as shown by the yellow dashed curves. In comparison, the ECT achieves both time and frequency marginal distributions that remain closely aligned with the analytical reference distributions for all parameter values, as shown by the red dashed curves. These results indicate that ECT maintains the inherent TF features of seismic signals while being less affected by window length.

#### 3.2. Noise robustness and TF fidelity analysis

We generate a synthetic seismic signal (Fig. 4(a)) by convolving a reflectivity series with a Ricker wavelet to evaluate the robustness and interpretability of the proposed ECT under different conditions. The corresponding TFRs are displayed in Fig. 4(b)–(e). The STFT result in Fig. 4(b) exhibits limited TF localization, with neighboring reflection events between 0.3 s and 0.6 s showing notable energy diffusion. Fig. 4(c) shows that SET improves frequency localization but primarily captures IF ridges, thereby degrading other spectral features. The TET can identify seismic reflection events, but noticeable distortions occur when events are closely spaced, as shown in Fig. 4(d). Compared to other methods, the proposed ECT (shown in Fig. 4(e)) produces a more concentrated TFR while preserving the intrinsic time-width and bandwidth characteristics of the seismic signal.

To further examine the computational efficiency of different methods, we evaluate their runtime using the same synthetic seismic signal and implementation environment. To reduce the influence of randomness in a single execution, each method is executed 20 times, and the average runtime is taken. As shown in Table 1, STFT has the lowest computational cost. The runtimes of

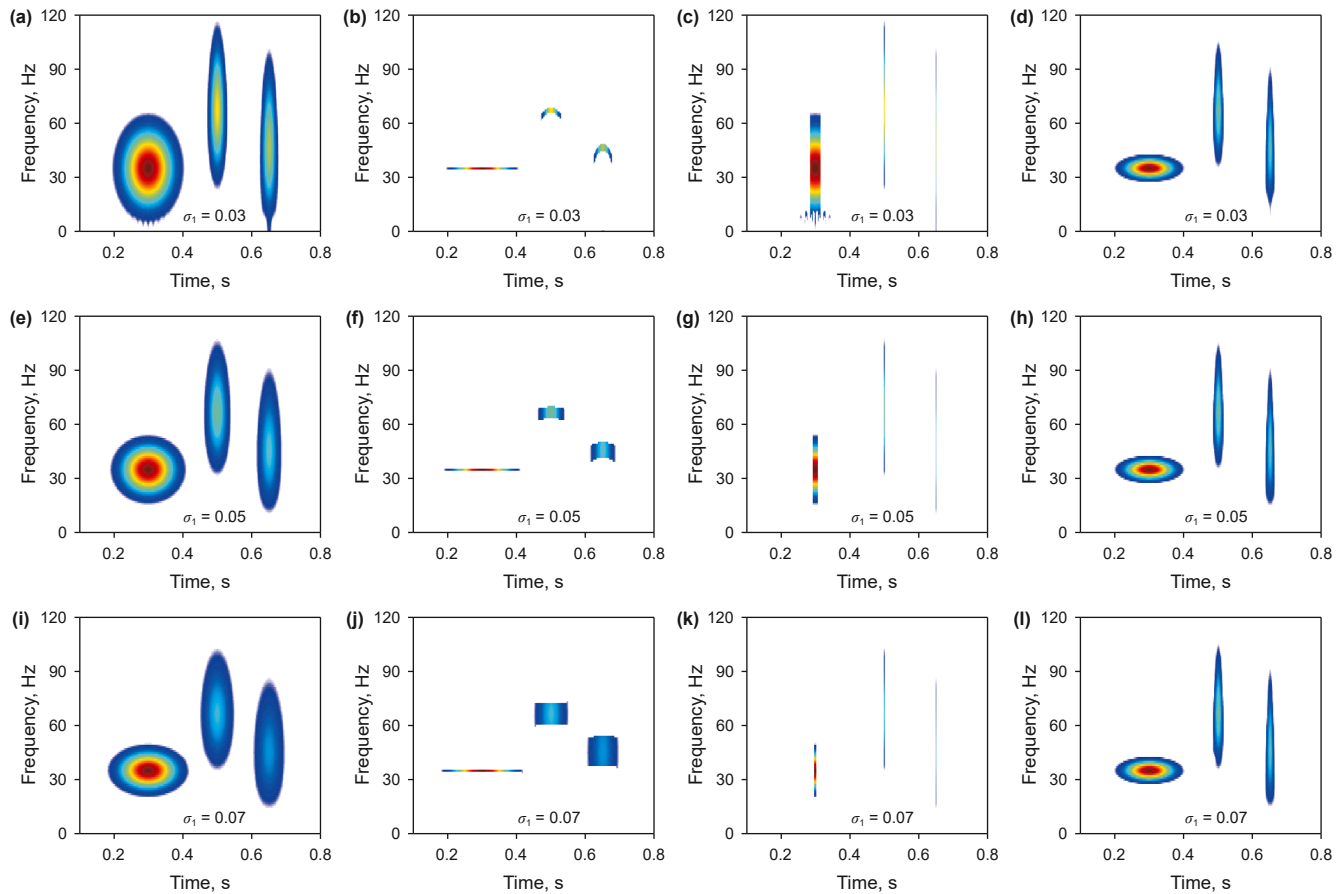


Fig. 2. TFRs of STFT (in the first column), SET (in the second column), TET (in the third column), and ECT (in the fourth column).

SET, TET, and ECT are comparable and remain within the same order of magnitude, with ECT exhibiting only a slight increase. Overall, the computational cost of ECT is moderate and acceptable for practical seismic signal analysis.

To assess performance under noisy conditions, noise derived from field seismic data is added to the synthetic signal, as shown in Fig. 5(a). The corresponding TFRs in Fig. 5(c) and (d) show that SET and TET are strongly affected by noise, making it difficult to distinguish signal from noise. By comparison, the STFT and ECT results exhibit lower noise sensitivity, with the ECT still providing a more energy-concentrated distribution, as shown in Fig. 5(b) and (e).

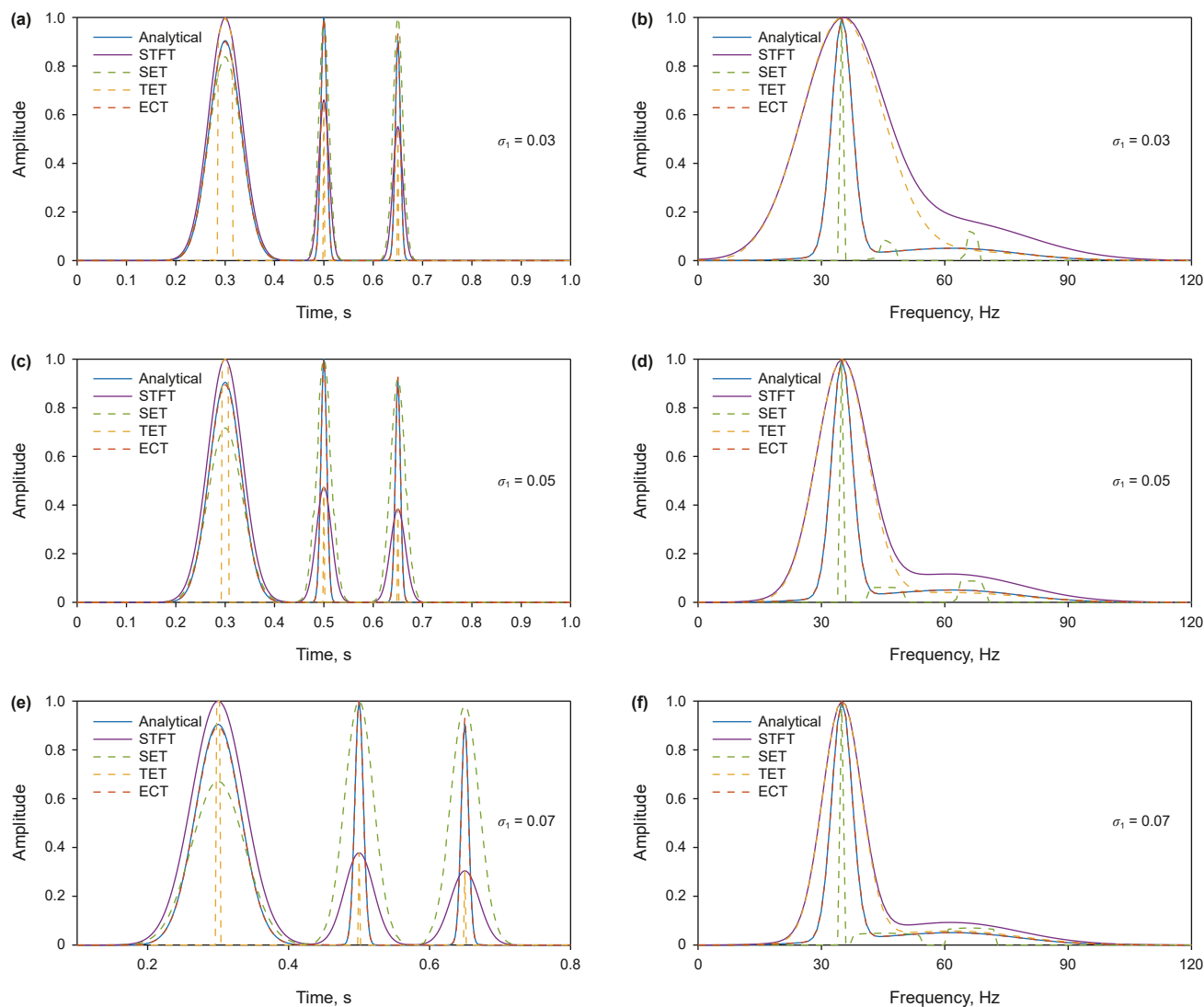
In order to quantitatively compare the performance of different methods, three evaluation metrics are adopted to jointly characterize noise robustness and TF fidelity. The normalized mean square error (NMSE) is computed between noisy and noise-free TFRs to quantify noise robustness. In addition, correlation coefficients between the time and frequency marginal distributions derived from each method and those derived from the ideal TFR are used to evaluate TF fidelity, reflecting the degree of alignment with the analytical reference distribution. Table 2 summarizes the NMSE and correlation coefficients, with each value representing the average over 50 independent realizations. The STFT yields the lowest NMSE, while SET, TET, and ECT exhibit higher NMSE values owing to the additional differential operators introduced within the STFT framework. Nevertheless, the NMSE of ECT remains only slightly higher than that of STFT and is substantially lower than that of SET and TET, indicating noise robustness comparable to

STFT. Moreover, ECT consistently achieves the highest correlation coefficients in both time and frequency marginal distributions under all tested conditions, indicating that its TFR remains closest to the analytical reference distribution. Overall, the proposed ECT demonstrates stable performance under both noise-free and field noise conditions.

#### 4. Field data example

In this section, field seismic data are used to evaluate the applicability of the proposed ECT for reservoir identification. Fig. 6(a) shows the seismic data comprising 1079 traces, with a sampling interval of 2 ms. Previous geological and production information confirms that the interval bounded by the black dashed lines corresponds to the target tight sandstone gas-bearing reservoir. Of the three wells shown in the section, wells A and C are gas wells, whereas well B is a dry well. To determine the frequency band for analysis, amplitude spectra from traces near the wells are examined, as shown in Fig. 6(b). The results reveal the primary spectral energy range of the reservoir response, providing a basis for subsequent TF analysis and frequency section comparisons.

We then conduct a systematic comparison of the 3D time-space-frequency representations generated by STFT, SET, TET, and ECT. As shown in Fig. 7(a), the limited frequency localization of the STFT results in blurred boundaries in the channel reservoir, reducing the distinguishability of geological features across different frequency sections. The SET result shown in Fig. 7(b)



**Fig. 3.** Time (in the first column) and frequency (in the second column) marginal distributions.

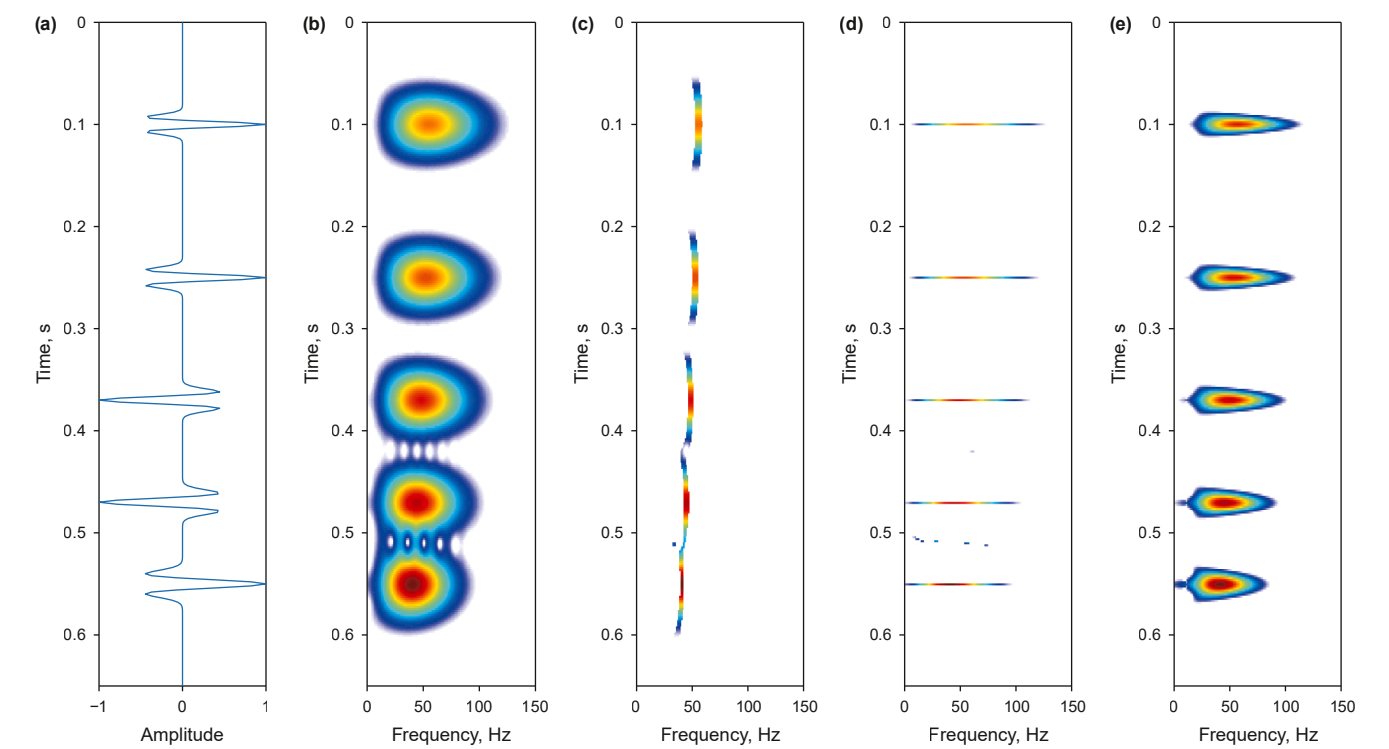
exhibits improved frequency localization, while retaining information around IF. As a consequence, the resulting TFR lacks explicit bandwidth information, which limits its ability to represent the full spectral extent of geological bodies. The TET uses a GD operator that can outline some seismic reflection interfaces and layers. However, the similarity among frequency sections indicates that TET's frequency localization is low, making the differences in frequency responses between reservoirs less distinguishable, as shown in Fig. 7(c). By contrast, the proposed ECT achieves a highly energy-concentrated TFR while preserving both time-width and bandwidth characteristics of the seismic signal, enabling a more complete characterization of the spectral responses of different reservoirs, as shown in Fig. 7(d).

To emphasize anomalies related to gas-bearing reservoirs, two representative frequency sections at the dominant frequency (30 Hz) and the high frequency (46 Hz) are compared in Fig. 8. The first column shows the 30 Hz sections, whereas the second column shows the 46 Hz sections. The vertical lines indicate the locations

of wells A, B, and C, and the dashed horizons mark the interpreted reservoir interval.

When comparing each 30 Hz section with its corresponding 46 Hz section in Fig. 8, STFT, TET, and ECT consistently show that strong amplitudes near wells A and C at 30 Hz decay significantly at 46 Hz, whereas no such attenuation is observed near well B. This pattern aligns well with the expected attenuation of seismic energy as waves propagate through gas-bearing reservoirs, suggesting that these three methods can identify attenuation anomalies associated with gas-bearing reservoirs. In contrast, the SET results show a markedly different behavior. Although the 30 Hz section (Fig. 8(c)) still shows an identifiable energy distribution along the reservoir interval, the channel response is not evident in the 46 Hz section (within the stratigraphic boundaries in Fig. 8(d)), making it difficult to determine whether the observed attenuation is related to the gas-bearing reservoir.

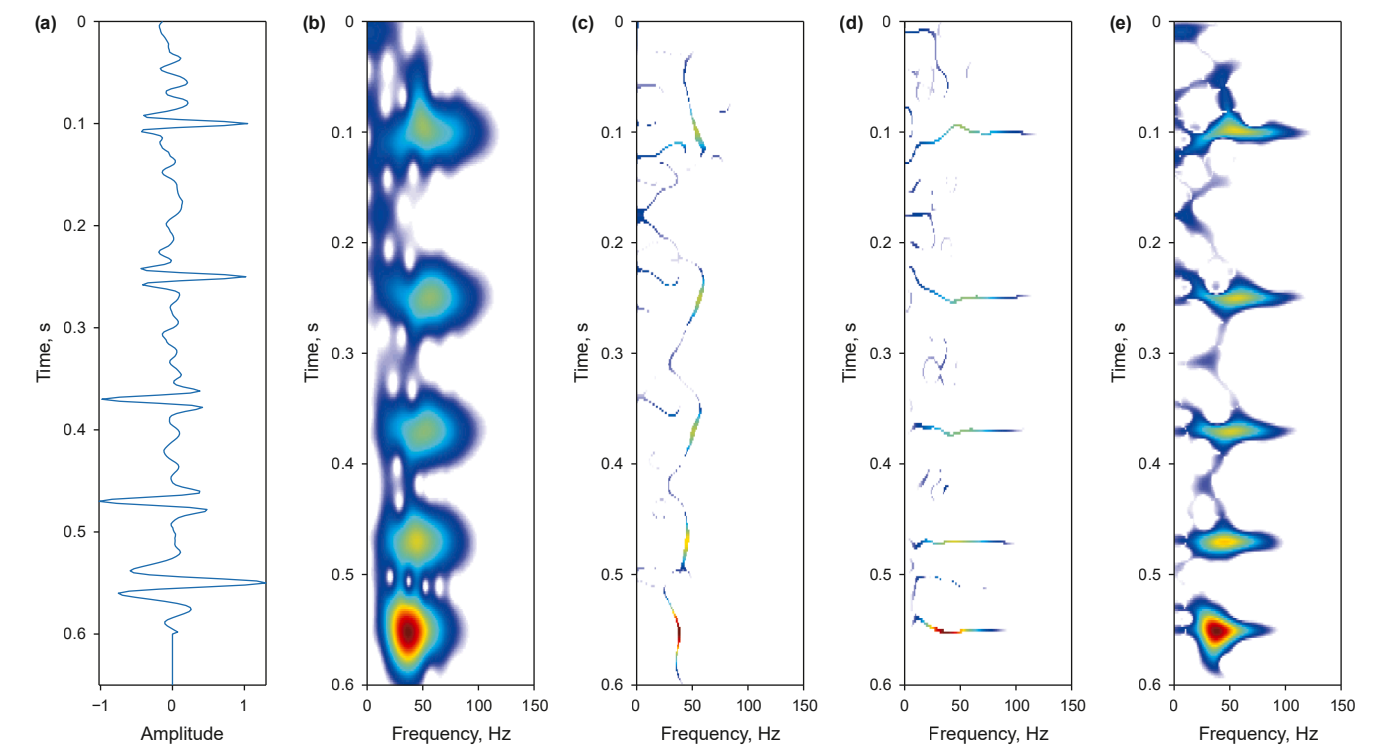
Although STFT, TET, and ECT all reveal attenuation anomalies through the 30 Hz and 46 Hz sections, their delineation performance differs. Owing to its limited TF localization, the STFT results



**Fig. 4.** (a) The waveform of the seismic trace. TFRs of (b) STFT, (c) SET, (d) TET, (e) ECT.

**Table 1**  
Comparison of computation time.

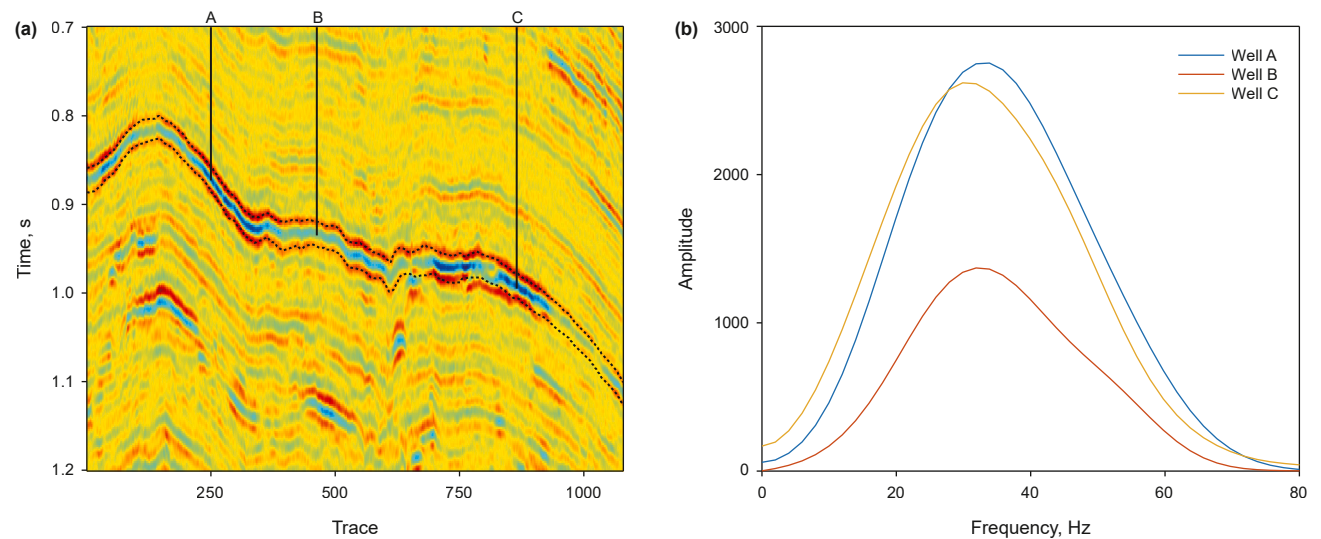
| Method              | STFT  | SET   | TET   | ECT   |
|---------------------|-------|-------|-------|-------|
| Computation time, s | 0.013 | 0.022 | 0.022 | 0.024 |



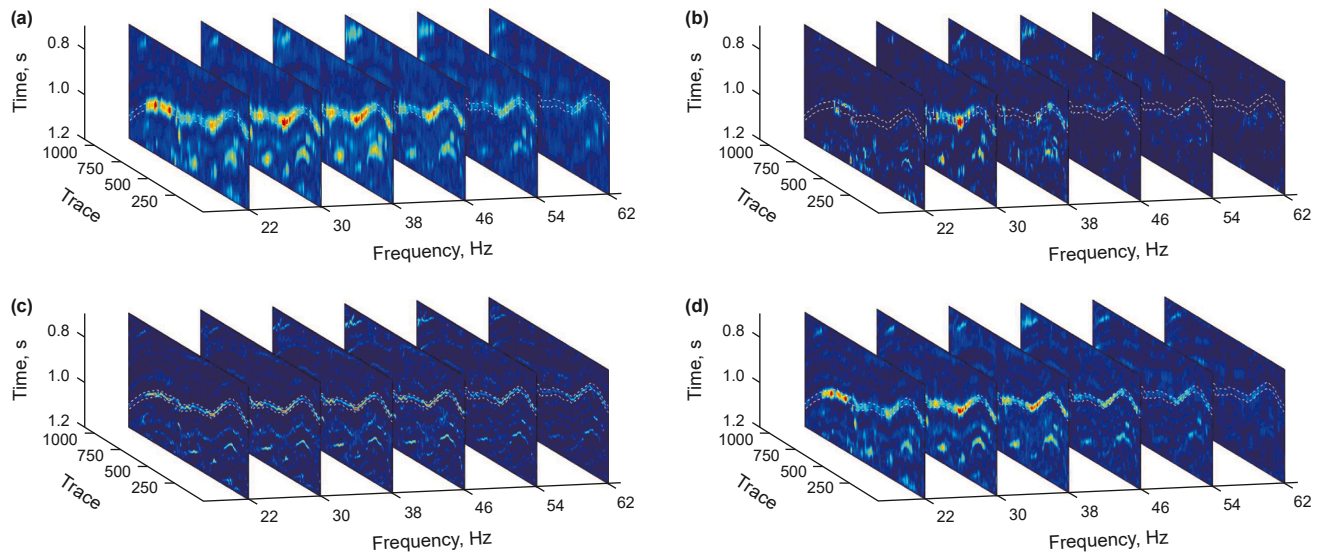
**Fig. 5.** (a) The waveform of the noisy seismic trace. TFRs of (b) STFT, (c) SET, (d) TET, (e) ECT.

**Table 2**  
Quantitative comparison of the four methods.

| Method | NMSE | Time marginal correlation coefficient | Frequency marginal correlation coefficient |
|--------|------|---------------------------------------|--------------------------------------------|
| STFT   | 0.27 | 0.89                                  | 0.90                                       |
| SET    | 1.56 | 0.79                                  | 0.57                                       |
| TET    | 1.32 | 0.62                                  | 0.86                                       |
| ECT    | 0.35 | 0.99                                  | 0.91                                       |



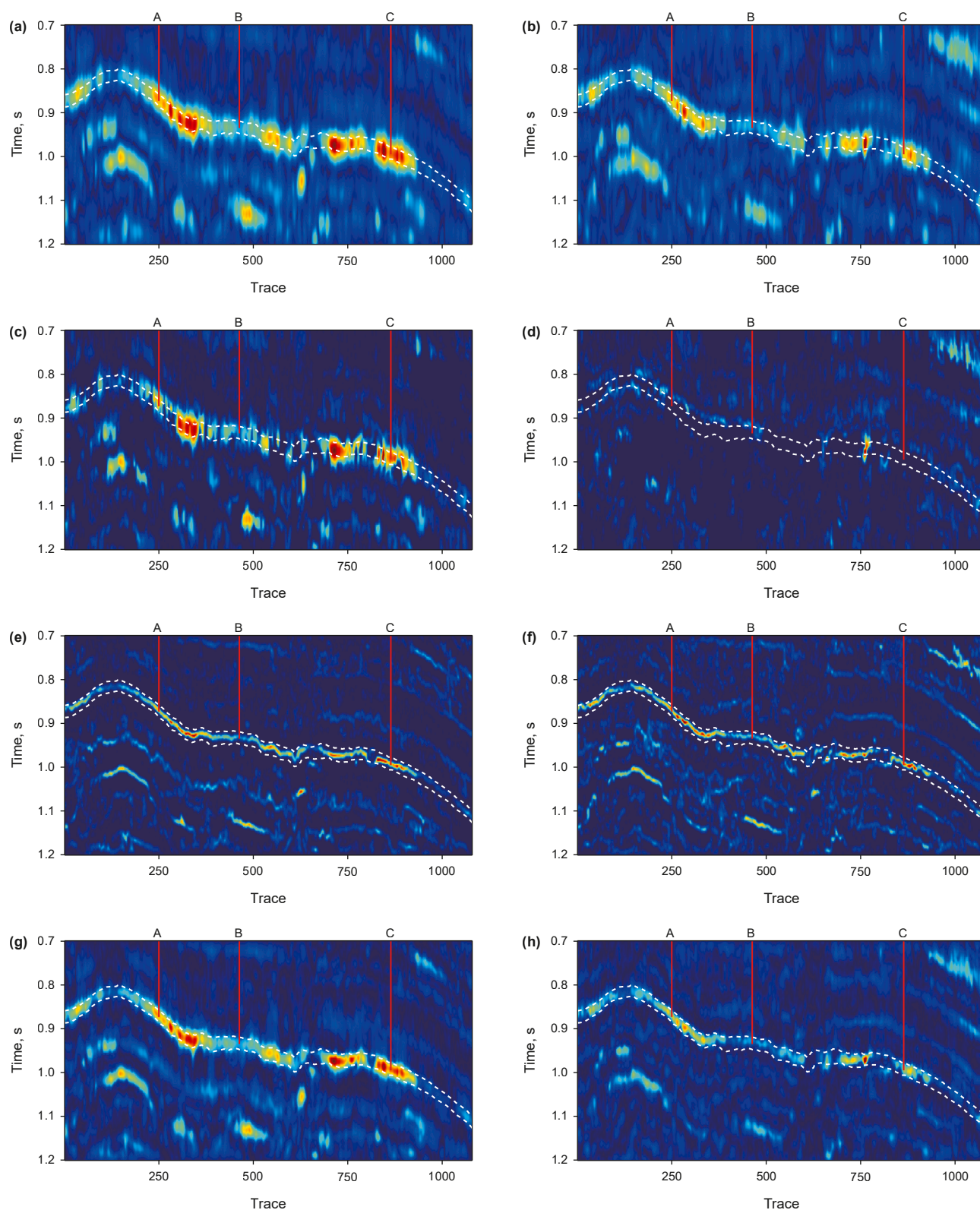
**Fig. 6.** (a) Field seismic data, (b) amplitude spectra of wells A, B, and C at the reservoir interval.



**Fig. 7.** The 3D time-space-frequency representations generated by (a) STFT, (b) SET, (c) TET, and (d) ECT.

(Fig. 8(a) and (b)) exhibit noticeable spectral smearing, thereby blurring the reservoir boundaries. The TET offers clearer boundary delineation, but its low frequency localization results in a less pronounced difference in spectral energy between the two

sections. Additionally, the high time localization of TET paradoxically masks the reservoir thickness, making the identified reservoir appear thinner than its known boundaries in Fig. 8(e) and (f). By comparison, the proposed ECT provides an energy-



**Fig. 8.** 30 Hz (in the first column) and 46 Hz (in the second column) sections generated by (a–b) STFT, (c–d) SET, (e–f) TET, and (g–h) ECT, respectively.

concentrated TFR in Fig. 8(g) and (h), in which both the attenuation contrast between 30 and 46 Hz and the reservoir boundaries are more clearly and consistently depicted. Overall, the proposed ECT provides a more precise and reliable depiction of gas-bearing reservoirs than other methods.

## 5. Discussion

A TF analysis method, ECT, is rederived and applied to gas-bearing reservoir identification. ECT suppresses energy diffusion in the STFT by incorporating second-order central moments, while preserving energy.

ECT exhibits weak dependence on window length. In the STFT, the window length directly influences time and frequency localization. For SET and TET, the TFRs depend not only on the STFT spectrum used for reassignment but also on the associated estimators, thereby altering the TFRs. By contrast, the effective concentration range in ECT is determined by statistical properties rather than by window length. Within a reasonable range of window lengths, the TFR of ECT remains comparatively stable, and both time and frequency marginals agree well with the analytical expectations for individual wavelets.

ECT exhibits higher noise robustness. In contrast, SET and TET rely on single ridge operators, which are susceptible to noise and therefore yield higher NMSEs than the other two methods. By avoiding reliance on ridge estimation and instead proportionally suppressing diffuse TF energy within a local neighborhood, ECT reduces the impact of noise on the TFR, yielding an NMSE close to that of the STFT while maintaining improved TF localization.

From the perspective of identifying gas-bearing reservoirs, TF analysis requires not only obtaining frequency variations but also achieving high time localization. Under this interpretive objective, STFT's TF localization limits the detection of abnormal features. TET's emphasis on time concentration offers limited benefit when frequency responses are of interest and may obscure thickness-related differences. SET compresses the original bandwidth at the algorithmic level through frequency redistribution, thereby weakening the spectral characteristics associated with gas-bearing reservoirs. By confining dispersed TF energy to specific time-width and bandwidth intervals, ECT achieves higher TF localization while capturing signal frequency variations, thereby offering advantages for gas-bearing reservoir identification. However, because ECT does not aim to estimate IF or GD with high precision, it may be less suitable for applications where such instantaneous parameters are the primary objective. In these cases, SET or TET may be preferable, whereas ECT is better suited to interpretations that require a stable TFR and reliable characterization of spectral variations.

In terms of computational cost, SET, TET, and ECT are all extensions of the STFT framework and therefore require more computation than STFT. Because ECT introduces additional operator constraints based on the concepts of SET and TET, its computational cost is slightly higher than that of the latter two methods. Given the benefits of ECT, which include high localization and better preservation of TF features, this additional cost remains acceptable.

## 6. Conclusion

This paper theoretically rederives the ECT for seismic signal analysis within the WVD framework and applies it to gas-bearing reservoir identification. This transform incorporates both the TF centroid and second-order central moments, thereby obtaining a TFR with time-width and bandwidth information for seismic signals. By concentrating diffused TF energy into an interval consistent with the signal's intrinsic characteristics, ECT achieves higher TF localization relative to STFT and provides a more complete description of TF energy distributions than SET and TET. Results from synthetic examples and field seismic data demonstrate that ECT produces a stable and energy-concentrated TFR while providing clear TF features associated with gas-bearing reservoirs. These results indicate that ECT can serve as a practical tool for TF analysis of seismic signals and identification of gas-bearing reservoirs.

## CRediT authorship contribution statement

**Si-Yuan Wang:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Ying Hu:** Funding acquisition, Methodology, Writing – review & editing. **Xu-Ping Chen:** Writing – review & editing, Investigation. **Hui Chen:** Writing – review & editing, Funding acquisition.

## Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix

### A. Derivation of $ESO(t, \omega)$

To derive  $ESO(t, \omega)$ , we first calculate  $D_t(t, \omega)$  and  $D_\omega(t, \omega)$ . According to Eqs. (13)–(16),  $D_t(t, \omega)$  and  $D_\omega(t, \omega)$  are composed of  $P(t, \omega)$ ,  $P_t(t, \omega)$ ,  $P_\omega(t, \omega)$ ,  $P_{tt}(t, \omega)$ , and  $P_{\omega\omega}(t, \omega)$ . We derive the results of  $P_t(t, \omega)$ ,  $P_\omega(t, \omega)$ ,  $P_{tt}(t, \omega)$ , and  $P_{\omega\omega}(t, \omega)$  under the STFT framework one by one.

$$\begin{aligned}
P_t(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau g\left(\tau + \frac{u}{2}\right) \overline{g\left(\tau - \frac{u}{2}\right)} x\left(t - \tau + \frac{u}{2}\right) \overline{x\left(t - \tau - \frac{u}{2}\right)} e^{-j\omega u} du d\tau \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{t - \alpha + t - \beta}{2} g(t - \beta) \overline{g(t - \alpha)} x(\alpha) \overline{x(\beta)} e^{-j\omega(\alpha - \beta)} d\alpha d\beta \\
&= \frac{1}{2} S^{tg}(t, \omega) \overline{S^g(t, \omega)} + \frac{1}{2} S^g(t, \omega) \overline{S^{tg}(t, \omega)}
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
P_\omega(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \\
&= \frac{-j}{2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( g'\left(\tau + \frac{u}{2}\right) \overline{g\left(\tau - \frac{u}{2}\right)} - \overline{g'\left(\tau - \frac{u}{2}\right)} g\left(\tau + \frac{u}{2}\right) \right) x\left(t - \tau + \frac{u}{2}\right) \overline{x\left(t - \tau - \frac{u}{2}\right)} e^{-j\omega u} du d\tau \\
&= \frac{-j}{2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( g'(t - \beta) \overline{g(t - \alpha)} - \overline{g'(t - \alpha)} g(t - \beta) \right) x(\alpha) \overline{x(\beta)} e^{-j\omega(\alpha - \beta)} d\alpha d\beta \\
&= \frac{j}{2} S^{g'}(t, \omega) \overline{S^g(t, \omega)} - \frac{j}{2} S^g(t, \omega) \overline{S^{g'}(t, \omega)}
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
P_{tt}(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau^2 g\left(\tau + \frac{u}{2}\right) \overline{g\left(\tau - \frac{u}{2}\right)} x\left(t - \tau + \frac{u}{2}\right) \overline{x\left(t - \tau - \frac{u}{2}\right)} e^{-j\omega u} du d\tau \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( \frac{t - \alpha + t - \beta}{2} \right)^2 g(t - \beta) \overline{g(t - \alpha)} x(\alpha) \overline{x(\beta)} e^{-j\omega(\alpha - \beta)} d\alpha d\beta \\
&= \frac{1}{4} S^{t^2g}(t, \omega) \overline{S^g(t, \omega)} + \frac{1}{2} S^{tg}(t, \omega) \overline{S^{tg}(t, \omega)} + \frac{1}{4} S^g(t, \omega) \overline{S^{t^2g}(t, \omega)}
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
P_{\omega\omega}(t, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi^2 W_g(\tau, \xi) W_x(t - \tau, \omega - \xi) d\tau d\xi \\
&= -\frac{1}{4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( g''\left(\tau + \frac{u}{2}\right) \overline{g\left(\tau - \frac{u}{2}\right)} - 2g'\left(\tau + \frac{u}{2}\right) \overline{g'\left(\tau - \frac{u}{2}\right)} + g\left(\tau + \frac{u}{2}\right) \overline{g'\left(\tau - \frac{u}{2}\right)} \right) x\left(t - \tau + \frac{u}{2}\right) \overline{x\left(t - \tau - \frac{u}{2}\right)} e^{-j\omega u} du d\tau \\
&= -\frac{1}{4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( g''(t - \beta) \overline{g(t - \alpha)} - 2g'(t - \beta) \overline{g'(t - \alpha)} + g(t - \beta) \overline{g''(t - \alpha)} \right) x(\alpha) \overline{x(\beta)} e^{-j\omega(\alpha - \beta)} d\alpha d\beta \\
&= -\frac{1}{4} \left( S^g(t, \omega) \overline{S^{g''}(t, \omega)} - 2S^{g'}(t, \omega) \overline{S^{g'}(t, \omega)} + S^{g''}(t, \omega) \overline{S^g(t, \omega)} \right)
\end{aligned} \tag{A.4}$$

where  $\alpha = t - \tau + \frac{u}{2}$  and  $\beta = t - \tau - \frac{u}{2}$ . Based on Eq. (A.1-A.4), we have

$$\begin{aligned}
D_t(t, \omega) &= \frac{P_{tt}(t, \omega)}{P(t, \omega)} - \left( \frac{P_t(t, \omega)}{P(t, \omega)} \right)^2 \\
&= \frac{\left( \overline{S^g(t, \omega)} \right)^2 \left( S^{t^2g}(t, \omega) S^g(t, \omega) - \left( S^{tg}(t, \omega) \right)^2 \right)}{4S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} + \frac{\left( S^g(t, \omega) \right)^2 \left( \overline{S^{t^2g}(t, \omega) S^g(t, \omega)} - \left( \overline{S^{tg}(t, \omega)} \right)^2 \right)}{4S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} \\
&= \frac{1}{2} \Re \left\{ \frac{S^{t^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{tg}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
D_{\omega}(t, \omega) &= \frac{P_{\omega\omega}(t, \omega)}{P(t, \omega)} - \left( \frac{P_{\omega}(t, \omega)}{P(t, \omega)} \right)^2 \\
&= -\frac{S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^{g'}(t, \omega)}}{4 S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} + \frac{2 S^g(t, \omega) \overline{S^g(t, \omega)} S^{g'}(t, \omega) \overline{S^g(t, \omega)}}{4 S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} - \frac{S^g(t, \omega) \overline{S^g(t, \omega)} S^{g''}(t, \omega) \overline{S^g(t, \omega)}}{4 S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} \\
&\quad + \frac{(\overline{S^{g'}(t, \omega)} S^g(t, \omega) - S^{g'}(t, \omega) \overline{S^g(t, \omega)})^2}{4 S^g(t, \omega) \overline{S^g(t, \omega)} S^g(t, \omega) \overline{S^g(t, \omega)}} \\
&= \frac{1}{2} \Re \left\{ \left( \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right)^2 - \frac{S^{g''}(t, \omega)}{S^g(t, \omega)} \right\}
\end{aligned} \tag{A.6}$$

Therefore,

$$\begin{aligned}
ESO(t, \omega) &= \sqrt{ESO_{\text{wvd}}(t, \omega)} \\
&= e^{-\frac{1}{4} \left( \frac{(t-t_c(t, \omega))^2}{D_t(t, \omega)} + \frac{(\omega-\omega_c(t, \omega))^2}{D_{\omega}(t, \omega)} \right)} \\
&= e^{-\frac{1}{4} \left( \frac{2 \left( \Re \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} + \frac{2 \left( \Im \left\{ \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \left( \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right)^2 - \frac{S^{g''}(t, \omega)}{S^g(t, \omega)} \right\}} \right)} \\
&= e^{-\frac{1}{2} \left( \frac{\left( \Re \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} + \frac{\left( \Im \left\{ \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \left( \frac{S^{g'}(t, \omega)}{S^g(t, \omega)} \right)^2 - \frac{S^{g''}(t, \omega)}{S^g(t, \omega)} \right\}} \right)}
\end{aligned} \tag{A.7}$$

When using window function  $g(t) = e^{-\frac{t^2}{2\sigma_1^2}}$ , we have

$$\begin{aligned}
ESO(t, \omega) &= e^{-\frac{1}{2} \left( \frac{\left( \Re \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} + \frac{\left( \frac{1}{\sigma_1^2} \Im \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{1}{\sigma_1^4} \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 - \frac{1}{\sigma_1^2} \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \frac{1}{\sigma_1^2} \frac{S^g(t, \omega)}{S^g(t, \omega)} \right\}} \right)} \\
&= e^{-\frac{1}{2} \left( \frac{\left( \Re \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} + \frac{\left( \Im \left\{ \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\sigma_1^2 - \Re \left\{ \frac{S^{i^2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^{ig}(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} \right)}
\end{aligned} \tag{A.8}$$

### B. Proof of consistency with the analytical time-width and bandwidth

Here, we consider a Morlet wavelet:

$$x(t) = e^{-\frac{(t-t_0)^2}{2\sigma_2^2}} e^{j\omega_0 t} \quad (\text{B.1})$$

where  $t_0$  is the time center, and  $\sigma_2^2$  characterizes the time-domain Gaussian envelope width. In the frequency domain, this Morlet wavelet can be expressed as:

$$\hat{x}(\omega) = \sqrt{2\pi}\sigma_2 e^{-\frac{\sigma_2^2(\omega-\omega_0)^2}{2}} e^{-j(\omega-\omega_0)t_0} \quad (\text{B.2})$$

where  $\omega_0$  is the frequency center, and  $\frac{1}{\sigma_2^2}$  characterizes the frequency-domain Gaussian amplitude spectrum width.

Combining Eq. (1) with Eq. (B.1), we have:

$$\begin{aligned} S^g(t, \omega) &= \int_{-\infty}^{+\infty} e^{-\frac{(\tau-t_0)^2}{2\sigma_2^2}} e^{j\omega_0 \tau} e^{-\frac{(t-\tau)^2}{2\sigma_1^2}} e^{-j\omega \tau} d\tau \\ &= e^{-C} e^{j(\omega_0-\omega)B} \int_{-\infty}^{+\infty} e^{-A\eta^2} e^{j(\omega_0-\omega)\eta} d\eta \\ &= e^{-C} e^{j(\omega_0-\omega)B} \sqrt{\frac{\pi}{A}} e^{-\frac{(\omega_0-\omega)^2}{4A}} \\ &= \sqrt{\frac{2\pi\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}} e^{-\left(\frac{(t-t_0)^2+\sigma_1^2\sigma_2^2(\omega-\omega_0)^2}{2(\sigma_1^2+\sigma_2^2)} + \frac{j(\omega-\omega_0)(\sigma_1^2 t_0 + \sigma_2^2 t)}{\sigma_1^2 + \sigma_2^2}\right)} \end{aligned} \quad (\text{B.3})$$

where  $A = \frac{\sigma_1^2 + \sigma_2^2}{2\sigma_1^2\sigma_2^2}$ ,  $B = \frac{t_0\sigma_1^2 + t\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$ ,  $C = \frac{(t-t_0)^2}{2(\sigma_1^2 + \sigma_2^2)}$ , and  $\eta = \tau - B$ .

After we obtain  $S^g(t, \omega)$ , we can calculate  $S^{tg}(t, \omega)$  and  $S^{t^2g}(t, \omega)$  as:

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$$\begin{aligned} S^{tg}(t, \omega) &= \int_{-\infty}^{+\infty} (t-\tau)x(\tau)\overline{g(t-\tau)}e^{-j\omega\tau}d\tau \\ &= -\sigma_1^2 \frac{\partial}{\partial t} \left( \int_{-\infty}^{+\infty} x(\tau)\overline{g(t-\tau)}e^{-j\omega\tau}d\tau \right) \\ &= -\sigma_1^2 \frac{\partial}{\partial t} S^g(t, \omega) \\ &= S^g(t, \omega) \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \left( (t-t_0) + j\sigma_2^2(\omega-\omega_0) \right) \\ S^{t^2g}(t, \omega) &= \int_{-\infty}^{+\infty} (t-\tau)^2 x(\tau)\overline{g(t-\tau)}e^{-j\omega\tau}d\tau \\ &= \sigma_1^4 \frac{\partial^2 S^g(t, \omega)}{\partial t^2} + \sigma_1^2 S^g(t, \omega) \\ &= \sigma_1^4 \frac{\partial}{\partial t} \left( -S^g(t, \omega) \frac{1}{\sigma_1^2 + \sigma_2^2} \left( (t-t_0) + j\sigma_2^2(\omega-\omega_0) \right) \right) + \sigma_1^2 S^g(t, \omega) \\ &= S^g(t, \omega) \left( \frac{\sigma_1^4(t-t_0)^2}{(\sigma_1^2 + \sigma_2^2)^2} + \frac{2j\sigma_1^4\sigma_2^2(t-t_0)(\omega-\omega_0)}{(\sigma_1^2 + \sigma_2^2)^2} - \frac{\sigma_1^4\sigma_2^4(\omega-\omega_0)^2}{(\sigma_1^2 + \sigma_2^2)^2} + \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \right) \end{aligned} \quad (\text{B.4})$$


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$$\quad (\text{B.5})$$

Therefore:

$$\begin{aligned}
 ESM(t, \omega) &= S^g(t, \omega) e \\
 &= S^g(t, \omega) e \left( \frac{1}{2} \left( \frac{\left( \Re \left\{ \frac{S^g(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Re \left\{ \frac{S^{2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^g(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} + \frac{\left( \Im \left\{ \frac{S^g(t, \omega)}{S^g(t, \omega)} \right\} \right)^2}{\Im \left\{ \frac{S^{2g}(t, \omega)}{S^g(t, \omega)} - \left( \frac{S^g(t, \omega)}{S^g(t, \omega)} \right)^2 \right\}} \right) \right. \\
 &\quad \left. - \frac{1}{2} \left( \frac{\left( \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right)^2 (t - t_0)^2}{\frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}} + \frac{\left( \frac{\sigma_2^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \right)^2 (\omega - \omega_0)^2}{\frac{\sigma_1^4}{\sigma_1^2 + \sigma_2^2}} \right) \right) \\
 &= S^g(t, \omega) e \\
 &= S^g(t, \omega) e \left( \frac{1}{2} \left( \frac{\sigma_1^2 (t - t_0)^2}{\sigma_2^2 (\sigma_1^2 + \sigma_2^2)} + \frac{\sigma_2^4 (\omega - \omega_0)^2}{\sigma_1^4 + \sigma_2^2} \right) \right) \\
 &= \sqrt{\frac{2\pi\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}} e^{-\frac{(t-t_0)^2}{2\sigma_2^2}} e^{-\frac{\sigma_2^2(\omega-\omega_0)^2}{2}} e^{-\frac{j(\omega-\omega_0)(\sigma_1^2 t_0 + \sigma_2^2 t)}{\sigma_1^2 + \sigma_2^2}} \quad (B.6)
 \end{aligned}$$

From Eq. (B.6), we can determine that the TFR of ESM is centered at  $t_0$  in time and  $\omega_0$  in frequency, showing the same TF centroid as the Morlet wavelet. Moreover, its time-domain and frequency-domain envelope extents are consistent with those of the Morlet wavelet. Therefore, the corresponding energy distribution of ESM has the same first-order moments and second-order central moments as the energy distribution associated with the Morlet wavelet. Since the energy recovery operator only compensates the spectral energy amplitude without altering the time-width or bandwidth range of the TFR, the same consistency also holds for the proposed ECT.

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